QUANTITY AND ECONOMY IN MANUFACTURE

BY

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PREFACE

The subject of economic lot sizes strikes to the very roots of the effective operation of a manufacturing plant. Some consideration of the principles underlying the determination of the best quantity to produce at any one time, no matter how superficial it may be in the beginning, is essential, as only thus can industrial executives maintain the leadership in their respective fields, which their particular enterprises may have achieved in the past. Upon closer examination it will be found that these principles permeate the whole structure of industrial management to such an extent that it is impossible to purchase raw materials, plan or schedule production, control inventories, coordinate sales and manufacturing operations, select the best process and equipment, adequately conserve working capital, and maintain a satisfactory earning power for the enterprise, without recourse in some manner to them. Evidence of these facts comes directly from industry itself, because industrial executives and consulting engineers of the most progressive corporations in this country during the past twenty years have adopted some practical method of determining the economical size of manufacturing lots or purchase orders and employed these quantities in improving the regulation of their operations.

The author has taken great care in the presentation of this subject in its relation to the economic aspects of manufacture, so as to emphasize in the very beginning the importance of a more scientific approach to the conduct of all manufacturing operations and the utility of economic production quantities. The first three chapters are designed primarily to interest the higher executives of a concern who may desire to learn how they or their enterprise can profit by the methods and technique described in the remaining chapters of the first part. Calculation sheets and graphs have been prepared and mechanical means of solution have been suggested, so that any practical plant executive or subordinate should find no difficulty in understanding and adopting any portion or all of them that may seem to be beneficial. No recommended procedures of this nature are complete which are not

accompanied with illustrations of the manner in which each may be properly utilized. Accordingly, the last chapter in the first part contains numerous examples drawn at random from industries where a reliance upon economic lots has proved valuable.

The second and third parts of this book are devoted to a thorough analysis of all factors which may contribute to a better understanding and more general acceptance of the basic principles. This general discussion of the whole subject was set apart from the description of its practical utility in order in no way to confuse those who wished merely to employ the conclusions as they appear in condensed form, in the prescribed technique, without enquiring into the fundamentals. An exposition of this kind would be incomplete, however, if the statements and procedures presented in the first part were not adequately supported by sound theory. Accordingly, each step toward the final development of the basic relationship, as well as those leading to a later simplification, contains, besides the discussion and interpretation of every factor, graphical illustrations and tables into which the mathematical analysis has been confined.

The author wishes to acknowledge the sincere interest and cooperation given him by members of the corporation and faculty of the Massachusetts Institute of Technology, particularly those of the Economics Department, who have made the completion of this type of research possible. He also wishes to express his appreciation to the American Society of Mechanical Engineers and to Mr. W. L. Conrad and Prof. C. W. Lytle, recent chairmen of its Management Division, for the support and encouragement they have given to this undertaking, through its whole period of development, and especially for their confidence in its ultimate value as shown by the establishment of the Management Formula Committee, who have placed their approval upon these findings. Continued contacts and interviews with industrial executives and plant engineers have been of the utmost assistance, especially that with Mr. R. T. Kent, of Bigelow, Kent, Willard, and Company of Boston, to whom the author is greatly indebted not only for his counsel but also for his criticism from the point of view of the practical-minded executive.

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INTRODUCTION

Heretofore management has been considered as far too intangible a subject to be classed as one of the exact sciences. Only recently has it been appreciated that, through an application of the engineering method, many of the indeterminate factors can be definitely related on a fundamentally sound basis,[1] even to the point where mathematics may be employed in the determination of the "one best way." Of course mathematics has been some utility from the beginning as a device for calculating certain factors, such as wages, bonuses, machine feeds and speeds, and the like, besides playing a major part in accounting; however, there is little evidence of any attempt in the past to analyze the functions of management in this manner on a purely scientific basis. Industrial executives seem to abhor any approach to the problems of management which savors of technical details. Accordingly, management has largely achieved its successes by attacking the most obvious difficulties, which were certain to yield the greatest return for the effort employed and to show at the end of the year a substantial increase in profits in the companies' balance sheets.

As time progresses and more of these difficulties are overcome, an increasing amount of effort will be required to obtain the same proportionate degree of success. Even at best a stage of diminishing returns will set in, unless management is provided with more precise tools in order to continue the search for those hidden costs which must eventually be brought to light, if the present rate of progress is to be maintained. Accordingly, the practical-minded executive must be willing to accept the product of research in management, similar to that which is being conducted at the Massachusetts Institute of Technology, and accustom himself to the newer modes of thought, even though the treatment may be for the moment somewhat unfamiliar. In this regard, mathematical formulae will be of considerable

[1] *Viz.*, Alford's "Laws of Management" and other similar writings.

advantage, because they can be employed to give expression to the basic relationships in a symbolic form, so that their full significance may be appreciated at a glance. Recourse to mathematical formulae as a means to a broader interpretation of management should not necessarily imply lengthy or tedious calculations, because, in the initial stages of their use, the greatest gain will come from the application of the fundamental principles involved in the determination of the best executive policies, and an actual solution according to specific data will only be required in the later stages, when it becomes necessary to establish a measure or standard of performance as a guide to those responsible for the continuance of achievement.

In the Economics Department of the Massachusetts Institute of Technology a program of research in management was adopted in order that a more exhaustive study of the fundamentals of management could be made, and management established through the medium of the engineering method on a plane comparable with the more exact sciences. As a result of this form of endeavor a new conception of the economic aspect of manufacture has been introduced, one phase of which is enlarged upon in the accompanying treatise. This has revealed that minimum-cost production is not the basic factor in the economical conduct of manufacture, because it has been demonstrated in the following pages that manufacturing operations cannot be economical unless capital can be conserved, and then the desired profits will be earned only when production is obtained at the lowest unit cost consistent with the situation thus imposed.

The primary purpose of such a research project is to analyze the more obscure problems of industry and to bring to light those factors which are most perplexing, in order that the industrial executive may be properly informed as to their true relationship. Thus a scheme of basically sound and perfectly coordinated facts may be created, which will comprehend all phases of any problem, and so provide adequate technical facilities adaptable to practical application to industry. With this greater knowledge it is hoped that the remaining uncertainties of management may be more readily overcome, and that the executive who is not supposedly a student of management will be able to devote

his energies more successfully to the immediate needs of his business. It is problematical, in the majority of cases, whether the ordinary executive, who is thus absorbed in the details of management, would have the time available to devote to the discovery of such facts by himself, unless there had been some extraordinary incentive, because such men have quite enough to occupy their minds without stopping to question presumably well-established and long-accepted principles of management, as apparently basic as, in this case, is production at a minimum unit cost.

PART I

THE UTILITY OF ECONOMIC LOT SIZES

QUANTITY AND ECONOMY IN MANUFACTURE

CHAPTER I

A SCIENTIFIC APPROACH TO PRODUCTION CONTROL

Management as a function of industry is primarily concerned with the direction of human effort toward a common purpose. True managerial ability is a rare quality even in this day of trained executives and depends for its success upon a keen perception and understanding of human nature so applied that men are stimulated through leadership to concentrate their efforts toward some productive end. Moreover, management has the added responsibility of utilizing and safeguarding the great resources of capital available in this country for industrial purposes and of guiding the equally great resources of human labor so as to transform raw materials into articles which may be universally distributed among our people for their common good.

Basic Facts Lead to Better Management.—Management cannot achieve its primary purpose successfully without a basic knowledge of the facts underlying all of the problems it has to face. With the ever increasing complexities of our modern industrial system can any executive, regardless of the extent of his abilities, be expected to direct his organization properly without such facts? Already management by exception has been recognized as an important principle in the administration of any enterprise, and the theory upon which this is based is founded upon the fact that executives should be concerned only with the abnormal, unforeseen, and unexpected problems as they arise, in order to capitalize to the fullest extent their creative faculties, and that all matters of routine nature, or those that can be made so, should be sifted down to an irreducible minimum and then so

coordinated as to provide a continuous and reliable flow of information from which the executives may draw to justify their decisions. Ideally, if all the contributary factors in management could be so reduced to routine facts, management would be in a position to exercise with perfect freedom its initiative and originality solely toward the most effective utilization of human ability and skill and to achieve a greater industrial and social prosperity.

Management as an Exact Science.—Throughout the greater part of the past decade phrases such as hand-to-mouth buying, mass production, reduction of inventories, greater rapidity in capital turnover, waste elimination, cost reduction, and the like, which are indicative of a new concept of the economic aspects of manufacture, have been familiar to every one. In fact the present industrial prosperity could not have been so rapidly achieved had it not been for the persistent search for better and better methods of management to supplement executive ability. This demand for more precise methods has increased to such an extent that supposedly satisfactory technique has to be constantly discarded and replaced by one of still greater refinement and of more effective simplicity. During this period of development in scientific methods of management, however, it is quite evident that the tendency has been to attack each phase of manufacture as an individual problem in accordance with the exigencies of the situation, without any particular regard to the manner in which the fundamentals underlying each phase may be related in a comprehensive and properly integrated scheme for the conduct of all manufacturing activities. Circumstances now warrant a thorough inquiry into the economic aspects so that all phases of the problem, as enumerated in part at the beginning of this paragraph, can be formulated into a basically sound theory which will explain the trend and justify a more scientific coordination of managerial technique. Moreover, on account of the obscurity which surrounds many of the factors now entering into the conduct of business, executives have found that successful management requires in addition certain devices by which they can measure the performance of the organization and determine the extent to which these improved methods are achieving the desired results. In other words, management, which has long been considered as depending upon too many indeterminate

factors to be susceptible to numerical evaluation, is in the process of being gradually transformed into one of the exact sciences.

Need for Scientifically Controlled Production Quantities.—Scientific methods of management have been developed and improved upon constantly to eliminate all possible waste in time, labor, and material. No fundamentally sound method, however, has been established for coordinating the sales demand with the requirements for working capital, the purchase of raw material, and the nature of the process. All of these factors are essential to the control of production and, if they are not properly related, will cancel the benefits which should be derived from other scientific methods. It has long been recognized that the unit cost of manufacture will vary with the quantity produced at any one time, and it can also be demonstrated that if the size of the lot is correctly chosen, the unit cost can be made a minimum. Similarly, it can be shown that the profit expected in return for the sale of each product depends upon the quantity produced in any single lot, and that for a certain quantity it will be a maximum when compared with the amount of capital required for its manufacture. If these facts are summarized it can be stated that *an economic-production quantity is that quantity which can be produced at the lowest total unit cost consistent with an economical use of capital in its manufacture, taking into consideration not only the preparation charges against each process order and investment charges on the capital involved, but also rental charges on the space occupied by the article when carried in stock, losses due to deterioration and obsolescence, and the nature of the process.*

Advantages Derived from Economic-production Quantities.—From the principles underlying the theory of economic-production quantities, clearly defined facts concerning the fundamentals of any manufacturing scheme can be directly provided, each one of which has been reduced to its simplest proportions so as to show its relation to the other factors and permit an accuracy of executive control which has heretofore been impossible. The ability of subordinate executives, who are in most instances practical men, to understand the principles of production and to apply them in their everyday activities is essential to efficient manufacture. Unless the relative importance of the component factors can be readily separated from the unimportant

ones, the education and general improvement of the whole organization cannot be so rapidly achieved. Once the uncertainties are removed each executive, whether of high or low rank, can assume much larger responsibilities and act with much greater assurance, because he need no longer doubt the facts and spend valuable time in ascertaining their reliability.

Utility of Economic-production Quantities.—With a broader understanding of these principles of economical manufacture, new and reliable means are provided for directing and controlling the output of any industry and for coordinating manufacturing and marketing programs. Definite production schedules and sales policies can be properly balanced so as to anticipate the seasonal as well as the major trends in the business cycle. The planning department can be provided with the most comprehensive data expressed in the simplest terms, and based on financial considerations of business as well as the conditions imposed by a specific manufacturing process. The mathematical treatment automatically disposes of the necessity for any individual consideration of the various factors and their relationship for every part to be processed. No longer need manufacturing schedules be maintained in estimates of an uncertain origin, generalizations, and the demands of insistent customers. Management can thus maintain a greater control of working capital, only tying up such amounts as the actual market conditions warrant, thereby maintaining inventories in the most liquid condition.

A Basis for the Economics of Manufacture.—Moreover, the theory of economic-production quantities as developed for fabricated parts can be equally well applied to control the assembly of a multitude of such parts into a final product or to the purchase of raw materials. This involves merely the proper substitution of items into the fundamental relationship in accordance with the specific conditions. As a result it is possible to present a graphic picture mathematically of all phases of manufacturing operations from the receipt of raw material to the shipment of the finished product in a way which will permit a rapidity and assurance in control that will be satisfactory to the selling organization as well as to those in control of finances.

Interpretation through the Use of Formulae.—The subject of economic-production quantities is far from new. From time

immemorial the lowering of manufacturing costs has been the aim of industrial executives, whether it was brought about premeditatively or merely by the exigencies of the trade. In the early stages of scientific management this objective took the form of an endeavor to produce goods at minimum cost, and with the advent of mass-production methods it was quickly realized that the lot size had much to do with the minimum-cost point of manufacture. As a result it was discovered that the production quantity could be determined mathematically by the use of formulae, and from that time on many simple expressions have been developed to meet the requirements of a specific industry. Since no one of these formulae included all the factors which govern the problem, none of them could be used as a general formula embracing all conditions that could possibly arise.

Resistance to Mathematical Technique.—The universal acceptance by production executives of such formulae has been greatly retarded by their reluctance to use any aids in their management which savored of mathematics or employed formulae. In a similar way it took a long time before graphical representation of facts was accepted or even the possibilities appreciated. It should be remembered that a mathematical formula is merely a symbolic representation of facts so set forth that their true relation may be taken in at a glance. A close parallel can be drawn between the adoption of formulae and that of charts, except for the fact that charts provide merely a static presentation of data, whereas a formula is much more dynamic in its application, because it represents an engineering or an economic law which is universal and is limited in no way to specific figures which change from day to day.

Mathematics a Recognized Scientific Tool.[1]—This is a surprising commentary upon supposedly progressive executives when in the next move they rely implicitly upon similar formulae which from an engineering standpoint have entered into the design of a part, the purpose of a product, the strength of a building or a machine, or even the chemical composition of the material employed. It will be evident throughout this entire

[1] See Chap. XVIII, p. 161, "Time and Motion Study," by Lowry, Maynard, and Stegemerten, published by McGraw-Hill Book Company, Inc., 1927.

study that formulae can be used to present facts in the most direct way, and that actual numerical values need be substituted only for the respective symbols in such cases as demand the closest attention. As manufacturing problems become more complex, it will be more generally appreciated that formulae will provide the most accurate tool for management to use in exploring their unknown portions, as it will be from these obscure situations that future savings are to come.

Mathematics also a Tool for Management.—Modern science certainly gives industry innumerable examples of just such instances. What human being can visualize the properties of steam at elevated temperatures or pressures, and yet steam boilers and turbines have been designed to operate efficiently at 3,200 lb. per square inch pressure and at temperatures of 1000°F. Where would industry be today were it not for the laws governing the flow of electricity expressed in formulae? As a result we have our power and light transported cheaply where it is needed and always ready for use, we have our telephones and our radios as well. Formulae have been the tools of science for controlling those forces which are too obscure to be handled directly as so much gold or steel or food. Can management longer afford to postpone the opportunity offered by a mathematical approach to its problem if the soundness of the economic structure which underlies any business enterprise can be more reliably determined by the same methods that the engineer employs in determining the safe loading and strength for each member in the structure of a bridge or the framework for one of our modern skyscrapers? As soon as the modern business executive can be convinced that formulae will provide him with a sixth sense which will enable him to arrive more accurately at the correct business policies for his company, their adoption as a common tool of management will be assured. Progress in management methods cannot cease; but if greater accomplishments are to be achieved, the best tools must always be in use.

Tedious Computations Can Be Avoided.—Even in the light of these facts further resistance has been encountered to the use of formulae in management due to the belief that tedious calculations are always involved, especially when a variety of problems of a similar nature have to be solved, because the initial effort may often have to be repeated as conditions change. Here

again the whole purpose of the formula has been lost sight of. It is so easy to forget the fact that it provides a direct means for obtaining an answer to a problem which otherwise might be only approximated by a process of trial and error with an indeterminate degree of success. When a decision of importance depends upon the reliability of the conclusions, can one afford to employ methods which do not offer the same degree of precision? Often the easier method will require in the end a greater effort of one sort or another because the executive has failed to comprehend all the fundamentals. Naturally, if a formula is to be universal in its application, it will tend to become complicated. Its appearance, however, should not prevent one from taking advantage of its utility. Moreover, a little experience in handling the formula may quickly demonstrate the possibility of simplifying the general expression to suit a specific case without sacrificing any of the desired accuracy, by eliminating those factors which are known not to apply. This is a perfectly legitimate procedure and may yield information of sufficient value not to warrant a complete solution until some other factor has been more thoroughly investigated. At any rate, a danger signal has been provided which, if disregarded, indicates that the executive has failed to consider all facts and to improve his power of management by taking advantage of each opportunity. Such a policy can only spell disaster sometime in the future, either for the executive or possibly for the whole organization.

Advantages of Simplification.—Later in this volume[1] an example of how simplification can be accomplished will be given. The same method can be applied to any other formula. Neither should this discussion imply that every formula has to be solved mathematically. The principles behind it may be used alone as a means of analysis with entire satisfaction, because the simplification process may in itself point out the facts with sufficient clarity to make it absurd to attempt to introduce data and get a numerical answer for a myriad of cases. It may be advisable merely to establish rules for the guidance of subordinates which will be even more satisfactory, because the whole picture can be briefly and correctly stated in terms familiar to them. If the production problem is too complicated for such procedure alone, and simple control figures for each part or product are available,

[1] See Chap. XIX and p. 280.

actual data can then be introduced into the formula and the best quantity for any manufacturing lot determined. The results can then be arranged in tabular form. In all probability the added effort and expense can be justified by the relative improvement in control. As a final step, the cost of obtaining the facts from the simplified formulae can be greatly reduced if mechanical means of solution are introduced, such as slide rules, graphs, nomographic charts, calculation sheets, and the like, provided the one best suited to the conditions is selected.

Importance of Capital Turnover.—A just criticism of all previous formulae for economic-production quantities has been the failure actually to conserve capital. In many cases the economic quantity when calculated from one of these expressions has been found to be larger than the quantity which common sense or experience would have indicated as being an appropriate size for the lot. This would probably necessitate an increase in inventories or even the borrowing of additional capital in order to maintain them. This has raised the question of whether the ideal quantity for a single lot should be that which can be produced for a minimum cost, or that which will consume the minimum amount of capital in its production and storage without incurring any serious increase in cost above the minimum. It can be demonstrated beyond doubt that the economic-production quantity is not the minimum-cost quantity and that, even if a slight increase in cost occurs, the saving which accompanies a more conservative use of capital will entirely offset any such loss, so that the expected return or gross profit is in no way impaired. Many facts point to this same conclusion. Indeed those industries which are able to maintain a high rate of capital turnover are in the main the most successful or have the soundest financial structure.

Dangers from Obsolescence.—Another factor which demands a similar consideration is obsolescence, a most indeterminate element which, for a long period of time, may not have any appreciable effect but may occur at any instant with distressing results. It is caused by changes in style, the demands of the customer, or improvements in design and engineering practice. How is the industrial executive to guard against the losses incurred by its appearance; by continuing the manufacture of the product in large volume to obtain the lowest cost; by tem-

porary sales propaganda; or by producing the largest quantity within a reasonable range of the minimum-cost quantity (and the range is very wide if one has really studied it) that will hazard only the smallest amount of capital? Naturally, the last course is the only legitimate one for any right-minded executive to pursue, no matter how progressive he may be. Accordingly, in this latest study of economic-production quantities the factor of conservation of capital has been most carefully considered.

Realization of Savings.—Other types of resistance to the use of scientifically determined production quantities have arisen from the opposite situation where the economic quantity is less than the previous quantity processed per lot. It has not been uncommon to hear an executive ask, what can be done with the storage space that will no longer be used, with the machine equipment that will be idle, with the men who will have to be laid off, and even with the capital released from production? Questions of this order are typical, but any right-minded man will know that if his business is not standing still or losing ground he will before long be glad of such men, space, time, or money. It is at least conceivable that these potential resources may be exceedingly valuable at a time of alteration or expansion because they can be employed to better advantage otherwise, and thereby postpone if not entirely eliminate the expenditure of large sums which otherwise might have to be secured from some outside source in order to achieve the same results. Moreover, if any of these resources have been made available for such purposes and for the present remain idle, it will give the progressive administrator a greater incentive to utilize them as quickly as possible in a manner which will yield a fair return. Such a situation is a challenge to ingenuity, and if a study of production quantities can stimulate a greater endeavor for further economies, much can be hoped for. The difficult part is to know where to begin; the second part of accomplishment is merely incidental.

A Scientific Approach Eliminates Errors.—Other objections have arisen from the superficial realization that there are many seemingly indeterminate factors which enter into any scientific determination of economically balanced production schedules. It is often claimed that the errors in forecasting sales or styles alone would prohibit any accurate determination; that rush orders, breakdowns, trade factors, the human element, slack

periods of production, limitations of the formulae themselves, and many other reasonable uncertainties remove the subject of economic-production quantities from practical or useful mathematical treatment. Of the fact that these variables are constantly occurring there is no moment's doubt, but not until a real study of the problem can be made does it become apparent that many of them can be either legitimately avoided or accurately accounted for.

Mathematical Technique Not Inflexible.—One misleading conception has been due to the belief that the results of any mathematical determination must from the nature of its origin be rigid and inflexible. So it is with all single numerical values for the economic-production quantity which have been determined by the minimum-cost method used in all previous formulae. As soon as conditions change, new values must be computed, and this involves repeated calculations of a large number of items in order that the control figures may be kept up to date. It will be demonstrated in the course of this discussion that the very nature of the problem lends itself to a flexible solution, dispensing with all previous objections. A range of production quantities can be established with the assurance that any quantity which lies within its limits may be produced for the same manufacturing advantage that, as formerly believed, could be obtained only for the minimum-cost quantity. This indeed is flexibility. Executive judgment need no longer be hampered by uncertainties; on the other hand it can be greatly supplemented by the clearer understanding of the facts thus provided, so that many varying situations and emergencies can be more readily anticipated.

CHAPTER II

ECONOMIC-PRODUCTION QUANTITIES

If one stops to review the primary causes underlying the present industrial prosperity of this country, much of the progress in recent years can be attributed to mass production. Those industries which have been able to profit by its adoption have found that many of the intricate production problems, which had previously given their executives much concern, could be more readily provided for, if not entirely avoided, through the resulting opportunity for large-scale operations. A greater degree of coordination could thus be achieved between sales programs and production schedules which has permitted a more economical use of capital and a reduction in inventories, because many situations which demanded the storage of material throughout the plant could then be dispensed with. Similarly, the operations which naturally enter into the fabrication and assembly of any product and which had heretofore been carried on in an unrelated manner could be brought together into their logical sequence so that a continuous flow of material might be achieved, with the result that production costs would be lowered at the same time that higher wages were being paid. Moreover, the indirect expenses of such a method of manufacture incidental to administration, control, service, and the keeping of records have been automatically reduced from those commonly found in the older types of manufacturing units bordering upon a job shop, because many of the complexities, which were a necessity under those circumstances, could be eliminated.

Industries Favored by Mass-production Methods.—Those industries which have been able to attain continuous processes of manufacture from one cause or another are fortunate indeed. They have achieved an ideal situation not so much from inductive economic reasoning as from sheer force of progress and the fortunate magnitude of the market which they served. Outside forces have caused a given situation to exist and the progressive industrial executive has merely taken advantage of his oppor-

tunities and by a process of trial and error (for there have been many false starts) and the experiences of others has evolved this remarkable prosperity.

Factors Prohibiting Mass-production Methods in Less Fortunate Industries.—Many other industries not so inherently fortunate, due to processes which apparently cannot be operated continuously, a large variety of products, a diversified or fluctuating or limited sales demand, style factors, rapidly occurring obsolescence and deterioration, have been forced to continue in the older methods of manufacture. In their case the natural solution is not so evident; nevertheless, given the proper tools of management with the facts, their way will be made easier at least to approximate the ideal conditions, even if they cannot be attained immediately.

Application to Continuous Processes.—The theory of economic-production quantities has no application to the ideal conditions where a continuous process is in effect. That does not imply that the theory is inapplicable, for in fact it is only through a conscious or unconscious application of the theory that the ideal has been achieved. It simply means that all conditions have been fulfilled and that the theory in itself has performed its purpose. Even if under these conditions the theory cannot longer be of service as a whole, there are many of its component parts which will provide means for still further improvement in the ultimate unit cost of production.

Application to Intermittent Processes.—Economic-production quantities are of utmost importance to all the less fortunate types of industry where intermittent processes exist. By that is meant a process which requires the setting up of the manufacturing equipment either for fabrication or assembly of a specific part or product at intervals, and owing to the fact that these units can be produced at a faster rate than they can be consumed, the process must be stopped when a sufficient quantity has been manufactured to supply the demand for a reasonable period of time from those units which have been placed in stock. Meanwhile, the manufacturing equipment is shifted to the production of other articles of a related nature, until the stock of the original article has been reduced to the order point, whereupon the cycle will repeat itself.

What Quantity Should Be Produced?—Where such conditions exist production executives are continuously in a quandary as to what quantity should be produced in any one manufacturing lot. There are two alternatives. On the one hand a large quantity can be produced in each lot for a single setting up of the manufacturing equipment, in order that the lowest possible manufacturing cost can be attained through a distribution of the preparation charges over the greatest number of units that can be conveniently placed in production. On the other hand, if the control of inventories is of greater importance than that of cost, a small quantity can be produced that will be just sufficient to meet the immediate sales demand and cover the time required to replenish the stocks of finished products. If the executive is production minded, he will tend to produce the largest quantity in order to obtain the lowest unit cost, and if he is financially minded he will tend to produce the smallest quantity, because he realizes that the invested capital can be more rapidly turned over Both types of executives are correct in their judgment as far as each has gone. Has the first appreciated the degree to which his policy will eventually increase inventories and consume capital, however, and has the second realized that he is impairing the company's profits by incurring an unnecessarily large allotment of the set-up or preparation costs in proportion to the cost of direct labor, material, and overhead for each unit produced? It should not be unjustly assumed that these executives have gone to extremes in carrying out their policies. Absurdities can easily be corrected because the error is so obvious. It is for the majority of cases which lie in a middle ground that the greatest care must be taken, for then the relation of the economic factors is most obscure. That is why measures of management should be of ever growing importance to industry.

The Economic Balance.—The fact is that an economic balance[1] should exist between the cost of preparation prorated to each unit in the lot and the cost of carrying that unit in inventory. If the size of the production lot can be so chosen that the unit allotment of the total machine set-up and production-control charges, incurred by the preparation for the manufacture of that lot, are equal to the unit allotment of the cost of carrying the average number of pieces in storage for the time that any

[1] See p. 163.

one of the pieces produced in that lot still remains in stock, the desired economic balance can be achieved and the unit cost will become a minimum. It has been upon this hypothesis that all previous formulae[1] for the economical size of lot have been established, and upon which all minimum-cost quantity determinations will always have to be made. This is a crude way of expressing the true relationship and later on in the more technical portions of this discussion the actual mathematical basis substantiating this fact will be fully described.

Mathematical Treatment.—As industries have become more convinced of the value of a rational means of controlling production on an economically balanced basis, it was found that the existing methods did not represent the specific conditions in certain cases and that other factors also influenced the problem. As a result various mathematical expressions for computing the lot size have been devised to care for each special case. If any one of these be employed without a complete understanding of its limitations or a knowledge of the conditions to which it only may apply, widely divergent and erroneous results will be obtained. This situation was instrumental in bringing about an exhaustive study of the whole problem, particularly with regard to those factors which are of importance in one case and which should be eliminated in another.

The Ultimate Unit Cost Defined.—The conditions which impose the economic balance, as stated above, require that neither the unit allotment of the preparation cost nor the unit allotment of all inventory charges or similar items should exceed each other. This means that the sum of their respective values, when equal, will be always less than their sum if either one is greater than the other. When the sum of these two unit charges is added to the total unit cost of direct labor, material, and overhead, the resulting total charge will represent the entire cost of manufacture of any unit of production, because no other charges can be assessed to it until it has passed from the control of the factory. Accordingly, the term "ultimate unit cost" will be used to designate the summation of all these charges and includes, besides the accumulated value of the unit at the time it leaves the process and enters stores, the cost of the capital involved during manufacture and storage, as well as any charges which may have been

[1] See Chap. IX, and Table XI, p. 145.

incurred by the actual storing of each unit. The ideal of all manufacturing operations is apparently to make this ultimate unit cost a minimum. Continuous production methods achieve this in themselves, but in the case of intermittent production some additional managerial effort must be applied to attain a similar situation. For such cases, then, the minimum-cost quantity must be specifically determined. Before this conclusion is accepted as final, however, there is another important consideration which cannot be overlooked.

Economic versus Minimum-cost Quantities.—In the first chapter[1] it was mentioned that lot production based upon minimum-cost quantities did not conserve capital. In contrast to this the exigencies of business, the keenness of competition, and the insistence of investors for greater profits in the face of declining prices demand production at minimum cost. How can these two opposing factors be reconciled? Fortunately research has disclosed the fact that the manufacturing advantage, which can be realized by the production of a minimum-cost quantity, can also be realized from a certain other production quantity even smaller in size than that. By the manufacture of this smaller quantity a certain amount of the capital which would normally have been employed in the manufacture of the minimum-cost quantity can be released and invested in the manufacture of other products having an equally good potentiality for a like return on the capital. The cumulative effect of the savings in capital employed during a year from all manufacturing operations will eventually show in the balance sheets and earning statements of the company a greatly improved financial condition, a more rapid turnover of capital, and a greater actual earning power for each dollar invested.

An Economic Range of Production.—In this method of approach the increase in manufacturing costs, due to a reduction in the lot size which makes it more difficult to absorb the total set-up charges, has not been overlooked. This increase in cost is offset by the greater rapidity in capital turnover, for in the last analysis it is the greatest aggregate number of dollars that can be earned in a year clear of all charges that count. When this portion of the problem is discussed in detail,[2] it will be shown

[1] See p. 10.
[2] See Chap. XI.

that over a comparatively large range of quantities within the neighborhood of the minimum-cost quantity, the increase in manufacturing cost is very small, and that a substantial reduction from that quantity can be safely processed in any one lot. Since there is but one quantity less than the minimum-cost quantity which has the same manufacturing advantage, this quantity will be known as the "economic-production quantity," for which a definition[1] has already been given.

The Point of Maximum Rate of Return.—Thus if there be a range of production quantities between the minimum-cost quantity and the economic-production quantity, and these two quantities have the same manufacturing advantage with regard to the return earned on the capital invested in each case, there must be a lot size somewhere near the center of this range for which there will be the greatest spread for the range between cost and price, so that a greater return than that normally expected can be earned. This proves to be the case and this production quantity has been designated as the "point of maximum return." Accordingly, it is possible, by a proper adjustment of production schedules, to obtain a better return and an economical use of capital at the same time. It is not recommended, however, that production schedules be maintained exactly at this point, as one immediately loses the desirable advantage of flexibility due to an economic range of production quantities. Since at other points in this range a greater rate of return can still be achieved even though the spread between cost and price will not be so great, depending upon the relation of the actual lot size to that for a maximum return, it is wiser in the long run to select the most convenient production quantity within the range than to adhere to a single ideal quantity.

The Basis of Economic-production Quantities.—The fundamental relationship of the various factors controlling the economic-production quantity can be obtained first by finding the expression for the quantity which can be produced at a minimum unit cost, and second, by introducing a corrective factor which takes into account the relative amounts of capital employed. Owing to a disagreement among accountants, two distinct expressions[2] can be developed for the economic-production

[1] See p. 5.
[2] See p. 168.

quantity. One is based upon the contention[1] that the cost of capital is an integral part of manufacturing cost and therefore should be included in the value given to any unit of production held in inventory. The other is based upon the grounds that articles in inventory should be evaluated only at the value which they have accumulated up to the instant that they enter stores as finished parts, subassemblies, or the completed product, and that the cost of capital whether incurred while in process or in stores should be charged against the cost of doing business and not be included in the inventory value of the unit of production.

The Practical Solution versus the Exact Solution.—The first contention has received very little support. The second has been universally adopted and is much more logical as it does not inflate inventories with charges which are of a corporate nature, and avoids the complication of compounding such charges as is necessarily the case when included in the value of articles in inventory. The expression derived from the first hypothesis has been called the "exact solution" to differentiate it from the second, which has heretofore been called the "approximate solution." Since the exactness in the first case is more academic than practical, and since the derivation of the formulae in the second case conforms to current accounting practice,[2] the latter will be recommended hereafter as the basis for all economic lot-size determinations, and all such expressions will be referred to in general as belonging to the practical type of solution. Accordingly, it will be demonstrated in the course of this treatise that the limits of the economic range of production and the point of maximum return can be computed from the fundamental relation Eq. (1) given in Table I.[3] Should occasion arise where a comparison of these two types of solution is desired, the corresponding expressions derived from the exact type will be found in Tables XXIX and XXX.[4]

Factor for Determining Critical Points within the Economic Range.—From an inspection of these equations, a close similarity will be noticed except for that group of terms which are multiplied to the first two elements in any one of them. This group can be

[1] See SCOVILLE, C. H. "Interest as a Cost," Chap. IV, pp. 21–36, The Ronald Press Company, 1924.

[2] See p. 165.

[3] See p. 37.

[4] See pp. 276 and 277.

reduced in most cases to a constant factor where it will have a specific value for either the economic-production quantity or the point of maximum return in contrast to a value of unity for the minimum-cost quantity. Thus if this group be represented by the symbol f_r, a single expression can be written for use in determining all lot-size problems, and when any one of the three important points in the economic range is to be determined, the value for f_r characteristic of that point can be employed. This opens up quite an opportunity for simplification in computing all three, because it is possible to apply the factor f_r as a correction to the appropriate element in the already calculated denominator of the minimum-cost quantity formula, without having to repeat any of the calculations.

Application to Assembly as Well as to Fabrication.—Any of these formulae can be used to determine the economical quantity for the production of a fabricated part or the assembly of a final product. None of these formulae, however, can be used to determine the quantity for both fabricating and assembling at the same time. This is obviously because in the assembly of a final product a number of parts are involved, each of which has a distinct and different economical quantity for its fabrication, and then any one of these component parts may be used in a variety of other assemblies. To transform[1] the formula for fabricated parts to that for assembled articles, the value of each part that it had accumulated upon being placed in stock plus the unit charges for storage, if a separate consideration of this factor is required, should be added to the similar value for all other parts used in the assembly, in due proportion to the quantity of each entering into the finished product, and this total used in the assembly formula as the value of the material used. All other items remain entirely unrelated and do not carry over, the capital charges being posted against the cost of doing business at each manufacturing step and not being cumulative with the progress of manufacture.

Economic-purchase Quantity.—In a similar manner the basic relationships expressed in these general formulae can be used to determine an economic-purchase quantity.[2] The transition of

[1] See pp. 165–168.

[2] See Davis, R. C., Minimum Cost Purchase Quantities, A.S.M.E. *Trans.*, January–April, vol. 50-MAN, No. 6, p. 41, 1928.

the formulae for use in purchasing is not as simple because different factors apply. Their general characteristics will be the same, however, and they will be found in relatively the same places. One additional feature is incorporated in the purchase formula which takes into account price concessions based upon quantity, so that if the price advantage warrants, larger quantities may be procured in advance of production requirements in proportion to the saving in capital expenditures, without incurring any greater costs for raw material by the time it is actually needed. Again it is possible to specify the intervals at which deliveries should be made so as to maintain the minimum of raw material in inventory. As previously stated, the cost of material used in the economic-production quantity formula is the accumulated costs of purchasing, transportation, and storage charges per unit added to the purchase price, or in other words, the total unit value of the raw material at the instant it is removed from stock for productive purposes. As the derivation of the details of this formula is a problem in itself, no attempt at this time will be made to go further with this particular phase of the major subject.

Errors Incurred by the Incorrect Selection of Data.—The theory of economic-production quantities when applied to actual industrial problems will not require any radical changes in accounting methods or production records. One precaution must be taken, however, and that is no item which appears in one element should be allowed to appear in the value of any other element employed in the numerator or the denominator, except where it may be specifically provided for in the formula. Its appearance in two different elements will yield absolutely fictitious results. Such an example is to be found in the practice of considering the set-up of the manufacturing equipment as the first operation in the make-up of the direct-labor costs per piece. If it is not removed from this item it will cause a serious error, as it is one of the major factors controlling the whole problem and belongs only in the numerator of the formula, as a part of the preparation costs.

Classification of the Controlling Factors.—In the application of the formulae there are three types of factors that need be considered, and it will be found that therein lies the only criterion for selecting and properly arranging the data. All items having

the same characteristics should be grouped together: first, those which are independent of the quantity processed in any one lot but which are dependent in some way upon the number of lots processed; second, those which are entirely dependent upon the quantity within each lot; and third, those which are independent of either the lot or the quantity or are constant at all times. Overhead distribution through burden rates is one of the factors typical of the third group, and many items can be consolidated into one factor in this classification that have neither characteristics of the first or second. Care must be taken, however, to be sure that no item which might easily be placed in the burden rate is required elsewhere in factors purposely segregated. In no event should any items be omitted which enter into the eventual cost of the article, as this will also cause a discrepancy in results. Knowledge of these facts will be particularly useful when the relationships expressed in the formula are used in the analysis of manufacturing methods without resorting to actual mathematical determinations.

Major Factors.—The major factors entering into the determination of an economic quantity are:

The average rate of consumption per day S_a, based upon forecasts of yearly requirements.
The total preparation costs P which include not only charges for the setting up and dismantling of equipment M but also the production-control expense O.
The unit cost of raw material m or component parts.
The unit cost of direct-production labor l and overhead o on direct-production time.
The unit process time t.
The rental charge for space occupied by articles in stores s.
The height h to which storage is permitted.
The unit volume or bulk of article b.
The interest rate i upon which the cost of capital is based.
The rate of return r expected from the capital employed.

Of all these ten major items not more than five and possibly only three will be actually required by any calculation for the economic quantity. It will be shown later,[1] that, given a specific case, only the most characteristic items need be used to obtain a reliable result. Which items to use is an important question that actually can be answered with great simplicity.

[1] See Chap. V, or Chap. XIX.

Minor Factors.—In obscure situations it is conceivable that other factors will arise which cannot be justifiably overlooked. These minor factors consist of:

Variable or seasonal demand S_{a_s}.
Anticipation of obsolescence θ.
Deterioration Δ and δ.
Delivery to stores rate D.
Batch factor n.
Container size q_c.
Correction factor to compensate for variations in wage rates between operators k_a.
Details of process time t_n, T_M, and k_d.
Average stock factor k_s.
Number of machines used in parallel in any operation n_m.
Bin factor k_b.
Capacity of bin q_b.

The fact that all these lesser items are listed here does not imply that every one or any one of them have ever to be used. The object of research is to supply the hidden facts, and in order to determine their relative importance they have been introduced. Some industrial condition may arise where a knowledge about some detail will be invaluable and to be complete these items have been included in the general expression.

For practical purposes the general utility of the theory of economic-production quantities will be set forth in the next chapter, showing how tangible results can be obtained by analysis which may or may not employ any calculations.

CHAPTER III

UTILITY AND APPLICATION

Whether the theory of economic-production quantities has been employed consciously or unconsciously by industrial executives, as the basis of their daily processes of thought in determining a manufacturing policy, one cannot avoid the fact that it is one of the fundamentals upon which all industrial progress must rest. Without question the criterion for successful industrial endeavor is the amount of profit which can be earned upon the capital invested in any particular enterprise. Since selling prices are determined by the extent of the market and the intensity of competition, it has been believed that the desired rate of return which is normally expected by those who have undertaken the risks of business can only be achieved through a consistent effort on the part of all to attain the lowest cost production. On the contrary, it is not sufficient to strive merely for the lowest manufacturing cost, because in many instances the advantages gained through highly refined processes using complicated automatic machinery, as well as other advantages arising from the application of approved scientific methods, can be entirely lost if the policies of the sales and financial divisions are not properly coordinated with those of the production division. It is now evident that the ideal situation can be achieved only where executive policies are based upon the attainment of the lowest ultimate unit cost for each specific product consistent with an economical utilization of capital resources. In selecting a suitable basis for defining the goal of all manufacturing operations, the ultimate unit cost should be preferred over any other cost item, because it is the only one which takes into consideration all the factors incidental to manufacture, which can possibly be attributed to a unit of production, and includes those which reflect the financial operations required to support the activities of the manufacturing divisions.

Economic-production Quantities the Basis for Operating Policies.—Accordingly, for intermittent production a successful

manufacturing policy can be determined only on the basis of economic-production quantities, because under these conditions alone can the lowest ultimate unit cost be attained with the least expenditure of capital. Executives who prefer to adhere to the older method of basing their manufacturing policies upon production at minimum cost will fail to realize the greatest benefit from the capital employed, because it can be shown that a greater expenditure of capital will be required in this case to obtain the same gross return, for a given period, than could be earned with less expenditure in the first case. Such a policy can be justified only when the nature of the product and process permit the adoption of continuous production methods. Even in this latter case the situation has not altered; the only difference is that under these conditions the minimum-cost quantity can be produced for a minimum expenditure of capital, whereupon it can be said that the economic quantity and minimum-cost quantity coincide.

Minimum-cost Quantities a Measure of Method Only.—Minimum-cost quantities, however, are the measure of the best manufacturing methods. If there be several acceptable processes available for the manufacture of a given unit of production, the best process can be selected by determining which will yield the lowest minimum cost under the conditions imposed by the existing sales demand. Thus, when the rate of consumption can be made to equal the rate of production through an aggressive sales promotion policy, it will be found that the lowest ultimate unit cost can be realized by the adoption of a continuous process, bringing one back to an earlier conclusion. If an intermittent process must be selected, instead, the best operating policy will then be determined by manufacture based upon economic quantities for that process, and, as minimum-cost quantities and economic quantities are related, one can still be assured of a greater gross return.

Margin of Profit in Relation to Capital Expenditures.—Possibly this whole situation can be more fully explained by considering a typical problem. Would it be wise to improve the manufacturing facilities for any unit of production and increase the quantity produced in each lot to such an extent should the expenditure of capital become so great that the increased margin of profit no longer represented the same, not to mention a greater,

gross return upon each dollar invested? It is doubtful whether any astute business man would sanction a cost reduction policy of this nature, especially if there were no opportunity to increase the rate of consumption or sales demand at the same time in order to maintain at least the previous rate of turnover of both inventories and capital resources. If this cannot be achieved a substantial part of these resources will then become tied up in slow-moving stocks of finished goods upon which no immediate profit can be made, and then a smaller gross return will have to be accepted upon a materially larger capital outlay. Thus it should be all the more evident that the earning power of a corporation depends most decidedly upon the return as measured by the margin of profit for the most suitable manufacturing method that requires the least proportionate investment of capital.

Financial Aspect of Minimum-cost Production.—From the foregoing discussion it should be recognized that the economic aspects of manufacture are much more closely associated with the theory of economic quantities than with the theory of minimum-cost quantities. Some day the executives who persist in following the older school of thought will be confronted, through competition, with the necessity of having to reduce still further the cost of manufacture in order to maintain the normal margin of profit, and in so doing will require additional capital which will not be available. Owing to the unfavorable financial position of the company, brought about by a disregard of economic factors, the usual methods of securing new sources of capital through the sale of securities of one kind or another cannot be relied upon. In such an emergency the new capital will actually have to be squeezed out of the business in the most effective way possible, and in the end economical quantity production will have to be resorted to. Gradually, through the liquidation of inventories, the scrapping of old equipment, and the elimination of waste, capital can then be accumulated for reinvestment in more economical equipment, so that the manufacturing operations can again be placed on a profitable basis.

Economic Quantities Explain Many Common Industrial Practices.—This sequence of events may suggest to some executives the years following the boom period of 1919 to 1920 which gave impetus to this new concept of the economics of manufacture. Many of the common practices followed in modern

manufacturing methods had their origin at that time. Thus it can be said that "hand-to-mouth buying" was the initial step unconsciously taken toward the application to purchasing of the theory of economic quantities and conservation of capital. Similarly, the basic elements of mass-production methods and the general urge for a sweeping reduction in inventories, which followed this period, can be attributed to the recognition of certain advantages in this same theory when applied to manufacturing operations. Further evidence of an appreciation of its economical aspects may be found in the growing demand for a greater rapidity of capital turnover from the financial executives of industry, which was brought about by a closer study of financial and earning power ratios in order to make more effective the economic utilization of capital resources.

Comprehension of Economical Aspects of Manufacture Essential.—As long as the attention of executives was directed toward achieving an objective of large-volume low-cost production as the only medium of increasing profits, there was little opportunity for them to appreciate the more fundamental aspects of economical manufacture as presented by the present theory. What industrial executive, under these circumstances, would have the incentive to make a thorough investigation, such as this, of the whole situation until the necessity had arisen, even though it might reveal a more beneficial manufacturing policy by means of which a gross return could be obtained upon the capital invested that would be more consistent with his desires for a suitable reward for undertaking the risks of business? In most instances the time of such a man is occupied by many other pressing and important matters in connection with the daily conduct of business, but, if it can be demonstrated that the success of his particular enterprise depends upon his ability to comprehend the true relation of volume of business, unit costs, and the rate of return, he will be in a position to recognize the advisability of adopting some of the more precise methods of management which are provided by the theory of economic quantities and others of a similar nature.

Protection against Obsolescence.—Another problem, having many perplexities, which can lend itself to similar treatment, is that of avoiding losses from obsolescence which may result from style changes or improvements in designs or processes.

This is the outcome of sales policies intended to increase the volume of business and break down the resistance of the purchaser quite as much as of the independent whim of the customer. The original purpose is excellent, but it throws an added responsibility upon the production-control division of the concern in order not to let manufacturing operations get out of hand and build up inventories, which, when the style or design does change, would become unsalable. Protection against this kind of obsolescence can be provided for by the production of the smallest quantity which lies within the economic range in a manner similar to that through which the conservation of capital is achieved. Moreover, the frequency of changes arising from a policy of progressive obsolesence can be controlled to a large degree through the use of the principles of economic quantities.

Transition from Intermittent to Continuous Production.—Having thus pointed out the importance of a thorough appreciation of the relation of the theory of economic-production quantities to the general policy of the corporation, much is yet to be gained by a consideration of its relation to the problems encountered in the planning and control of production. Take for instance the case where the volume of sales is large, but where it does not equal the eventual capacity of the manufacturing equipment. Here the ultimate unit cost is approaching the limit it would reach could the intermittent process be transformed to a continuous process. It would be feasible to make this transformation if it could be demonstrated that the cost of the idle equipment would be less than the accumulation of preparation charges over the same period of time. An analysis of the factors entering into the ultimate unit cost for each type of process would determine once for all the volume of sales for which it would be worth while to establish permanent set-ups and operate the process continuously. The advantages of the continuous process will not be fully realized, of course, until all requirements for storing raw material, fabricated parts, or finished products are dispensed with. Nevertheless, if the initial step can be taken, the others can be made to follow as rapidly as any increase in the sales demand will permit.

Control of Abnormal Peaks in Continuous Production.—Again, in plants operating upon continuous processes where parallel lines of manufacturing equipment are employed, the

seasonal nature of the demand may make it advisable to withdraw one or more of these lines periodically from production. For instance, if two lines of identical equipment are manufacturing the same article continuously and the demand is fluctuating sufficiently to require a third line in operation at intervals in order to meet the peaks, this line may be considered as operating upon an intermittent process. It may be inadvisable to operate it only for the excess quantity immediately required, as the expense of starting and stopping production on this line will multiply rapidly as the peaks come and go. If a larger quantity can be produced once this line is in operation, these preparation charges can be spread over a greater number of articles even though some must be held in reserve for future consumption, as the accumulation of investment charges will in all probability not be so great as would the too frequent recurrence of the preparation charges. The principles of economic quantities can be applied in this case to smooth out the production for these abnormal points in demand. Moreover, if the operating schedules as determined for this line provide an opportunity for utilizing the manufacturing equipment, in the intervals between the production of one type of article, for the manufacture of another type, duplication of equipment can be dispensed with, the cost of the idle equipment lessened, and the capital represented by the value of such machines employed for other purposes.

Selection of an Economical Process.—The selection of the most suitable process depends directly upon the theory of economic quantities, as that process for which the ultimate unit cost at the minimum point is least will be the most economical. In cases where the actual operations are the same the difference between costs will depend upon the degree of coordination between operations and the freedom provided for the flow of work. Numerous instances have been observed where potentially low cost, due to highly efficient tools and equipment, has been lost owing to the fact that semifinished material is allowed to accumulate unnecessarily between operations. Sometimes it is impossible to realize the full effectiveness of coordination owing to the nature of the processes, as in the case of non-continuous production where all articles in the lot must be processed as a single group, and no one article of that group can be shifted to a succeeding operation until all have passed through the preceding one. If, however,

the lot can be arbitrarily subdivided into batches, so that each batch can be processed independently, operations may be made to overlap with a resulting marked decrease in total production time, and a proportionate saving in the cost of capital invested in work in process. Finally, if each unit of production can proceed through the process from one operation to another entirely independent of the other units, a condition approximating continuous production is achieved for the intermittent process while it lasts.

Selection of Economical Material-handling Methods.—A study of the elements of the processes with these facts in mind may lead to more effective forms of material-handling equipment. A determination of the lot size for the various products passing through a similar sequence of operations will aid materially in determining the most economical form of equipment which should be employed. Similarly, the theory of economic quantities may be applied to determine to what extent it is reasonable to set aside space on the manufacturing floor or in finished-parts or assembly stores. Often the space provided at each operation for material undergoing production or that in any department which is set aside for material awaiting the start of a process is limited, and overcrowding will result if the flow of work is not properly coordinated. All of these factors contribute largely to an orderly process of manufacture, and it is only through an application of the theory that one can appreciate the full extent to which delayed material may represent a considerable capital investment for which the financial interests demand a reasonable turnover.

Coordination of Sales, Financial, and Manufacturing Policies.—In the scheduling of production other advantages can be realized beside those which a flexible range of economic quantities has to offer over the older method of a single inflexible quantity, or the indeterminate method of continually attempting to match production schedules with the requirements of the sales department. When sales forecasts of reasonable accuracy are available, machine-load budgets can be established and production schedules prepared in advance so that a certain article will enter into production on the same day in any manufacturing period. This will provide another opportunity for simplifying departmental routine, making possible a reduction in the preparation costs

arising from production control, and will leave the schedule clerks greater time to concentrate upon the exceptional situation or unforeseen rush orders. The preparation costs can be further reduced if articles, the process for which requires similar set-ups, can be scheduled in sequence so that the least number of tool changes and readjustments are required for each. Consequently, larger quantities can be processed for the same or even smaller unit costs and a saving in operating time realized which can be utilized to increase the available machine capacity, as less time will be consumed in unprofitable machine set-ups.

Basis for Predetermined Standard Costs.—The process engineers can derive much assistance from the principles of economic quantities in their analyses of manufacturing methods by utilizing the economic quantity as a standard of comparison. Predetermined costs can be computed on this basis with greater assurance that they will be more representative of actual operating conditions. In selecting the most desirable process, these considerations should not only be given to the time allotted to the movement of material but also to the average unit time of production, as the cost of capital employed in work in process depends entirely upon this factor. A method,[1] later described in detail, has been established which can be employed to much advantage in determining the minimum standard of performance. Moreover, the relation of the application of value through the efforts of labor during one part of the process to that in other parts may be the deciding factor in determining in what condition the unit of production should be stored. If storage is necessary it should precede a costly operation when the added labor value is high in order not to enhance unduly inventory values.

Joint Economic-production Quantities.—A unit of production as far as the manufacturing operations are concerned may consist in a combination of closely associated fabricating and assembly operations, for which it would be inadvisable to consider any of the elements as distinct, because the unit may itself be but one of a number of intricate parts entering into a final assembly. The best method of production in this case may be tested in advance by the comparison of the ultimate unit costs resulting from various sequences of operations. In such a situation the determination of a joint economic quantity is permissible pro-

[1] See p. 95, Table VIII.

vided proper consideration is given to the relative rates of consumption should any of the elements be required for assembly in other similar units.

Selection of Manufacturing Equipment.—The selection of the best types of manufacturing equipment will depend upon the size of lots to be processed, especially when their adoption depends upon a justification of idle time. Need for multiple units of equipment may be determined by a comparison of the machine-load budget and the economic lot size of a proposed process. In the design of expensive tools, jigs, and fixtures, economic quantities can be used to determine whether the current rate of consumption is sufficient to insure that this cost, which might easily be returned through large-volume production many times over, would be earned before the utility of the article changed because of design obsolescence. Likewise the space allotted to the manufacture of any article depends upon the type of equipment employed, as well as the quantity of material that is to be placed beside each machine during its operation; therefore all phases of the layout of production depend upon the economic quantity.

Economical Storage of Parts or Final Products.—The storage of material, whether it occurs during a process or after the fabricating or assembly operations have been completed, is at best undesirable, even though it is necessary in the coordination of unrelated rates of production and consumption. The ultimate unit cost of any article which must be held in stores inventory is affected not only by the actual cost of storage but also by the cost of the capital that is temporarily represented by its total value. If a series of fabricating and assembly processes for a given article can be perfectly synchronized with each other, or with the purchase of raw material in the first instance, or with the specific requirement of the customers in the second, articles, even though produced by an intermittent type of process, can flow uninterruptedly through the sequence at the proper intervals, so that all storage periods can be dispensed with and the resulting costs eliminated. Other articles similarly produced and synchronized can be processed over the same manufacturing equipment during the periods that the first article is out of production so that idle machine time can be avoided. In this manner the ultimate unit cost can be made to approximate its limiting value,

or that cost which could be achieved if the volume of business permitted the adoption of a continuous process.

Economical Control of Manufacturing Reserves.—Similarly the banks of material which accumulate during the processes in advance of each operation can be studied so that the quantity required at any point will be only sufficient to take up any slack in the flow of work over the shortest possible period of time. Only when the processes become continuous can the storage and investment charges be divorced from the ultimate unit cost for a particular article, because in that case the banks are only required as a reserve against an emergency, and the resulting costs, which are inherently smaller, are no longer assessable to the product but are instead charges against the management which take the form of insurance against unavoidable interruption. Here too the ultimate unit cost has been reduced by a shrinkage in the investment charge which is due not only to a shortening of the storage time imposed upon material in process but also to a lessening of the quantity required for reserve.

Characteristics of Processes Requiring Economic Quantity Control.—Naturally economic-production quantities are only applicable to intermittent processes, because in continuous production the perfect coordination of manufacturing schedules and the sales demand has obviated the need for machine changeover from one unit of production to another and the subsequent storage requirements. In general the determination of an economic quantity will be of importance in cases where the total set-up and preparation costs are a considerable proportion of the total manufacturing cost of the lot. This may be due to the fact that the size of the lot has not been properly determined or that the rate of consumption is too low to obtain a reasonable production quantity, even though the ratio of the preparation costs to the unit manufacturing cost lies within all normal limits. Again this may be due to some characteristic of the processes and then only, when the rate of consumption is large, can a satisfactory distribution of these preparation costs be made which will yield a desirable ultimate unit cost. On the other hand, economic quantities are of equal importance in cases where the unit manufacturing cost is high, and the rate of consumption is insufficient to insure a rate of inventory turnover that will keep the investment charges within reasonable limits. Even when

all items seem to be fairly well proportioned, a particular situation may involve the problem of conservation of capital or protection from obsolescence which cannot be solved without recourse to economic quantities, if the ultimate unit cost is to be the deciding factor.

High preparation costs are usually found to exist in any complicated manufacturing process involving numerous machine operations. These charges accumulate not only from the large number of individual set-ups but also from the increased amount of supervision and control required. The more nearly automatic the equipment may be the greater are the costs of changeover, not only because the set-ups are more intricate but also because a greater number of lots of various articles can be produced in the same length of time. This is due to the fact that the rate of production is much greater than that for simpler types of machines capable of performing the same operations. Style factors or special designs to suit customers' requirements may interfere with a process, which might otherwise be continuous, through the necessity for machine changeover or the storage of finished articles awaiting shipment on the customers' schedules. Even if shipments can be made directly from the end of the production line, storage of some sort, even of raw material, will be required to absorb the changes in demand, and whatever the situation may be an excellent opportunity is presented for economic quantity control.

Economical Control of Service or Repair Parts Division.—Industries which maintain service divisions will also find much utility from economic quantities. Service or replacement parts are usually ones required for products which are no longer current, and it is advisable to provide separate facilities for their manufacture. Although there may be a fairly uniform demand for such parts, it does not depend upon the activity of the sales department and a considerable degree of probability enters into its determination, especially as the time for complete obsolescence approaches. Under rare circumstances is the volume even sufficient to warrant continuous production, and so an intermittent process must be employed which will yield as low a manufacturing cost as practicable, depending upon the economical lot size, without entailing too large an investment charge for carrying these parts in inventory to meet this more or less indefinite demand.

Economical Number of Repetitions.—It is almost without question that an infinite variety of situations can be found throughout industry in general, where economic quantities can be applied. In fact other uses for the basic theory have been developed. For example, it can be stated fundamentally that the economic number of repetitions of any nature is equal to the square root of the ratio of the expenditure of effort in preparation for the occurrence to the expenditure per unit of time of effort to obtain each single repetition or its resulting effect, whichever may be desired. In many instances the mathematical formulae employed to express this relationship are exceedingly simple and in others much more complicated. The difficulty in the determination of the final result depends entirely upon the number of individual items which enter into the composition of the two major factors composing this ratio. When applied to production the various methods of approaching the problem from a mathematical standpoint are outlined in the following chapters of this section in such a manner that the one which requires the least effort in computation can be easily selected with definite assurance of the accuracy of the results that can be obtained.

CHAPTER IV

THE SIMPLIFIED FORMULA

If the determination of the economical size of a production lot, or the limits of the economic range, required the use of any one of the fundamental formulae derived in Chap. XVIII, industrial executives would in their present state of mind undoubtedly prefer to continue their past haphazard methods of arranging production schedules rather than to adopt a scientific method which involved so complicated a technique. Fortunately, the majority of manufacturing problems requiring this type of solution have certain inherent characteristics which will permit the use of simplified forms of these fundamental equations. In a few isolated cases, however, it will be found, where the relationship of the controlling factors is more than ordinarily obscure, that one must resort to these fundamental formulae in order to obtain reliable facts for the control of the manufacturing operations. In case any one of these formulae should have to be employed to obtain the lot size, a single general equation has been prepared as shown in Table I, which may also serve to illustrate the close parallel between the equations for each of the important points in the economic range, and the relationship between the various factors which must be considered.

Fundamental Formulae Not Required in All Cases.—Since a thorough knowledge of the influence and importance of each factor has been gained by the evolution of these complex relations, one should be more willing to employ one of the simpler forms of these equations and disregard certain of the unimportant factors with a great deal more confidence. For the most part, each problem to be investigated will have certain characteristics which are sufficiently evident upon the surface to indicate those items that can be disregarded. For example, a unit of production is normally quite compact and will occupy a relatively small space in proportion to its value, and, if this be so, one can quite reasonably assume that the unit charges accruing from the space occupied by the article in stores will have a negligible effect upon

TABLE I.—FUNDAMENTAL RELATIONS FOR ALL ECONOMIC LOT-SIZE DETERMINATIONS

$$Q = \sqrt{\frac{P \cdot F \cdot Sa_s}{(K_s + K_w) \cdot fr + K'_v}}, \tag{1}$$

where,
for the minimum-cost quantity

$$f_r = 1,$$

for the point of maximum return

$$f_r = \left(1 + \frac{r}{i}\right),$$

for the economic-production quantity

$$f_r = 1 + 2\frac{r}{i} + \frac{r^2}{i^2\left(1 + \dfrac{K_v}{K_s + K_w}\right)},$$

and the elements in the denominator

$$K_s = c \cdot i \cdot \left[\frac{k_s}{\dfrac{\delta}{\theta \cdot Sa_s} + 1} - \frac{\theta \cdot S_{a_s}}{2 \cdot D} \cdot \left(1 - \frac{1}{n}\right)\right],$$

$$K_w = \left[\left(\frac{1}{k_a} - 1\right) \cdot m + c\right] \cdot k_a \cdot k_d \cdot \theta \cdot S_{a_s} \cdot t_p \cdot i,$$

$$K_v = \frac{s \cdot b}{h} \cdot k_b \cdot \left[\frac{1}{\dfrac{\delta}{\theta \cdot Sa} + 1} - \frac{\theta \cdot S_{a_s}}{D} \cdot \left(1 - \frac{1}{n}\right)\right],$$

$$F = \left[1 + \frac{T_M \cdot k_m \cdot i}{8} + \frac{1}{\dfrac{\theta \cdot Sa_s}{\delta} + 1}\right].$$

For an interpretation of the symbols see text Chap. II, pp. 22 and 23, or Table V, or Appendix XIII, p. 349.

For simplification and special forms see Chap. V, Tables III and IV, or Chap. XIX, Table XXXI.

the ultimate unit cost. Similarly, if the actual process time for each unit is relatively short and finished parts or products can be removed rapidly from work in process to stores, only a small portion of the total cost of capital employed will accumulate from this source. This will indicate immediately that the cost of carrying articles in inventory is the factor which largely controls the size of the lot in relation to the preparation costs.

Manufacturing Analysis Permits Simplification.—Not always can one so easily segregate the various factors, but a similar conclusion can be reached if a general survey of manufacturing operations is made like that which is suggested in the following chapter.[1] Often a simple analysis will accomplish the desired results which will in no way involve any computations, if one can only follow out the logic behind the method of selection. As a result it is possible to derive a simple expression involving not more than three variables which can be grouped in a number of ways so as to shorten the final calculations, or some mechanical device can be employed similar to a slide rule or chart so that any subordinate in a production-control or process-engineer's department can rapidly obtain reliable values for the production quantities. The deviation in the answer from that which would have been obtained from either of the complete formulae in any such case, whether it be employed in the determination of economic- or minimum-cost quantities, will be of little consequence, as the results are to be used as limits for a range of reasonable practice rather than as a single inflexible quantity, which must be adhered to at all times. For the same reason repeated determinations of the production quantity for varying rates of consumption can be avoided, because any quantity can be arbitrarily chosen to meet this demand which lies between the economic quantity and the minimum-cost quantity, with the assurance that it will have the same manufacturing advantage that could be realized should the minimum-cost quantity have been actually processed instead.

Simplified Expressions.—For a manufacturing problem similar to the one cited above, which has been found to be typical of those more generally encountered, comparatively simple expressions can be employed for determining each of the limits of the eco-

[1] See pp. 67–71.

nomic range which merely depend upon an economic balance between the total preparation costs and the total charges for capital invested in inventories The simple form for the economic-production quantity or the lower limit of the range, which includes the expected return upon the investment reduces to

$$Q_{e_s} = \sqrt{\frac{P \cdot S_a}{c} \cdot k_r}, \qquad (2)^1$$

and that for the minimum-cost quantity becomes

$$Q_{m_s} = \sqrt{\frac{P \cdot S_a \cdot 2}{c \cdot i}}. \qquad (3)^1$$

In deriving this expression it was assumed that the consumption rate over the year will remain fairly uniform, and that no occasion demands a consideration of losses from deterioration or obsolescence other than that provided for through the resulting conservation of capital. The effect of the rate of delivery to stores upon the average quantity in stock has been disregarded, as that is another special case which had best be considered separately.[2]

Financial Policy Reflected in the Factor k_r.—One executive decision is required before the form for a specific manufacturing plant can be released for production-control purposes. The constant k_r must be determined which will represent the financial policy of the concern with regard to the interest rate i to be used in determining the cost of capital, and the rate of return r normally to be expected upon the capital invested with proper reference to the risks of doing business, in excess of the interest rate. Once this constant is established, by substituting the proper values for these two items in the expression

$$k_r = \frac{2}{i\left(1 + \frac{r}{i}\right)^2}, \qquad (4)^1$$

the formula for the economic quantity can be used indiscriminately by any clerk without any danger of revealing confidential facts concerning the financial policy of the company. As an aid to judgment it may be stated that the "cost of capital"

[1] For an interpretation of symbols see pp. 22 and 23, or Appendix XIII, p. 349.

[2] See p. 68.

is usually figured on an interest rate i of 6 per cent while a value of 18 per cent is quite representative of the normal rate of "return on capital," although this latter figure may vary from 12 to 30 per cent, depending upon the risks encountered in a specific industry.

Precautions in the Selection of Data.—The economic-production quantity for all articles manufactured, either fabricated or assembled, may be determined by selecting the correct values for the three remaining items in the formula [Eq. (2)] directly from current production or accounting records and from sales forecasts. One precaution must be observed and that is no duplication of cost figures can be permitted. All costs independent of the quantity must appear in the preparation cost item P, and all unit costs dependent upon the quantity must appear in the item c. Care must also be taken to see that machine set-up, dismantling costs, and production-control costs do not appear in the overhead distribution or in the direct-labor cost per unit produced. It is permissible in this case to include all storage space charges in overhead, as the use of this form assumes that they are quite small in proportion to the other unit charges. The reasons for this together with an explanation of the errors resulting from a misinterpretation of the data have been enlarged upon in Chap. VII.[1]

Adaptation of Formulae to Accounting Practice.—The time element upon which sales estimates are based varies with nearly every concern. In some instances it is most convenient to specify the rate of consumption S_a in terms of years, in others in terms of months, weeks, or even days. Some standard basis must be used in connection with either formula, because, if it is expressed in terms other than years, a factor must be introduced into the numerator in order to provide a suitable correction for the time element used as a basis for the interest rate and the rate of return on capital which are normally figured on a yearly basis. This factor will be equal to 12 if the consumption rate S_a is expressed in months, 52 if S_a is in terms of weeks, and 300 or some other equivalent figure[2] if S_a is in terms of days. It should be noted that the expressions given on page 39 have

[1] See p. 91.

[2] See also interpretation of data accompanying the calculation sheets on p. 59, Table V, under notes 1 and 2.

been written on a yearly basis. Once this correction has been made, data from current records can be used directly in the formula.

Recommended Technique for Simple Problems.—In order to provide a definite means of introducing the use of economic-production quantities to any manufacturing plant, the accompanying calculation sheet has been devised. By following the instructions for the collection of data and introducing the correct values in the spaces provided for calculation, the appropriate production quantity can be quickly computed with the aid of an ordinary slide rule or a table of square roots.

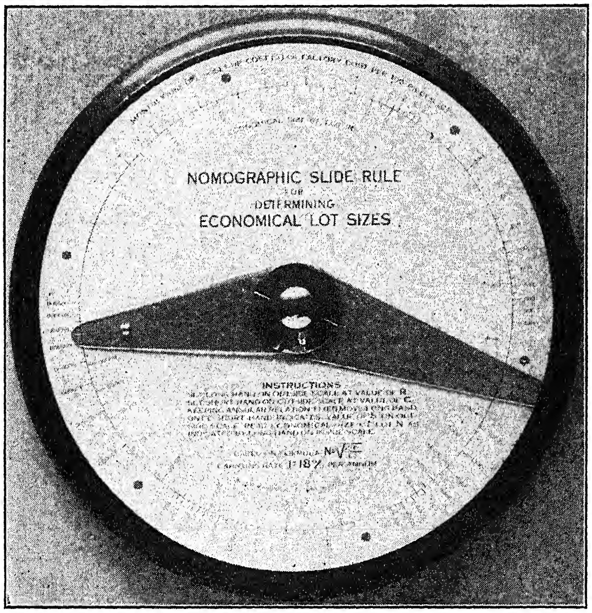

FIG. 1.—A single-purpose circular slide rule for the direct computation of the economic lot size from the simplest formula, developed by Benjamin Cooper for the General Electric Company.

Mechanical Means of Solution.—Some concerns have found that a special slide rule graduated specifically for this purpose will simplify the work still more. Two types are illustrated by Figs. 1 and 2; the first can be used merely for determining the economic lot size; the second, being a double-purpose rule, may be used to obtain the value of the annual volume of sales in addi-

TABLE II.—CALCULATION SHEET EMPLOYING SIMPLIFIED FORMULA

Item	A. DATA	Symbol
1.	*Unit Production Cost* (in dollars per piece): Add to the total unit cost of the material used in fabricating the part, taken at the time it enters the process or is set down at the first operation, all labor and overhead charges incurred in fabricating or assembly operations on the individual piece, except charges arising from the set-up and dismantling of the machines which perform these operations and charges for production control entered in item 2, which should be removed from overhead distributed upon operating time, if it has been the practice to include it there.	c
2.	*Preparation Cost* (in dollars per lot): This item includes the total labor and overhead charges involved in setting up the machines for fabrication of the parts or the assembly line and dismantling the set-ups after parts or assemblies have been produced as well as the cost of writing and issuing the production order, production-control costs, or any total charges directly pertaining to the lot and not dependent upon the quantity produced in the lot.	P
3.	*Consumption Rate* (pieces per year): Obtain the most reliable estimate possible for the total consumption of this part or assembly, whichever the case may be, for the current year, and include whatever allowance is necessary for service or replacement parts, inventory shrinkage, or other legitimate requirements.	S_a
4.	*Interest Rate* (percentage per year, expressed as a decimal): Obtain the figure for the interest rate in percentage per year which ordinarily would be paid on borrowed capital and which would be used to compute the cost of capital employed including, if need be, an allowance for the expense of the financial operations.	i
5.	*Expected Normal Rate of Return* (percentage per year expressed as a decimal): Obtain the figure in excess of the interest rate which represents the desired rate of return per year in percentage for capital employed in your business with due regard for the risks of business. This item may vary from 12 to 30 per cent or more, depending upon the risks involved.	r

B. COMPUTATIONS

Item

1. Multiply P by S_a by 2 $= \cdots \times \cdots \times 2 = \cdots\cdot$
2. Multiply c by i $= \cdots\cdot \times \cdots\cdot = \cdots\cdot$
3. Divide item 1 by item 2 $= \dfrac{\cdots\cdot}{\cdots\cdot} = \cdots\cdot$
4. Find square root of item 3 $= \sqrt{\cdots\cdot} = \cdots\cdot = Q_{ms}$. The minimum-cost quantity
5. Add i to r $= \cdots\cdot + \cdots\cdot = \cdots\cdot$
6. Divide i by item 5 $= \dfrac{\cdots\cdot}{\cdots\cdot} = \cdots\cdot$
7. Multiply item 4 by item 6 $= \cdots\cdot \times \cdots\cdot = \cdots\cdot = Q_{es}$. The economic quantity.

For an interpretation of symbols see text Chap. II, pp. 22 and 23, Table V, p. 58 or Appendix XIII, p. 349.

tion. Similar slide rules may be constructed to suit the conditions existing in any plant, or a slide rule of the circular type may be purchased on the market from manufacturers of calculating devices of this nature. Charts may be drawn to achieve the same purpose and in some cases may prove much more satisfactory, as the setting of a slide rule is avoided. A

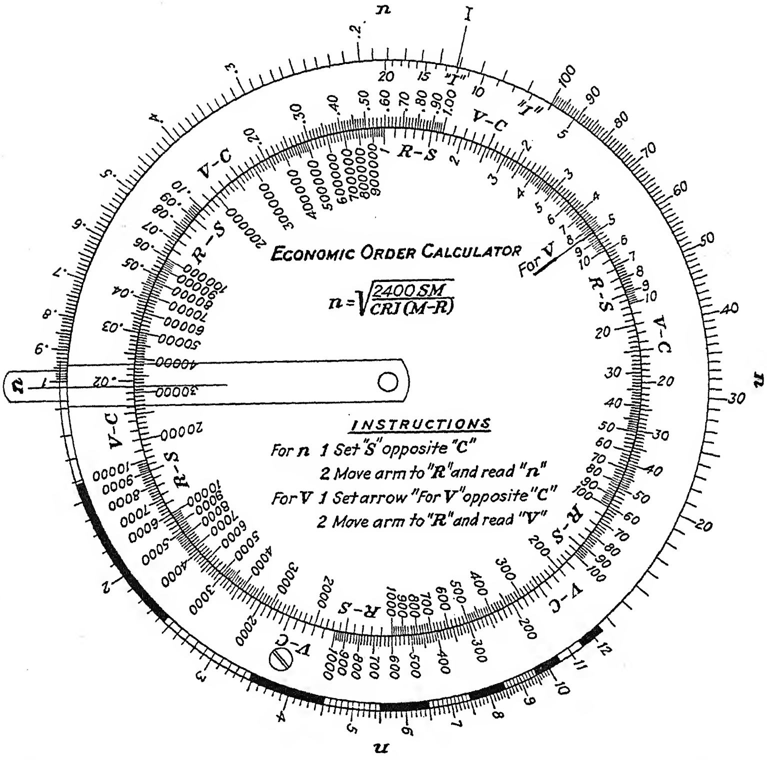

FIG. 2.—A double-purpose circular slide rule for the direct computation of the economic lot size or the value of the annual sales demand, developed by the Western Electric Company.

mechanical nomographic chart is illustrated in Fig. 3 and an ordinary graph drawn on standard coordinate paper is shown in Fig. 4.

Introduction of Constant Factors.—In some instances it may be advisable to subdivide the expressions given in Eq. (2) and (3) for greater ease in handling the calculations, especially where a large number of parts enter into a product which has a fairly

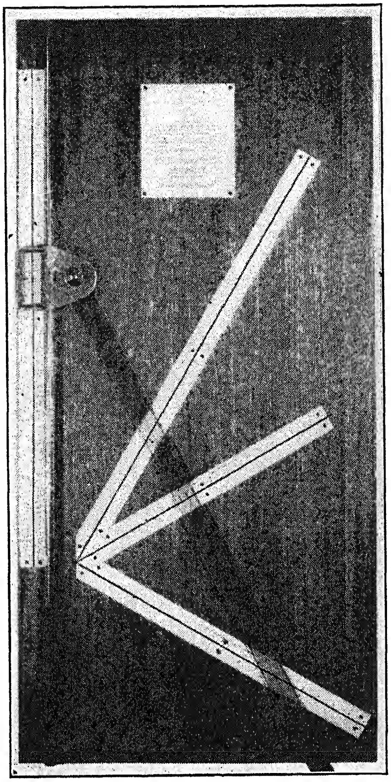

FIG. 3.—A mechanical nomographic chart for the direct computation of the economic lot size from one of the more complicated special formulae, developed by Benjamin Cooper for the General Electric Company.

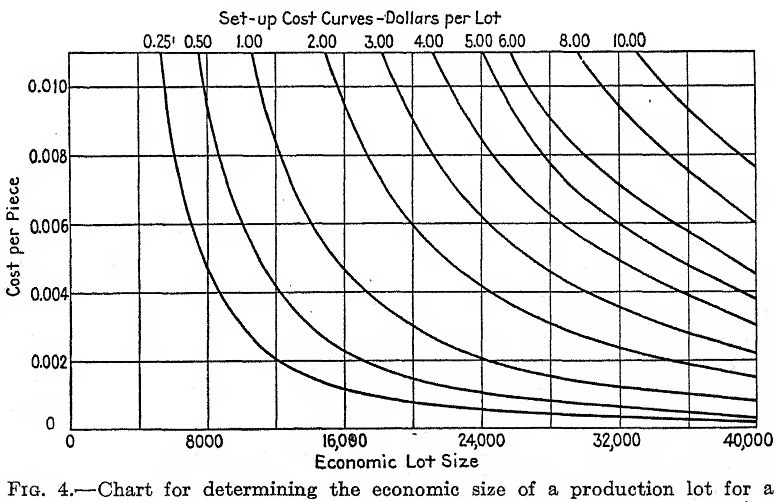

FIG. 4.—Chart for determining the economic size of a production lot for a specified rate of consumption (5,000 per month).

uniform demand. If, for instance, the product of the constant $k_r = 2.222$ and the sales demand S_a which, for a certain article is 9,000 pieces per year, is 20,000, and two units of a given part

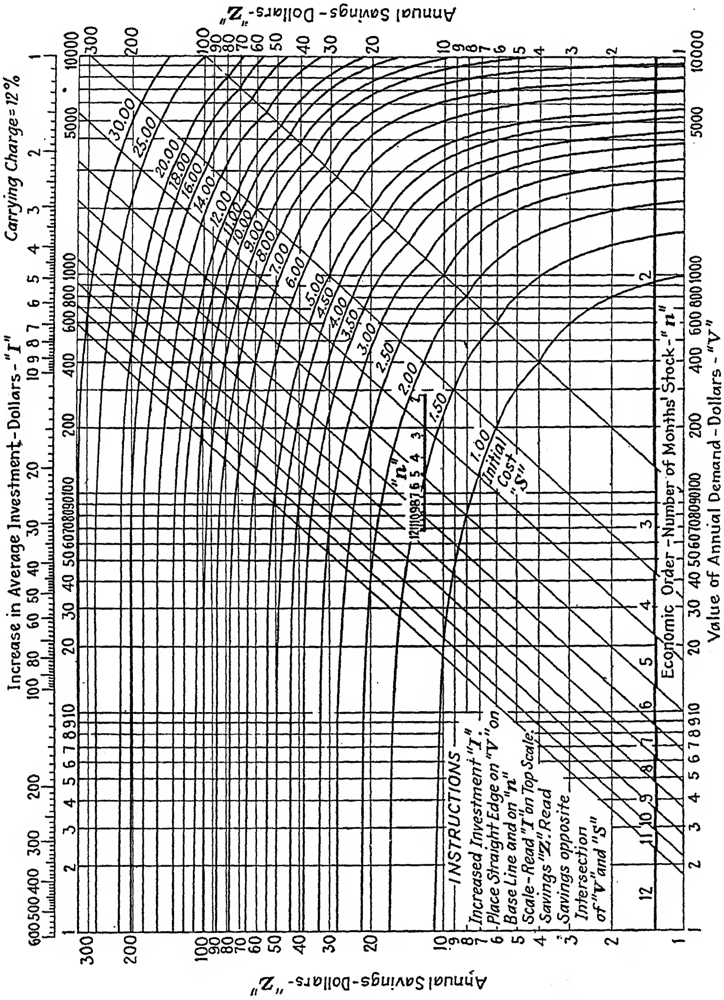

FIG. 5.—A special chart developed by the Western Electric Company for computing economic size of order and the resulting annual savings.

enter into its construction, as indicated by the multiple $\sqrt{2}$, the expression on page 39 [Eq. (2)] may be written so that

$$Q = \sqrt{20{,}000 \cdot \frac{P}{c}} \cdot \sqrt{2}.$$

A simple chart may be drawn up giving a curve for Q in terms of the ratio P/c when the corrective factor has been reduced to the

form $\sqrt{2}$, so that the proper value for the quantity to produce can be quickly determined when the consumption rate of the part is twice that of the basic rate. For other multiples such as $\sqrt{3}$, $\sqrt{4}$, etc., similar curves can be drawn on the chart to account for any other situation, as shown in Fig. 6. The fact that a representative rate of consumption was arbitrarily chosen in constructing this chart in no way diminishes its value, because a conversion table or chart (Fig. 14) can be arranged which can be used to correct the value of an economic quantity for any change in the demand that may occur. Such a device will have

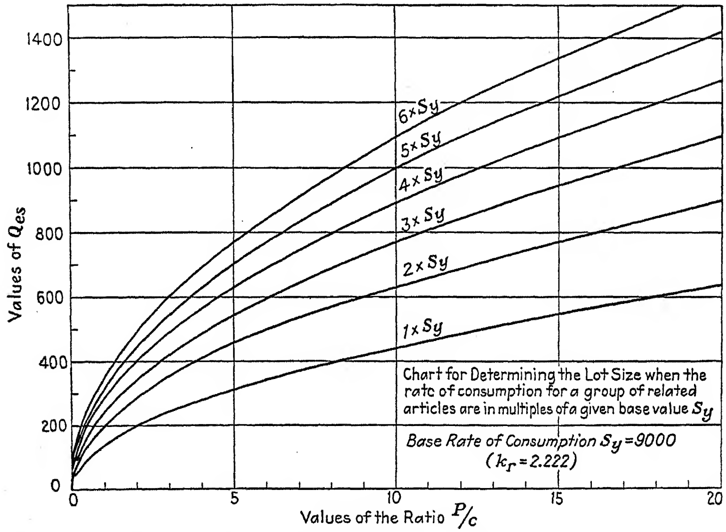

FIG. 6.—Chart for determining the lot size when the rate of consumption for a group of related articles is in multiples of a given value for the rate of consumption.

little application until the consumption has increased to a point when it becomes desirable to exceed the upper limit of the range or the minimum-cost quantity.

Determination of the Limits of Economical Production.—As the economic-production quantity is the smallest quantity that should be manufactured at any one time, the maximum quantity to be produced without incurring a reduction in the expected return on capital employed in its manufacture will be the minimum-cost quantity. No similar calculations are required to obtain this upper limit for production control because the two

are very closely linked through a constant factor. If Q_m is allowed to represent the minimum-cost quantity, then the economic quantity Q_e can be transposed into Q_m by the use of a factor f'_r where

$$f'_r = \left(1 + \frac{r}{i}\right)^2$$

in the relation

$$Q_m = \sqrt{f'_r} \cdot Q_e.$$

This factor will be constant for all manufacturing operations in a given plant, and when once specified as a part of the manufacturing policy no correction ever need be made. For example, if the values assigned to the interest rate i and the rate of return r, respectively, are 6 and 18 per cent, then the factor f'_r will be

$$\left(\frac{0.06 + 0.18}{0.06}\right)^2 = \left(\frac{0.24}{0.06}\right)^2 = 4^2 = 16.$$

Hence the production schedules can vary between one and four times the economic quantity for any product or component part without fear of exceeding the economic range, and provision can be made within this range for normal fluctuations in demand. When other values are used for i and r, the economic range will be different. This example may illustrate quite readily the reason for differentiating between the minimum-cost quantity and the economic quantity. The production-minded executive may be justified fundamentally in producing the minimum-cost quantity, but not so in actual practice, because in the first case that quantity would demand the expenditure of four times the capital required in the second. Obviously the minimum-cost quantity can no longer be called the economic quantity, as the conservation of capital is of equal if not greater importance than low-cost production.

The Influence of Capital Turnover.—One other word of caution might well be introduced here to the executive who may be overdesirous in obtaining conservation of capital. If a reasonable turnover of capital amounting to twenty to twenty-five times a year or even more can be obtained for a minimum-cost quantity, there is little to be gained through an increase in this rate of turnover by selecting a production quantity within the range which lies near to the economic quantity, unless funds are not available for the processing of a larger lot at one time.

Too small a lot will increase the number of changeovers in the year, which will lessen the available machine time for actual production purposes and will increase the indirect expense as a closer control and a greater amount of supervision may be required. In the last analysis it is preferable for one to employ that quantity within the economic range which will give a reasonable turnover of capital and inventories for which funds are available. The economic quantity marks the limit to which one can go in this direction without impairing the expected rate of return, and will be the ideal production quantity in all cases except where an appropriate turnover can be attained with a larger quantity.

CHAPTER V

SPECIAL FORMULAE

The simple formulae discussed in the last chapter for both the economic-production and the minimum-cost quantities will be found to be productive of excellent results in a large number of instances. Nevertheless in their derivation certain assumptions were made which if not wholly true will cause sufficient errors to make their use questionable. The economic range as well will become unreliable if its limits are uncertain, because its real value lies in an unrestrained freedom of action within definite limits which insure protection from a too rapid accumulation of hidden costs.

Individual Cases Require Separate Treatment.—In fact the formulae presented in the previous chapter belong to a group of special forms of the general expressions, each of which have a particular significance under certain stated conditions. These two were segregated from the others because they will be found to have in all probability a more universal application. No one should assume from this fact, however, that these or any other of the simplified forms should be employed without first having made reasonably certain that the form selected meets the requirements of the case. It is conceivable that instead of basing either of these quantities upon the cost of capital in stocks of finished units of production, they might be more effectively based upon cost of capital invested in work in process or even in extraordinary cases upon the charges arising from the storage space occupied. The author has found a number of instances where the most reliable approximation to the results obtained from the general formula can be achieved by the use of one or more of the other elements of the denominator in preference to that one used in the derivation of Eqs. (2) and (3). Certain corrective factors may also have to be introduced to account for special features in the process or the methods of conducting the business. Records may present the data in one form in one company, and in another they will be entirely different. By

simple changes in the make-up of each element, either of the basic formulae can be altered to suit each specific case so that the resulting form will be the one most adaptable to the circumstances and require the least effort to apply.

Characteristics of the Process Determine the Specific Form.—The best form for use in a given case may depend upon the type of process, the design of the product, or the manner in which materials are handled. The rate at which articles can be delivered to stock in relation to the rate of withdrawal to meet sales schedules may alter the production quantity. A long production period may be the deciding factor or even the bulk or unit volume of the product. The intrinsic value of the final article or the rate of accumulation of value during the process can have its effect as well.

Simplification of the General Formulae.—In cases where production schedules must take into account variable demand, deterioration of the product, style changes, rapidly occurring improvements in design, or other factors which demand special provisions to insure against undue losses from obsolescence, the general formula for the practical solution, as presented in Eq. (1), Table I,[1] must be used in its entirety, as there is nothing to be gained from simplification except where individual items are found from experience to have little or no effect. In the majority of cases where such special considerations are not needed, however, this general formula, when employed in determining the economic-production quantity, can be reduced to the form

$$Q_e = \sqrt{\frac{P \cdot S_a}{\left[\frac{c}{2} \cdot i \cdot f_p + \frac{m + c}{2} \cdot S_a \cdot t \cdot i\right] \cdot f_r + \frac{s \cdot b}{h} \cdot f_p \cdot k_b}} \qquad (5)$$

where

$$f_p = \left(1 - \frac{S_a}{D}\, k_p\right)$$

or

$$= \left[1 - \frac{S_a}{D}\left(1 - \frac{1}{n}\right)\right] \qquad (6)$$

is a corrective factor for the rate at which articles are delivered to stores, and k_b is a corrective factor to account for possible

[1] See p. 37.

savings in storage space through a rotation of bins from one article to another which is equal to

$$\frac{1}{2}\left(1 + \frac{1}{n_b} \cdot f_p\right)$$

and

$$f_r = \left\{ 1 + 2\frac{r}{i} + \frac{r^2}{i^2\left[1 + \dfrac{2 \cdot s \cdot b \cdot k_b}{i \cdot h\left[c + \dfrac{(m + c)\ S_a \cdot t}{f_p}\right]}\right]} \right\} \tag{7}$$

is the factor which provides for the proper conservation of capital. Similarly, the general formula for the minimum-cost quantity can be reduced to the form

$$Q_m = \sqrt{\frac{P \cdot S_a}{\dfrac{c}{2} \cdot i \cdot f_p + \dfrac{m + c}{2} \cdot S_a \cdot t \cdot i + \dfrac{s \cdot b}{h} \cdot f_p \cdot k_b}}. \tag{8}$$

Basic Elements of Economical Production.—In order to carry the process of simplification still further the major elements in these equations may be set forth if new expressions are written in which a characteristic symbol is assigned to each element. Accordingly,

$$Q_e = \sqrt{\frac{P}{(K_s + K_w) \cdot f_r + K_v}} \tag{9}$$

$$Q_m = \sqrt{\frac{P}{K_s + K_w + K_v}}, \tag{10}$$

where the element K_s represents the factors depending upon the time of storage, the element K_w represents the factors depending upon the process time, and the element K_v represents the factors depending upon the space occupied by articles in stock. Six combinations of these elements can be arranged as shown in Tables III and IV, wherein it will be noted that the forms Q_{e_s} and Q_{m_s} are equivalent to the simple expressions recommended for common use.

TABLE III.—SPECIAL FORMS OF EQUATIONS FOR THE ECONOMIC-PRODUCTION QUANTITY

$$Q_{e_s} = \sqrt{\frac{P}{K_s \cdot f'_r}} = \sqrt{\frac{2 \cdot P \cdot S_a}{c \cdot f_p \cdot i \cdot \left(1 + \frac{r}{i}\right)^2}}.$$

$$Q_{e_w} = \sqrt{\frac{P}{K_w \cdot f'_r}} = \sqrt{\frac{2 \cdot P}{(m + c) \cdot t \cdot i \cdot \left(1 + \frac{r}{i}\right)^2}}.$$

$$Q_{e_v} = \sqrt{\frac{P}{K_v}} = \sqrt{\frac{P \cdot S_a}{\frac{s \cdot b}{h} \cdot f_p \cdot k_b}}.$$

$$Q_{e_{sw}} = \sqrt{\frac{P}{(K_s + K_w)f'_r}} = \sqrt{\frac{2 \cdot P \cdot S_a}{[c \cdot f_p \cdot i + (m + c) \cdot S_a \cdot t \cdot i] \cdot \left(1 + \frac{r}{i}\right)^2}}.$$

$$Q_{e_{sv}} = \sqrt{\frac{P}{K_s \cdot f''_r + K_v}} =$$

$$\sqrt{\frac{P \cdot S_a}{\frac{c}{2} \cdot f_p \cdot i \cdot \left[1 + 2\frac{r}{i} + \frac{r^2}{i^2 \cdot \left[1 + \frac{2 \cdot s \cdot b \cdot k_b}{c \cdot i \cdot h}\right]}\right] + \frac{s \cdot b}{h} \cdot f_p \cdot k_b}}.$$

$$Q_{e_{wv}} = \sqrt{\frac{P}{K_w f'''_r + K_v}} =$$

$$\sqrt{\frac{P \cdot S_a}{\frac{(m + c)}{2} \cdot S_a \cdot t \cdot i \cdot \left[1 + 2\frac{r}{i} + \frac{r^2}{i^2 \cdot \left[1 + \frac{2 \cdot s \cdot b \cdot k_b \cdot f_p}{h \cdot (m + c) \cdot S_a \cdot t \cdot i}\right]}\right] + \frac{s \cdot b}{h} \cdot f_p \cdot k_b}}.$$

For an interpretation of symbols see calculation sheets Table V, p. 58, or Appendix XIII, p. 349.

TABLE IV.—SPECIAL FORMS OF EQUATIONS FOR THE MINIMUM-COST QUANTITY

$$Q_{m_s} = \sqrt{\frac{P}{K_s}} = \sqrt{\frac{2 \cdot P \cdot S_a}{c \cdot f_p \cdot i}}.$$

$$Q_{m_w} = \sqrt{\frac{P}{K_w}} = \sqrt{\frac{2 \cdot P}{(m + c) \cdot t \cdot i}}.$$

$$Q_{m_v} = \sqrt{\frac{P}{K_v}} = \sqrt{\frac{P \cdot S_a \cdot h}{s \cdot b \cdot f_p \cdot k_b}}.$$

$$Q_{m_{sw}} = \sqrt{\frac{P}{K_s + K_w}} = \sqrt{\frac{2 \cdot P \cdot S_a}{c \cdot f_p \cdot i + (m + c)S_a \cdot t \cdot i}}.$$

$$Q_{m_{sv}} = \sqrt{\frac{P}{K_s + K_v}} = \sqrt{\frac{P \cdot S_a.}{\frac{c}{2} \cdot f_p \cdot i + \frac{s \cdot b \cdot}{h} f_p \cdot k_b}}.$$

$$Q_{m_{wv}} = \sqrt{\frac{P}{K_w + K_v}} = \sqrt{\frac{P \cdot S_a}{\frac{m + c}{2} \cdot S_a \cdot t \cdot i + \frac{s \cdot b}{h} f_p \cdot k_b}}.$$

For an interpretation of symbols see calculation sheets Table V, p. 58, or Appendix XIII, p. 349.

The selection of any one of these forms implies that the actual ultimate unit cost for any product will not be reduced by any serious amount if one or two of the elements are omitted from the calculations. This may be demonstrated graphically by reference to Fig. 7 where it will be seen that the curve U for the ultimate unit cost,[1] when only one element is used, is but slightly lowered in position from that which it occupies when all elements are considered in computing the points for the curve. The effect

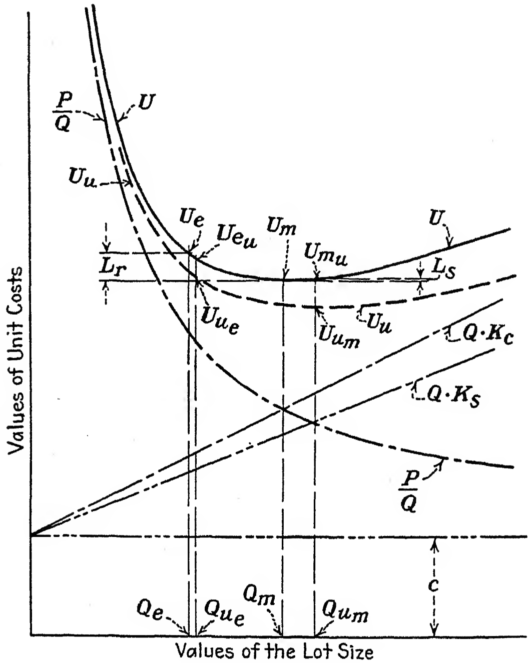

FIG. 7.—Effect of simplification upon the ultimate unit cost and the critical points of the economic range.

of this is to move the minimum-cost point to the right which will cause the value for the economic quantity as well to be slightly greater than it would be under ideal conditions. This increase in value is of little consequence as it is in the right direction. If, on the other hand, it had caused a decrease in the economic quantity, the lower limit for the economic range would be reduced below the value which yields the same economic value

[1] See definition p. 16 or derivation as presented in Chap. XII, and Table XIII.

in return upon capital, and would involve an increase in unit cost that would endanger satisfactory profits from manufacturing operations.

Deviation from Accurate Results Permissible.—The extent to which the values for both the economic quantity and minimum-cost quantity can be increased within reason depends upon the flatness of the curve *U* about the minimum-cost point, and that in turn is a function of the specific problem. As any such variation results in an increase instead of a decrease in the lot size, one might have some concern with regard to the effect upon the minimum-cost point, as it is the upper limit of the economic range. There is little chance for serious error from this source, however, because of the fact that the unit-cost curve slopes upward from its minimum point much more gradually for larger quantities than it does for smaller ones, with the result that for a given error in the value of the unit cost for the economic quantity a lesser error will be found to exist for the minimum-cost point, with much less likelihood of any impairment of the expected profit at that point. Moreover, if an increase in the sales demand should warrant the production of a quantity in the upper portion of the range, this fact alone would compensate for the apparent deviation. Even though the evidence would seem to point to a greater likelihood of the error in the economic-production quantity having a more serious effect, the fact that the error has increased the quantity and not reduced it is sufficient safeguard to settle any doubt in the matter, especially as the approximate quantity will in all cases still fall within the theoretical economic range. In any event it is possible to determine the permissible amount of variation for these values from the exact values in each case, and when this has been done the allowable variation in the data may be found from it in order to provide a measure[1] for selecting the simplest form that will satisfy requirements. It would seem that any method of simplification which involved such additional computation would be valueless. There is a great similarity, however, between the ultimate unit-cost curves for products of a related nature, and this permits the determination of a single index of variation in the data which will be applicable to a large number of problems. In fact it may be possible that a single index can be found which will apply to all

[1] See Chap. XIX for derivation of the method of simplification.

problems within a given concern. It is interesting to note that the resulting variation in either production quantity will be smaller than the allowed variation in the data. This is due to the fact that the error in the results varies as the square root of the errors in the data, a gratifying situation indeed.

The Selection of Special Forms.—As long as a measure can be provided to prevent excessive errors from affecting the results, one can proceed with the simplification of the general formula with much more assurance. A series of index ratios for each element have been arranged so that, by an inspection of their relative values, the appropriate element may be selected for use in the denominator of the desired special form. The one which has the largest value should be used in each case. The computation of the values for the various index ratios has been greatly simplified by the use of a common denominator, which device makes it possible to contrast the controlling factors in their logical order of importance and necessitates the evaluation of only two out of the three ratios required, the third being a constant in all cases. If it is desired to check the probable effectiveness of the simplified form thus obtained, the sum of the three ratios may be found and used as the denominator of another ratio, in which the numerator is the index for the element selected. This may be compared with the index of allowable variation in the data, and, if greater in numerical value, the resulting quantities may be relied upon.

The Calculation Sheets.—In order to simplify the procedure, calculation sheets (Tables V, VI and VII) have been prepared which can be used directly in any planning or production-engineering department without requiring any particular knowledge of mathematics or specialized engineering training. The proposed method[1] has been so arranged that the minimum amount of figuring is required,[2] in fact the major part of the work may be accomplished by a rapid inspection of the values for each index ratio, which may be calculated with sufficient accuracy by means of a common slide rule, using not more than two significant figures. The summation of the ratios and the determination of the corrective factor f_c is optional and need

[1] See Chaps. XIX and XX for derivation.

[2] See p. 103 for solution of an actual problem.

not be carried out, when familiarity with the method has been acquired, unless exact values for the economic-production or the minimum-cost quantities are desired.[1]

A Graphical Method of Selection.—If those responsible for the determination of economic quantities are accustomed to the use of charts, all computations connected with the selection of the appropriate form may be dispensed with. A universal chart for such purposes is illustrated in Fig. 9 by means of which the values of the index ratios may be not only directly found but also compared with the index for the maximum allowable

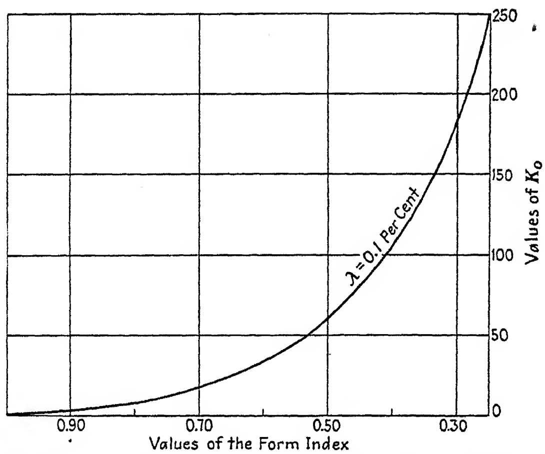

FIG. 8.—Curve for selecting the appropriate value of the form index f_c when the loss factor is specified as 0.1 per cent.

variation, so that the most effective form of all may be chosen. In many cases a chart of this order may be of much benefit, because it visibly demonstrates the interplay of one value upon another, and in the end provides means for analysis of manufacturing operations which could not be obtained in any other manner.

How to Employ the Chart.—To operate this chart, the value for the unit bulk b of the article in question should be located on the bottom scale to the extreme left; from which a vertical line should be followed until it intersects a sloping line represent-

[1] See also method of determining the process time t_p for the average piece, Table VIII, p. 95.

TABLE V.—INTERPRETATION OF DATA

Item	Definition	Symbol
1.	*Unit Material Cost* (in dollars per piece): This is the total unit cost of the material used in making the article taken at the time it enters the process or is set down at the first operation.	m
2.	*Unit Production Cost* (in dollars per piece): Add to the unit material cost m, item 1, all direct labor and overhead charges incurred in fabricating *or* assembly operations on the article, *excepting* charges arising from the setting up and dismantling of the machines or equipment which perform these operations, and charges for production control included in the preparation cost P, item 3, as well as any charges which may be included in the cost of the storage space s, item 8, both of which should be removed from overhead, distributed on direct operating time, if used separately. NOTE.—Production costs for fabrication and assembly cannot be combined.	c
3.	*Total Preparation Cost* (in dollars per lot): This item includes total labor and overhead charges involved in setting up the machines or equipment for fabrication of the articles *or* the assembly line and dismantling these set-ups after the articles have been processed, as well as the cost of writing and issuing the production order, production-control costs, and any other charges pertaining to the lot as a whole and not dependent upon the quantity produced in the lot. (See also note under item 2.)	P
4.	*Unit Production Time* (a decimal figure in years):[1] Determine for each operation the time consumed in performing the operation upon one piece. If time study or operation cost data are available these may be used instead. If the unit operation time is expressed on an hourly basis, it should be divided by the number of hours in the average or normal working year. If the actual production rate in pieces per day is known, the reciprocal of the production rate per day for each operation will give the unit production time for each operation which then must be divided by the number of working days per year. Add the resulting decimal figures together to obtain the unit production time for all operations in the process. If a more accurate figure is desired, follow the procedure outlined in Table VIII, page 95 for t_p.	t
5.	*Consumption Rate* (pieces per year):[2] Obtain the most reliable estimate possible for the total consumption of this part or assembly, whichever the case may be, for the current year. In addition to anticipated sales volume add allowances for service or replacement parts, inventory shrinkage, or other legitimate requirements.	S_a
6.	*Interest Rate* (percentage per year expressed as a decimal):[2] Obtain the figure for the rate of interest in percentage per year which would ordinarily be paid on borrowed capital and which would be used to compute the cost of the capital employed; and then reduce it to a decimal figure by dividing by 100.	i
7.	*Normal Rate of Return* (percentage per year, expressed as a decimal):[2]	r

Item	Definition	Symbol

Obtain the figure in excess of the interest rate which represents the desired rate of return per year in percentage on the total capital employed in your business with due regard for the risks of business exclusive of the cost of capital as defined in item 6. This item may vary from 12 to 30 per cent depending upon the risk involved.

NOTE.—If the business policy will not permit the giving out of such a figure the accounting division can furnish a value for the ratio r/i or the factors f_r or f'_r, as the case may demand.

8. *Space Charge* (dollars per square foot per year):[2] — s

Add together all total charges resulting from taxes, insurance, maintenance, depreciation, interest, etc., for the building, equipment, fixtures, heat, light, service, etc., and supervision, storekeeper's wages, etc., if not included in the general plant overhead accruing during the year to the particular storage area or department, and divide the total for the year by the total floor area available for storage purposes, aisles, etc., excluded, to which these charges apply.

9. *Unit Storage Space Required* (cubic feet per piece): — b

Determine the overall space required by each article as placed in stores with due allowance for clearances, voids in stacking, handling space, etc. A representative figure may be obtained by measuring the space occupied by a number of these articles as they lie in storage and then reducing it to a unit value.

10. *Average Permissible Height for Storage* (feet): — h

Measure the height from the floor to the top of the highest bin, or, if no bins are provided, this will be the height to which articles may be conveniently and safely stacked one upon the other or one container upon another. In all probability this figure will be a constant for a given storage area.

(Optional data required only in determining the factor f_p.)

11. *Rate of Delivery to Stores* (pieces per year):[2] — D

When the corresponding manufacturing and sales periods overlap, determine the number of articles per day which can be removed from the process as fast as the fabrication or assembly of each is completed. If no overlap occurs, the factor f_p will be unity.

12. *Number of Batches per Lot:* — n

When a lot is divided into batches in the last operation for ease in processing, n should represent the number of subdivisions. If the lot can be processed continuously, $n = 1/0$, $k_p = 1$, and $f_p = 0$. For an explanation of the factor k_p see Chap. XIV, page 209.

[1] If the value for the consumption rate S cannot be conveniently expressed in pieces per year when inserted in any one of the formulae and it is preferable to express it as pieces per month or per week or per day, the usual value for t in hours per piece should be divided by the number of working hours per month, per week, or per day, instead, as the case may require.

[2] If it is the practice in any company to express the rate of consumption S in terms other than pieces per year, such as pieces per month, week, or day, the values as specified in items 5, 6, 7, 8, and 11 should be divided by the number of months, weeks, or days normally worked in a year, overtime or short time omitted, as the case may be.

TABLE VI.—CALCULATION SHEETS

PART I. METHOD OF SELECTING SPECIAL FORMS OF THE GENERAL EXPRESSIONS FOR THE ECONOMIC-PRODUCTION QUANTITY AND THE MINIMUM-COST QUANTITY. (OBTAIN DATA FROM TABLE V)

Procedure	Space Reserved for Computations

A. PRELIMINARY FACTS

Item 1. *Plant Constant* ϕ (need be calculated but once):

Divide s by i and multiply by $2/h$ $\quad = \frac{\cdots}{\cdots} \times \frac{2}{\cdots} =$

Item 2. *Process Factor* f_p (use only when f_p is less than 0.9):
Subtract the ratio S_a/D multiplied by

k_p from one $\quad = 1 - \frac{\cdots}{\cdots} \times \cdots =$

where,

for non-continuous production

$$k_p = 0,$$

for semicontinuous production

$$k_v = 1,$$

for batch production

$$k_p = \left(1 - \frac{1}{n}\right)$$

Item 3. *Flow of Material Factor* f_m (normally employ f_m in case a):

(*a*) When raw materials and finished articles remain in the process from start to finish. . . . . $f_m = \frac{m}{c} + 1$

Special Cases

(*b*) When only raw materials flow continuously into the process and finished articles remain to the end. . . . . $f_m = 1$

(*c*) When only finished articles flow continuously from the process and all raw material is delivered before starting. $f_m = \frac{m}{c}$

(*d*) If the process time for the average price t_p is used in preference to t, the unit process time (Table V, item 4). . . . $f_m = \frac{m}{c} + 1$

Divide m by c and add one if necessary . . . . . $= \frac{\cdots}{\cdots} + (?) = \cdots$

B. COMPUTATION OF RATIOS

Item 4. Divide b by c $\quad = \frac{\cdots}{\cdots} = \cdots$

Procedure	Space Reserved for Computations
Item 5. Multiply item 1 by item 4	$= \cdots \times \cdots = \cdots = R_v$
Item 6. Divide S_a by f_p	$= \frac{\cdots}{\cdots}$
Item 7. Multiply item 3 by item 6 by t	$= \cdots \times \cdots \times \cdots = \cdots = R_w$
Item 8. (A constant for all problems)	$= \underline{1.0} = R_s$
Item 9. Add items 4, 7, and 8	$= \quad = R_c$

Item 10. (*a*) Select, by inspection, any one (or two if necessary) of the index ratios (items 4, 7, or 8) so that the total value of those selected will be at least ⅔ the value of the index ratio R_c (item 9).

(*b*) Check off here index ratios selected:
R_v ( ), R_w ( ), R_s ( ).

Item 11. *The Special Form:*

Place in the denominator of the special form the following expressions according to the index ratios checked above.

If R_v is chosen, use $\frac{s \cdot b}{h} \cdot f_p$.

If R_w is chosen, use $\left(\frac{m + c}{2}\right) \cdot S_a \cdot t \cdot i \cdot f_r$.

If R_s is chosen, use $\frac{c \cdot i}{2} \cdot f_p \cdot f_r$.

If f_p is greater than 0.9, assume $f_p = 1$.

Then the special form will be

$$Q = \sqrt{\frac{P \cdot S_a}{(?) + (?) + (?)}},$$

where, for the minimum-cost quantity Q_m,

$$f_r = 1,$$

for the point of maximum return Q_R,

$$f_r = \left(1 + \frac{r}{i}\right),$$

for the economic quantity Q_e, when the index ratio R_v is negligible,

$$f_r = \left(1 + \frac{r}{i}\right)^2,$$

or when the index ratio R_v demands consideration of the element K_v

$$f_r = 1 + \frac{2r}{i} + \frac{r^2}{i^2} \cdot \left(1 - \frac{R_v}{R_c}\right).$$

TABLE VII.—CALCULATION SHEETS

PART II (OPTIONAL). METHOD OF DETERMINING THE FORM INDEX AND THE ALLOWABLE VARIATION IN THE LOT SIZE. (OBTAIN DATA FROM TABLES V AND VI AND FIGS. 8 OR 13)

	Procedure	Space Reserved for Computations	
Item 12.	Multiply item 9 by i by 2	$= \cdots \times \cdots \times 2$	$= \cdots$
Item 13.	Divide item 6 by item 12	$= \dfrac{\cdots}{\cdots}$	$= \cdots$
Item 14.	Divide c by P	$= \dfrac{\cdots}{\cdots}$	$= \cdots$
Item 15.	Multiply item 13 by item 14	$= \cdots \times \cdots$	$= \cdots$
Item 16.	Find square root of item 15	$= \sqrt{\cdots}$	$= \cdots$
Item 17.	Add one to item 16	$= 1 + \cdots$	$= \cdots = K_o$

Item 18. Refer to Fig. 8,[1] page 57, or Fig. 13, page 76.
Select curve in portion of chart to the right of line M for an allowable increase λ in unit cost over the minimum cost and locate point on bottom scale below point of intersection of the appropriate curve for λ with the horizontal line drawn through the value of the problen index K_o (item 17) on the vertical scale. Read off the decimal figure for this point on scale f_c which will be the value for the form index. . . . . $= f_c$

NOTE.—The value obtained for f_c (item 18) may be used in place of the fraction $2/3$ employed preferably in item 10, Part I, Table VI, for selecting the appropriate special form, if greater accuracy is desired, or

Item 19. Multiply item 9 by item 18 $= \cdots \times \cdots = \cdots = f_cR_c$

Item 20. Then turning to Part I, Table VI, select the index ratio (or any group of two which has a greater total value than that for f_cR_c, item 19, and check these off in the appropriate spaces provided in section b of item 10. The appropriate special form can then be obtained as before from item 11.

[1] Use Fig. 8 only when a value for λ equal to 0.1 per cent is acceptable. For other values of λ use Fig. 13.

ing the simple unit manufacturing cost c, depending upon the cost of material and direct labor with its overhead. A horizontal line should be then followed to the right until it intersects another sloping line in the adjoining space representing the value of the ratio ϕ of twice the space charge to the interest rate i (expressed as a decimal) and the height h to which storage is permissible. From this point proceed vertically (in some cases upward and in others downward) to the line AA and from this intersection follow a horizontal line to the scale where the value of the index ratio R_v may be read directly.

In a similar manner locate on the scale at the extreme upper right the value for the consumption rate S_a and follow a horizontal line to the left to its intersection with the sloping line for the appropriate value for f_p. Then drop vertically from this point to the point of intersection with the sloping line below representing the value of the unit production time t_p. Now proceed horizontally to the left into the adjoining space until this line intersects another sloping line representing the factor f_m which accounts for the method of handling the material to and from the machine. From this point, as before, proceed either upward or downward to the line BB and then horizontally to the scale where the value of the index ratio R_w may be immediately found. The third index ratio R_s need not be calculated, as it has a constant value of unity.

The center space on this chart is a nomographic chart so constructed as to represent the relation

$$R_v + R_w + R_s = f_c \cdot R_c.$$

where f_c is the factor of allowable variation in the data and R_c is the index ratio for the sum of all three elements as they appear in the general formula. The center scale used in the nomographic chart represents the values of f_cR_c for the particular value of f_c which is permissible for the industry or type of product. Where this chart is being used entirely by one concern or in a specific plant, the appropriate scale may be inserted if it varies from that appearing in the chart in Fig. 9, which has been arbitrarily chosen for a value of f_c equal to 0.666[1] because it has proved to be satisfactory for a large number of cases. To facilitate the determination of the appropriate value for f_c and the construction of the proper scale, reference may be had

[1] That is, the fraction $\frac{2}{3}$ appearing in item 10, Table VI.

NOTES

Index Ratio R_s always equals 1.
The value of f_c used to construct center scale was arbitrarily chosen as 2/3.

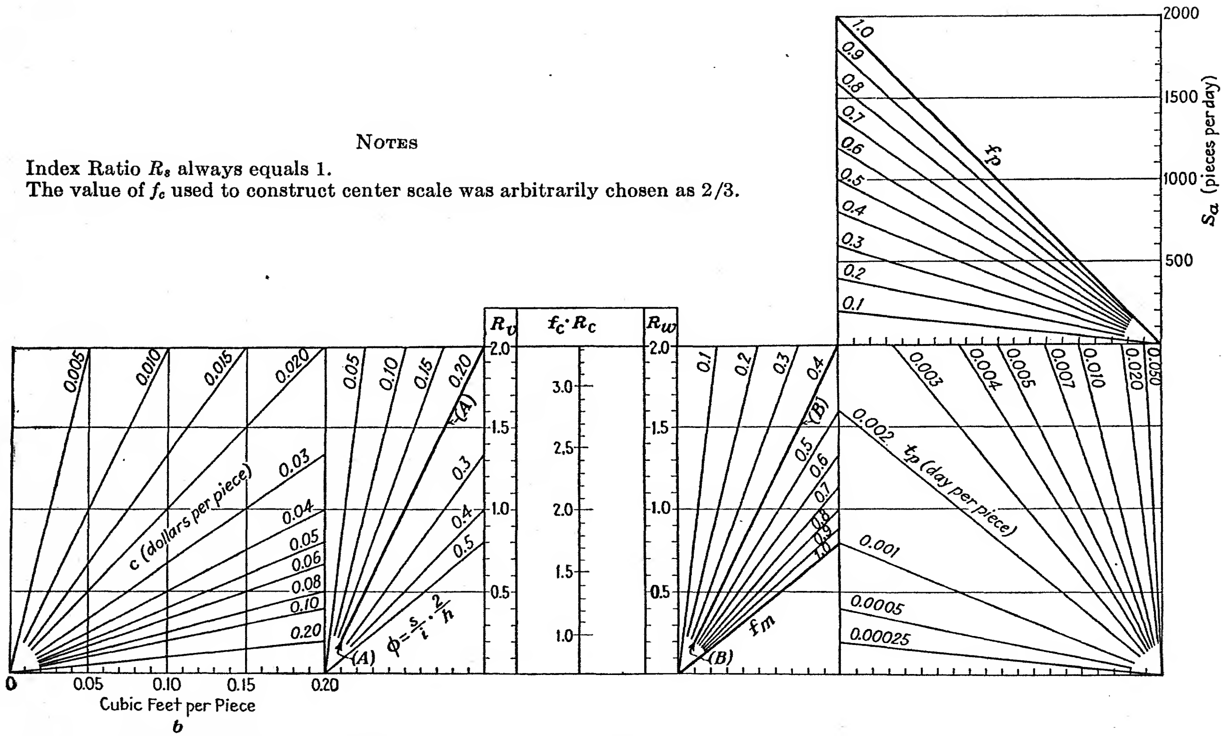

FIG. 9.—Graphical method of selecting special formula for the determination of the economic lot size.

APPLICATION

To obtain the value of the Index Ratio R_v:

1. Enter chart at left (bottom) with value of b.
2. Locate point of intersection of line for value of c and ordinate from b.
3. Move horizontally to right to intersection with line representing value of ϕ.
4. Move up or down vertically to line A-A.
5. Move horizontally to right and locate value of R_v on scale.

To obtain the value of the Index Ratio R_w:

1. Enter chart at right (top) with value of S_a.
2. Locate point of intersection of line for value of f_p and abscissa from S_a.
3. Move downward vertically to intersection with line representing value of t_p.
4. Move horizontally to left to intersection with line representing value of f_m.
5. Move up or down, vertically to line B-B.
6. Move horizontally to left and locate value of R_w on scale.

To locate limiting value of $f_c \cdot R_c$:

1. Draw diagonal line from value of R_v on left hand scale to value of R_w on right hand scale.
2. Read off value of $f_c \cdot R_c$.
3. Simplest form of equation will be indicated by the single index ratio or group of two which will just exceed the value of $f_c \cdot R_c$.

To select simplest form of equation:

Under the conditions where			Use the form indicated by Table III or Table IV	
$f_c \cdot R_c < 1.0$	....	....	Q_{e_s}	Q_{m_s}
$f_c \cdot R_c > 1.0$	$R_w > f_c \cdot R_c$	$R_v < R_w$	Q_{e_w}	Q_{m_w}
$f_c \cdot R_c > 1.0$	$R_w < R_v$	$R_v > f_c \cdot R_c$	Q_{e_v}	Q_{m_v}
$f_c \cdot R_c > 1.0$	$R_w > R_v$	$R_v < 0.5$	Q_{e_w}	Q_{m_w}
$f_c \cdot R_c > 1.0$	$R_w < 0.5$	$R_v > R_w$	Q_{e_v}	Q_{m_v}
$f_c \cdot R_c > 1.0$	$R_w > R_v$	$R_v > 0.5$	$Q_{e_{sw}}$	$Q_{m_{sw}}$
$f_c \cdot R_c > 1.0$	$R_w > 0.5$	$R_v > R_w$	$Q_{e_{sv}}$	$Q_{m_{sv}}$
$f_c \cdot R_c > 1.0$	$\left\{\begin{matrix} R_w > 0.5 \\ R_w < f_c \cdot R_c \end{matrix}\right\}$	$\left\{\begin{matrix} R_v > 0.5 \\ R_v < f_c \cdot R_c \end{matrix}\right\}$	$Q_{e_{wv}}$	$Q_{m_{wv}}$

to the chart given in Fig. 10 from which the correct graduation for any scale required may be determined.

The selection of the correct form may be obtained at once from the nomographic chart by laying a straightedge upon the points on the right- and left-hand scales, representing, respectively, the values of the index ratios R_w and R_v, and noting the value upon the center scale for f_cR_c. If this value is less than 1.0, the form Q_{e_s} or Q_{m_s}, as given in Tables III and IV, will apply. If the value of f_cR_c is greater than 1.0, two elements will be

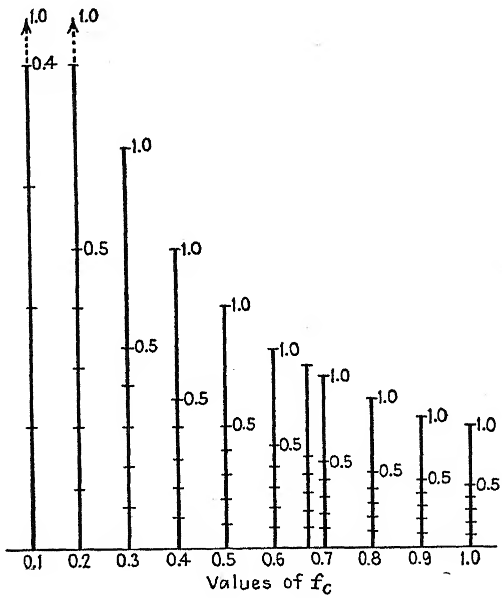

FIG. 10.—Sample chart for selecting graduations for scale of $f_c \cdot R_c$ for use in Fig. 9, in accordance with the value of the form index[1] computed by the method in Table VII.

required, except in such cases where the value of the larger of the two index ratios R_v or R_w exceeds the value for f_cR_c, whereupon elements representing this larger ratio may be used alone. If it is R_v use the forms Q_{e_v} or Q_{m_v}; if it is R_w use Q_{e_w} or Q_{m_w}. When two elements are required and one of the index ratios R_v or R_w is less than 0.5, the form to use will be $Q_{e_{sv}}$ or $Q_{m_{sv}}$ if R_v is greater, and $Q_{e_{sw}}$ or $Q_{m_{sw}}$ if R_w is the greater of these two. Should both the index ratios have values in excess of 0.5 but no

[1] Scale graduation for $f_c = 0.66$ corresponds with that employed in Fig. 9.

one of them greater than the value for $f_c R_c$, the forms $Q_{e_{wv}}$ or $Q_{m_{wv}}$ must be used.

A Short-cut Method for Accurate Results.—A number of occasions may arise where it is desirable to obtain quickly the true value of the minimum-cost quantity, and one may be unwilling to take the time that would be required to solve the general formula, especially where an approximate value has been previously obtained from a simplified form. In such a case the index of allowable variation in the data f_c can be employed to much advantage as a corrective factor, because it represents the amount of error between the values of the denominator in a simplified form and that in the general expression. Accordingly, a short cut can be devised[1] when

$$Q_m = Q \cdot \sqrt{\frac{1}{f_c}},$$

where Q stands for any approximate value, or, if the form Q_{m_s} be inserted for Q,

$$Q_m = \sqrt{\frac{2 \cdot P \cdot S_a}{c \cdot f_p \cdot i \cdot f_c}}.$$

Selection of Special Form Need Be Performed but Once.—It is not contemplated in this proposition that the selection of the appropriate simplified form must be performed every time an economic quantity is to be determined. The real purpose of this method is to provide means of analyzing the manufacturing operations and the nature of the product, when the use of economic lot sizes is first contemplated, in order to ascertain to what extent simplification may be achieved. Once this has been carried out, no further use need be made of this method of analysis, unless some new line of product is introduced for which the lot size must be determined. It is realized that a brief survey of manufacturing operations as proposed in Chap. IV[2] will ordinarily suffice to indicate the form to use. Should the results appear to be out of line with what should be good practice, a more technical study along the lines outlined in this chapter will serve to establish the correct method of handling the special cases. It does prove beyond any point of argument, however, that a simple form may be provided which will yield reliable

[1] See Table XXXV, p. 294.

[2] See p. 38.

facts for controlling production without recourse to the general form and the longer computations required for its solution, where a large number of fabricated parts and assemblies must be considered.

The Correction Factors, f_r and k_b.—Throughout the discussion in this chapter it will have been noticed that certain contributary factors have been introduced but no particular mention has been made as to their identity. Up to the present it has been necessary only to indicate their relation to the problem as a whole so as not to confuse the more important issues. The factor f_r, for instance, represents the items which control the conservation of capital and, as previously stated, when once determined by the executive policy will have no variable effect upon the results, as it will be constant for all problems. The factor k_b has been introduced to indicate how and when to make the corrections for the rotation of storage space between articles held in stock at different times and depends, likewise, on executive policy. It enters into the method of selection of the appropriate form in only such cases where this special method of handling the storage facilities exists. In other instances it may be totally disregarded.

The Process Factor f_p.—The factor f_p will probably have a more decided effect upon the economic quantity than any other if the nature of the process tends to approximate continuous production, particularly when the production rate may be very nearly equal to the consumption rate. The only obstacle in such instances to the adoption of continuous production is the fact that the production rate is greater than the consumption rate, and that if production is allowed to continue over too long a period, articles will accumulate in stock and eventually tie up large amounts of capital in inventory, unless the process is discontinued for a period of time in order to work off the excess stock. This assumes critical proportions when the production period overlaps the sales turnover period to a large extent, because, if the rate of delivery of articles to stores is not accounted for, serious errors will be introduced in determining the economic quantity that will prohibit taking advantage of this opportunity for low-cost production without increasing inventory values. Inspection of Fig. 11 will show that as the factor f_p,[1] which

[1] See Eq. (6), p. 50.

may be expressed as $\frac{1}{1 - \alpha k_p}$ for the moment where $\alpha = \frac{S_a}{D}$, approaches unity, it has a rapidly increasing effect on the production quantity Q. In the beginning when the values of the ratio of the consumption rate to the delivery to stores rate is less than 0.9, no attention need be paid to this situation, but if in the long run it seems advisable, the correction factor may be introduced into the formula wherever the element K_s is employed. In previous formulae for the lot size it was customary to use the rate of production for the whole process instead of the delivery to

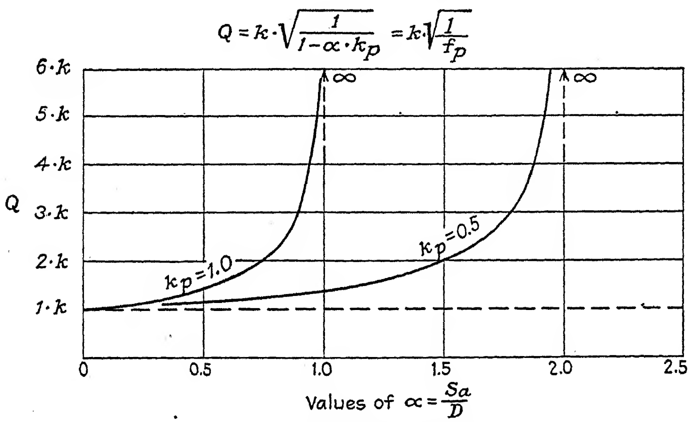

FIG. 11.—The influence of the process factor f_p on the lot size.

stores rate or the rate at which articles may be removed from work in process, but analysis of many situations has shown that this practice led to errors, because the rate of production did not always indicate the rate at which stocks were being built up. By using the delivery to stores rate a more comprehensive understanding of the factors can be achieved, and then if this rate should correspond to the production rate, the latter may be safely used indiscriminately.

The Flow of Material Factor f_m.—The factor f_m,[1] which takes into account the method of handling the material to and from the process, has had to be introduced into the calculation sheets in order to provide the necessary correction in the value of capital employed in production resulting from the rapidity of the flow of material. If the process is of the non-continuous

[1] See Table XXXIII, p. 289, or text, p. 290.

type, all articles remain in process for the entire production period, in which case

$$f_m = \frac{\frac{m + c}{2}}{\frac{c}{2}} = \frac{m}{c} + 1.$$

If the process permits the withdrawal of finished articles at the instant they are removed from the machines, the amount of capital introduced through the application of labor to each article will not accumulate in work in process inventories and the value of the factor will be

$$f_m = \frac{m}{c}.$$

Again, if it is advisable to introduce the raw material to the process only as it is required, even though the finished articles must remain in process until all have been completed, the value for the factor f_m will be unity. Finally, if both raw materials and finished articles can flow continuously to and from the process, continuous production is in effect for the time being and the factor f_m becomes zero. This indicates that the element K_w may be omitted from further consideration. With regard to the effect of this factor upon the general form of the element K_w, as used in any of the formulae, it may be stated that it provides a means of simplifying the general form in accordance with the flow of material due to the type of process. If the conditions set forth for the second case apply, the item c for the direct unit cost of labor and overhead may be omitted from the element K_w, leaving only the unit cost of material m as the basis of capital employed in work in process which affects the economic quantity. If the conditions indicate continuous flow of material, even though the process is intermittent, the element K_w may be omitted from further consideration.

The Ultimate Utility of Simplification.—The value of special forms of the general formulae cannot be underestimated, not merely from the fact that they simplify the procedure but also because the method of their selection provides a better understanding of the factors which control any methods of manufacture and permit a visualization of what actually occurs. Often study of a process, with regard to its characteristic ele-

ments, will suggest improvements which will eliminate waste in both time and human effort sufficiently to far outweigh that spent in making the mathematical analysis. Once a problem has been properly studied, slide rules, charts, or tables can be constructed, for the use of subordinates, which will dispense with all computations. One should remember throughout the full discussion of lot sizes that the mathematics are of least importance and are merely an analytical device employed to present the facts in a concise manner so that they can be rapidly translated into a serviceable technique for daily use.

CHAPTER VI

MAXIMUM- AND MINIMUM-PRODUCTION QUANTITIES

All previous methods that have been devised for determining the economic size of a production lot have contemplated the use of a single quantity for production-control purposes. Such a quantity is inflexible and offers no guide to the exercise of judgment in planning production schedules. It is absurd to expect that a quantity of this nature can be applied indiscriminately to account for the normal fluctuations in business conditions, and to provide for the production of the correct number of articles in advance of the expected sales demand. If the management of manufacturing operations is to be effective, individual initiative cannot be stifled by the adoption of arbitrary procedure, no matter how fundamentally sound the basic idea may be. If broad limits can be set within which any production quantity may be selected in accordance with the sales demand, with definite assurance that a reasonably low production cost can be obtained together with the proper conservation of capital, the desired freedom of action can be attained.

Recognition of Transitory Deviations in Data.—Naturally a production quantity should be employed which will maintain an economic balance between the preparation costs and the charges accruing to the lot for capital involved in its manufacture and storage, as well as from the space occupied in stores. Production schedules based upon such a quantity will meet the ideal situation; however, for any other where the rate of consumption may be slightly different, the minimum-cost quantity which applies in the first case will no longer be correct. Moreover, the error in forecasting this one item alone is liable to impair the accuracy of any single value of the minimum-cost quantity, because the actual sales demand at the time of processing the lot will undoubtedly be somewhat different from that which was originally estimated. Other factors will also vary: wages paid to labor employed in producing these articles may not be

the same as those which were paid the last time it was processed; and it is also quite probable that the overhead distribution rate will not remain constant from month to month. Likewise, only by good fortune is the time required to produce any article the same, because the very idea of piece rates and bonuses is to stimulate more effective manufacture in less time than that set as a standard.

It is not surprising, then, that production executives are prone to criticize any method of determining the best lot size which would seem to depend upon data that was susceptible to so many variations. One should not overlook the fact that many of these variations are caused by circumstances that are of a transitory nature, however, and that the specific values which may be recorded from time to time merely deviate from some standard by a relatively narrow margin. If the problem be studied with this in mind, standards of performance can be evolved together with appropriate limits of usefulness so that the presence of these variations can be properly recognized within a reasonable range of the standard, to provide a desirable amount of flexibility, and then, when they exceed these limits, to give warning of the approach of an exceptional situation which will require special executive attention.

The Utility of an Economic Range of Production.—Accordingly, a great deal can be gained by the determination of an economic range within which any quantity produced will so closely meet the desired minimum cost that the effect of these errors may be disregarded. The fact is that for quite a range of quantities around the minimum-cost point the change in the ultimate unit cost is very slight, and, if the extent of this range can be specified, production quantities may be varied to suit the immediate circumstance. Previous study of this subject has failed to take advantage of the permissible variation from the minimum cost in the derivation of a formula for the economic quantity. Some executives have recognized this fact to the extent that they have plotted the ultimate unit cost curve for a number of production quantities around the minimum-cost point, and then arbitrarily adopted that quantity which they believed to be the best. In no case, however, have they taken into account the actual loss incurred by the increase in unit cost and its effect upon the eventual profit. By inspection of

Fig. 12 it will be seen that if the quantities are increased, this curve, as one moves to the right, begins to rise slowly at first but later on more rapidly, until it seems to assume the slope of a straight line passing through the origin. If the quantity is decreased below the minimum-cost point, the curve will be found to rise much more sharply for a much smaller change in quantity, and for fairly small quantities will be rising quite rapidly. It is these increases in ultimate unit cost that are to be guarded against. Owing to the fact that they do not vary uniformly in each instance, it is difficult for men in charge of

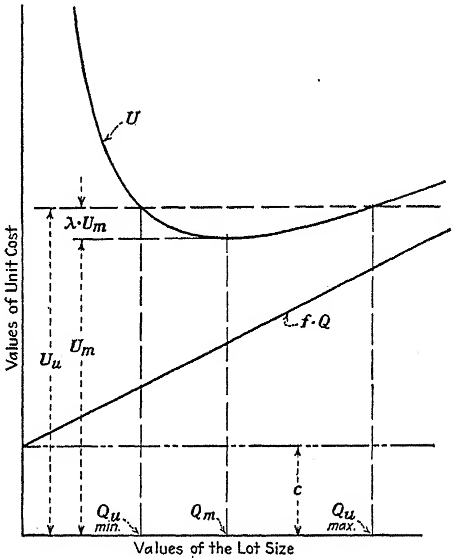

Fig. 12.—Diagrammatic representation of the cost factors entering into the computation of the ultimate unit cost.

production control to visualize the situation for a variety of products sufficiently so that they can always obtain the lowest cost under constantly changing conditions. The economic range definitely sets the limits for their guidance and provides the necessary data to aid them in meeting extraordinary demands. Moreover, the older methods of determining the economic lot size required a recalculation of the quantity every time a new lot had to be processed, but now through the use of an economic range no such necessity remains, and the task previously imposed upon the production-control division can be considerably lightened.

The Measure of Permissible Variation.—The measure to be employed in determining the appropriate economic range is the amount to which an increase in cost over the minimum cost is permissible under the existing conditions of business. Naturally, if the increase is too great the margin of profit expected from the sale of the product will be impaired. Two methods are available for determining the extent to which costs may be justifiably increased. The first, which has been outlined in the interpretation of the economic quantity, contemplates offsetting the increase in cost by a reapportioning of the expected return upon the capital invested through a more rapid turnover of inventories. The second, which involves an executive decision, provides means, through a comparison between past and present practice, for the determination of the percentage variation in cost from the minimum that will be permissible in order to gain the desired flexibility in planning. As the first method is in reality a special case of the second, the latter will be explained to begin with in detail.

The Limits of Permissible Variation.—By reference to Fig. 12 it will be seen that for any ultimate unit cost U_u not equal to the minimum cost two quantities may be produced, one greater and one lesser than the minimum-cost quantity. If the difference between this cost and the minimum cost is expressed as a percentage λ of the latter, a simple mathematical relation can be obtained for evaluating the limits for the range, if the variation is expressed as

$$E = 100 \cdot [(K_o \cdot \lambda + 1) \pm \sqrt{(K_o \cdot \lambda + 1)^2 - 1}] \qquad (11)^1$$

where

$$K_o = 1 + \frac{Q_m}{2} \cdot \frac{c}{P}.$$

If the minus sign $(-)$ in this equation is used, the resulting value for E will give the variation in percentage of the minimum-cost quantity which is allowable for the minimum-production quantity, λ having been specified. Similarly, the plus sign $(+)$ will give a value for E in percentage which may be used to determine the maximum-production quantity. It so happens that the terms represented by the factor K_o are constant for any article or unit of production as long as its design or method of manufacture

[1] For derivation see Table XXXVII, p. 301.

remains unaltered, and for this reason they can be grouped into one item which has been called the *problem index.*

Graphical Determination of the Limits of Variation.—The determination of the maximum and minimum limits for the

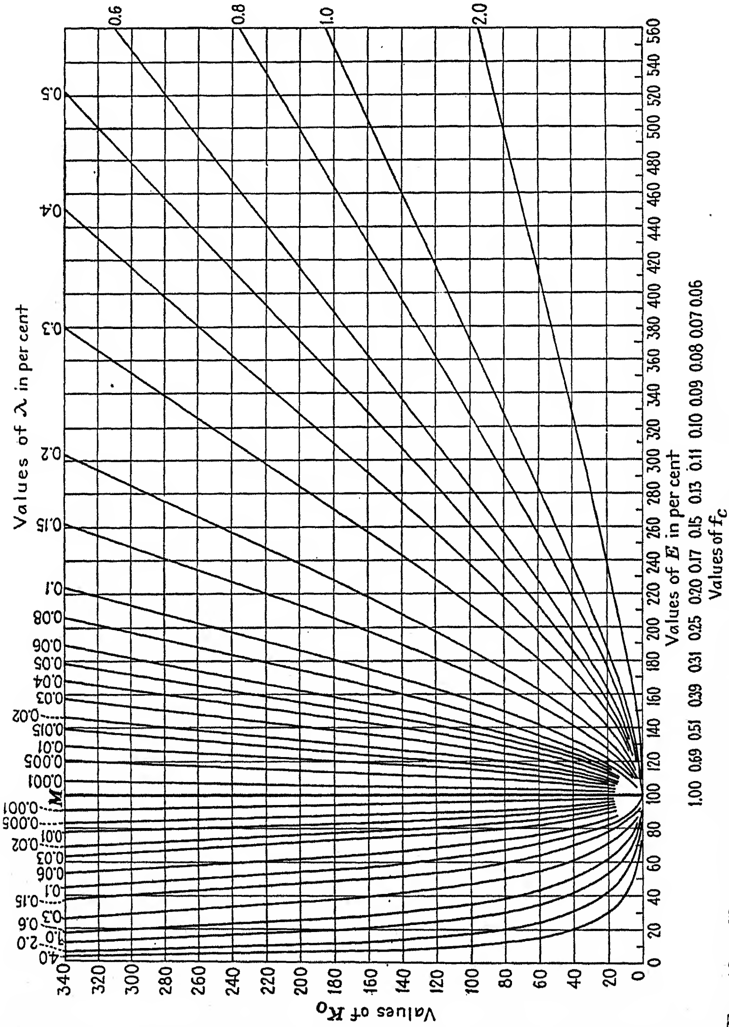

FIG. 13.—Chart for determining the form index f_c and the maximum- and minimum-production quantities.

production quantities can fortunately be simplified by the use of the chart illustrated by Fig. 13. As long as the values for e, E, and λ are expressed in percentages of the best practice and all characteristics of the problem are represented by K_o, this

method has a universal application. The chart has been drawn so that the minimum values for E appear upon the scale at the bottom of the left-hand space and the maximum values for E appear on a similar scale to the right. The vertical scale represents the values for the problem index so that a horizontal line through the appropriate value for K_o will intersect twice each of the curved lines representing the values of λ. This factor enables one to make a direct comparison of present practice with that which may be desired and to select a higher or lower value for the production quantity, whichever may seem to be the best.

Operation of the Chart.—By taking the quantity normally processed and dividing it by the minimum-cost quantity the corresponding value for E may be found when translated into a percentage figure. Then by locating this value on the scale, either on the right or the left, and following a vertical line to its intersection with the horizontal line through the appropriate value for K_o, the percentage increase over the minimum-cost quantity may be found and used as a basis for selecting the smallest quantity that can be produced for the same cost, or a quantity that can be produced at a less cost and still meet the sales schedules. Moreover, this latter scheme may be used to permit a proper executive decision as to a value of the maximum variation from the minimum cost, because the chart given in Fig. 13 will show the savings that can be realized from a scientific determination of the economic range. Once the value for λ is fixed, a similar chart may be constructed where the vertical scale is graduated for values of the minimum-cost quantity, and a family of curves may be drawn, each one of which represents a value for the ratio of the unit cost of material, direct labor, and overhead to the total preparation costs. This obviates the necessity of computing the value of K_o for each article.

Standards of Measurement for Production Control.—Given a permissible variation in percentage of minimum cost and a chart similar to either one just described, the production-control department will possess all the necessary implements for determining the best lot size under any set of conditions. The characteristics of each problem will be contained in the values calculated for the minimum-cost quantity and the problem index. These two items will have to be determined only once and that can best be performed by the process engineers at the time that the unit

of production is first approved and accepted as a regular item of manufacture. Moreover, this procedure suggests the possibility of employing the minimum-cost quantity as a standard of performance to be used not only as described above but also as a basis for predetermined costs and other figures used in the control of production.

Mathematical Determination of the Limits of Variation.—On the other hand if it is more desirable to obtain exact limits for the economic range which do not involve the arbitrary judgment of the individual, a definite value for the permissible variation E from the minimum-cost quantity can be determined through a consideration of the rate of capital turnover with respect to the desired return normally expected upon the capital thus employed, so that the resulting increase in cost will be entirely offset and in no way affect the ultimate gross profit to be derived from the sale of the article over the normal sales period. It will be of interest to note at this point when the value for E is thus determined from Eq. (11), wherein the minus sign is used for obtaining the minimum-production quantity or the lower limits of the range, that the resulting quantity will be the same as the economic-production quantity previously referred to. In this case the rate of return will be identical to that obtainable for the minimum-cost quantity, which would indicate that this latter quantity should be employed as the upper limit of the range if the expected rate of return is not to be impaired.

The Maximum-production Quantity.—If the method of reasoning which was used in connection with the graphical solution of the limits can be extended to apply in this case as well, a maximum-production quantity can be calculated by employing Eq. (11), wherein the positive sign is used, which will represent the quantity that can be safely produced in cases of emergency or sudden increment in the demand without exceeding the minimum allowable rate of return r_m. Accordingly, the maximum-production quantity can be computed from the following expression where

$$Q_{\max} = Q_m \cdot \frac{f_n}{f_d}\left[1 + \sqrt{1 - \frac{2 \cdot f_d}{f_n^2} \cdot \frac{P \cdot F}{v_s \cdot t_s \cdot i}}\right] \qquad (12)^1$$

[1] For derivation see p. 310, and Table XXXIX.

where

$$f_n = \sqrt{\frac{P \cdot F}{v_s \cdot t_s \cdot i \cdot R_c}} \cdot \left[2 \cdot R_c + \frac{r}{i} \cdot (1 + R_w)\right] + \frac{r}{i} \cdot \frac{C'_F}{v_s \cdot t_s}(1 - \rho)$$

$$f_d = 2 \cdot R_c + 2 \cdot \rho \cdot \frac{r}{i} \cdot (1 + R_w)$$

by determining the minimum return on capital employed that is advisable under the circumstances, and expressing it in terms of the normal expected return on capital through the ratio $\rho = r_m/r$.[1] The resulting value for $Q_{\max}$ on this basis may appear to be unreasonably great and out of proportion with practical experience. Therefore it may be wiser to select the maximum-production quantity to correspond with the value of λ for the economic quantity by using the chart on page 76, Fig. 13. A check on these two methods may be had by reversing Eq. (12) for $Q_{\max}$ and solving it for the rate of return that will result from the manufacture of the larger number of pieces as selected by the graphical method.

Normal Limits for the Economic Range of Production.—In most cases there will be sufficient latitude for planning production schedules if the maximum-production quantity be made the minimum-cost quantity and the minimum-production quantity the economic-production quantity, because within this range the normal expected rate of return can be maintained. Then only in extraordinary cases where it will be necessary to exceed the minimum-cost quantity may the more general limit as described in the previous paragraph be applied, with the full recognition of the fact that the normal return on capital will be reduced.

Temporary Corrections for Fluctuations in the Sales Demand.—Another phase of the situation arises when the production quantities are periodically revised upward on account of expanding business and increased manufacturing operations. Until such a time as the nature of the process is changed in order to take advantage of more effective methods of manufacture, made possible by the increase in the volume of business for a given product, the allowable increase in the minimum-cost quantity can be determined by the introduction of a factor f_s

[1] For the determination of a suitable value for r_m see p. 304, where r_m will be the smallest permissible value that can be legitimately assigned to r_u.

which depends upon the square root of the ratio of the new rate of consumption to the previous rate of consumption. To avoid any of the implied computations the value of this factor may be obtained directly from the curve given in Fig. 14. If one should stop to analyze further the effect of a change in the sales demand upon the production quantity it will be found that a certain limiting value will be reached beyond which it is inadvisable to go on account of the fact that the cost of capital employed in work in process as represented by the element K_w will assume such proportions as to overshadow the effect of the other two elements K_s and K_v.

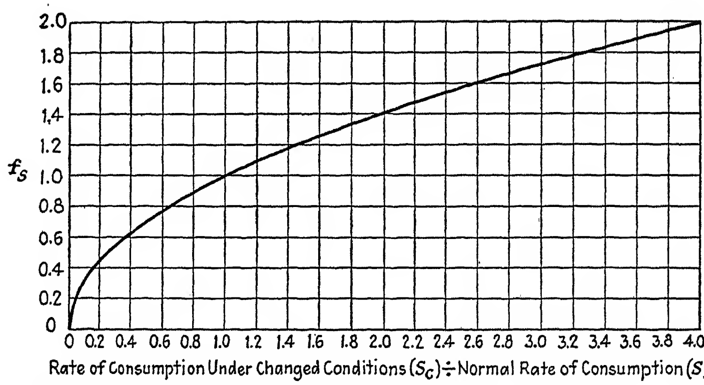

FIG. 14.—Correction chart for variations in the rate of consumption.

Practical Limits of Variation or Expansion.—This can be demonstrated graphically by reference to Fig. 15, where three curves have been plotted showing the change in values of the minimum-cost quantity for increasing rates of consumption. Curve *a* shows this effect upon these values obtained from the expression which depends upon all the elements in the denominator. Curves *b* and *c* show the similar effect when only the elements K_s or K_v or both and K_w, respectively, are employed in a simplified form. From an inspection of these three curves it becomes evident that the correction factor f_s can be used only to a limited degree when either the element K_s or K_v is the controlling factor, because the influence of the element K_w, which plays a more important part for large changes in the sales demand, has been disregarded. This is not serious because one can utilize the fact that the curve *a*, which represents the actual situation,

approaches as a limit the production quantity that would be employed if the element K_w had been the only one used in the denominator in order to establish a similar limit for this special case. In other cases the factor f_s will be found to be of use over a much larger variation in demand. Whatever the situation may be, it can be stated that this constant quantity which is represented by the curve c will be the practical limit for the maximum-production quantity, until such time when it becomes advisable to take advantage of the increase in the demand in order to obtain a reduction in cost through improvements in the product or process which could not have been previously achieved.

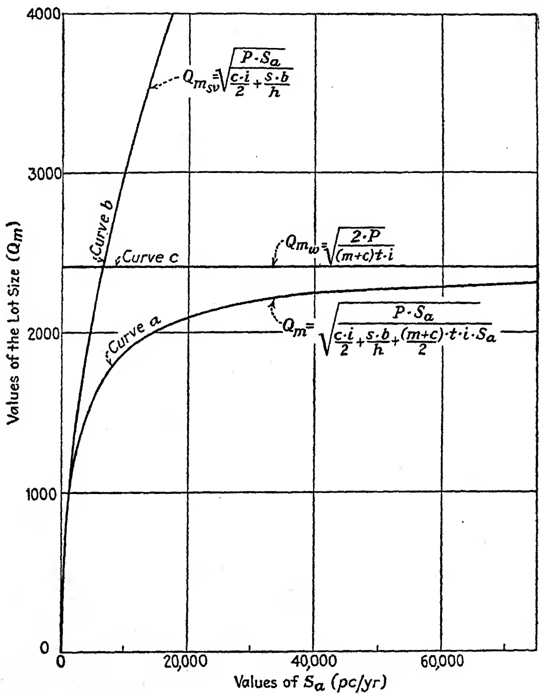

FIG. 15.—Effect of changes in the rate of consumption S_a upon the minimum-cost quantity Q_m as determined from the complete expression or one of its simpler forms.

Limiting Case for the Maximum-production Quantity.—The reason for this lies in the fact that the elements K_s and K_v are independent of the consumption rate and that the element K_w

is dependent upon it. When the element K_w is used alone in the formula for the production quantity, however, the consumption rate will appear both in the numerator and the denominator and may be cancelled out. As this is, from a mathematical point of view, identical with the condition which arises when the rate of consumption is large, the maximum quantity that should be produced in any one lot can be expressed by the relation where

$$Q_M = Q_w = \sqrt{\frac{P}{\frac{c + m}{2} \cdot i \cdot t_p}}.$$

Moreover, this limit will indicate the extent to which an increase in sales volume will benefit manufacturing operations, because if this situation can be brought into effect it becomes evident that the charges which naturally control lot production have been eliminated, and that only those which apply to continuous production remain. From this point on it is but a simple step to achieve the transformation whenever the consumption rate becomes equal to the production rate.

The Minimum-production Quantity.—In the foregoing discussion it has been assumed that the spread between price and cost will always be sufficient to yield the expected return upon the capital employed. When competition is keen, however, this may not always be possible, and then the only chance to obtain a reasonable gross return over any given sales period in view of the existing narrow margin of profit is so to adjust production schedules as to achieve the greatest turnover of the capital employed in these particular manufacturing operations that the conditions will warrant. In this situation it will be imperative to establish the practical limits for the minimum-production quantity. A formula has been derived for this purpose, based upon the fact that there is a certain lot size which will yield a maximum gross return consistent with the stipulated minimum sales price, and which also makes the necessary provision for the increased preparation charges resulting from the larger number of machine changeovers caused by the more rapid rate of turnover. The maximum rate of return in this case should not be confused with the expected rate of return previously employed, because it is only the maximum which can be obtained under very limited circumstances. The relation of the

controlling factors in this instance will be quite different owing to the assumptions made with regard to the narrow margin of profit, and will have no bearing whatsoever upon the economic balance which underlies the theory of the economic lot size.

The Economic-turnover Quantity.—To differentiate this quantity from the other production quantities it will be designated as the "economic-turnover quantity" and its value may be calculated from the expression

$$Q_t = \frac{P}{p_c - c} \cdot \left[1 + \sqrt{\frac{1 + \left[1 + \left(1 + \frac{C_f}{P} \right) \cdot \frac{S_a \cdot t_p}{150} + \frac{C_s \cdot b}{P \cdot s} \right] \cdot \left(\frac{p_c}{c} - 1 \right)}{1 + \frac{k_p \cdot t_p \cdot S_a}{300}}} \right] \quad (13)$$

where

p_c = price set by competition,

C_f = portion of fixed capital invested in manufacturing facilities utilized,

C_s = portion of fixed capital invested in storage facilities utilized,

and other symbols represent items used elsewhere in this book, an interpretation of which may be obtained by reference to Appendix XIII, page 349. No attempt will be made in connection with this study to explain further the principles upon which it is based as it would lead into a discussion that would be quite removed from the one now under consideration.

Adequate Information Assures Reliable and Flexible Control.—In summarizing the contents of this chapter it becomes evident that definite limits can be assigned to the quantities which can be profitably produced in any manufacturing lot, and that flexibility can be introduced to aid those in control of production schedules which will simplify departmental routine to a greater degree than would be possible when a single inflexible quantity must be employed. An economic range can be established within which any quantity can be produced for the same manufacturing advantage (that is, with respect to the expected return on the investment) that could be obtained for production at minimum cost, the limits of which are the economic quantity and the minimum-cost quantity. Moreover, if an extraordinary

situation exists, these limits may be exceeded in either direction as long as the lot size is neither so large or so small as to impair or prohibit the earning of a reasonable return. Under such circumstances a determination of the maximum- or the minimum-production quantity, whichever the case may demand, will indicate to what extent a change in production schedules will be of advantage or give warning that the earnings of the corporation will be adversely affected by unintelligent planning. In the latter case one would only resort to the economic-turnover quantity formula where the price spread obviously becomes the dominant factor.

CHAPTER VII

MISINTERPRETATION OF DATA AND ITS EFFECT

The adoption of economic-production quantities as a means to a better control of capital, inventories, and production has not met with immediate approval on account of the apparent absurdity of the results obtained from testing out some of the simpler formulae. The cause for this attitude among executives has for the most part been due to a misinterpretation of the data required in the formulae. In most cases the data have been taken directly from current cost and production records without sufficient appreciation of their actual composition and their ultimate relation to other values used in the same problem. This does not imply that accounting methods are wrong, but it does emphasize the fact that a survey of the accounting practice should be made to ascertain whether the method is adaptable to the formula, or whether it would be better to alter the formula where possible to suit the method. Duplication of cost items will cause one of the most serious errors, as will also the wrong classification of factors. Moreover, indefinite or too general definitions have accompanied some of the early expressions for the economic size of lot and have caused a misunderstanding of what was intended.

Misconception of the Interest Rate.—Few originators and users of formulae have ever had a true comprehension of the effects upon the final quantity of the value assigned to the item i representing the interest rate. The author has seen cases where values have been assigned ranging from the mere cost of capital taken at 6 per cent up to values as high as 30 per cent or even higher under special circumstances. In the beginning it was customary to use only the cost of capital, but it was soon appreciated that other items pertaining to the costs of doing business ought to be added to it. Such values, where they amount to 10 or 15 per cent, usually reflect the burden of taxes, storage space charges, depreciation losses, and allowances for obsolescence in

addition to the cost of capital. Following this came the use of values from 18 to 25 per cent into which were accumulated a multitude of similar charges under the heading of a percentage allowance for the risks of business. From this conception has grown the idea of the real importance for a special treatment of the problem upon the basis of the expected return upon capital employed.

Cost of Capital versus the Expected Return on Capital.—Production quantities obtained as a result of using the wrong value for the capital charges, have indicated that the lot size could be increased with ample assurance that the fundamental balance would create an economic situation. The fact is that the cost of capital i, when employed alone in the formula, yields quantities four times greater than they should be. Similarly, even when an allowance r for the risks of doing business is added to the cost of capital, that is, $i + r$, quantities can be obtained that are twice the real economic quantity. It is no wonder, then, that the practice of quartering or halving the results obtained from any of the previous expressions for the best lot size was adopted, as it was apparently necessary to bring the production quantity into line with that which experience would seem to indicate as being a more rational representation of the actual situation. This procedure manifestly is a contradiction of the principle that production at minimum cost is the most economical, and if the judgment of the individual is to be substituted for the facts which are an outcome of a scientific use of the economic balance between certain manufacturing costs, there is little to be gained from any use of a formula for the best lot size. This inclination on the part of executives to correct the production quantity can only be justified by the fact that the rate of capital turnover is of greater importance. If this is true, then any formula for the economic-production quantity must recognize this fact and incorporate the underlying principle in its derivation. Accordingly, as a result of the present investigation, it has been found that the expression for the minimum-cost quantity, which, incidentally, is based upon the cost of capital alone, should be corrected by a factor in the denominator which is equal to

$$f_r = 1 + 2\frac{r}{i} + \frac{r^2}{i^2} \cdot K_r$$

where

$$K_r = 1 - \frac{R_v}{R_c},$$

a constant depending upon the elements in the denominator, in order that production can be attained at the lowest cost consistent with an economical use of capital. Should it be found in the selection of the appropriate special form that the element represented by the index ratio R_v has but little influence, the ratio R_v/R_c can be assumed to be zero, and then $K_r = 1$, so that

$$f_r = \left(1 + \frac{r}{i}\right)^2.$$

If a formula is employed which contains this factor, there will be no longer any need for arbitrary adjustments, because the desired turnover can be attained for the fewest number of set-ups or production periods.

Factors Must be Treated According to Their Characteristics.—Turning to the general formula, Eq. (1),[1] which shows the relation of elements and major factors, it will be seen that all charges which are independent of the lot size appear in the numerator grouped under the preparation costs P, and that all those which are dependent upon the lot size are in the denominator and go to make up the various elements K_s, K_w, and K_v. Other charges[2] which have neither characteristic may legitimately remain in the distribution of overhead, or be incorporated with one of the constants used in the determination of the economic or minimum-cost quantities. In no case should any of the factors, which have either one or another of the chief characteristics, be included with the distribution of overhead except after a thorough analysis of both manufacturing operations and accounting practice has been made to ascertain that if so placed they will have no serious effect upon the results. Simplification of this order is quite justifiable, but it must be first determined for a given plant that such items are of little consequence.

Fallacy of Including Preparation Charges in the Direct Cost of Production.—The outstanding example of this type of error is contained in the accounting practice of including the machine changeover costs in the unit cost of production. The set-up and

[1] See p. 37, Table I.
[2] See p. 163.

dismantling costs which make up this item are naturally total charges against the production or job order and will be the same for any quantity produced. When these charges are reduced to a unit basis for the actual quantity produced and are added to the material, direct operating labor, and overhead cost per piece, the direct production cost c will be larger than it should have been had these latter three items been employed alone in its correct evaluation. This will result in a distortion of the ratio of the total charges to the sum of the unit charges which is the basis for any lot size determination. The effect of this is to reduce the economic quantity and uselessly increase the cost

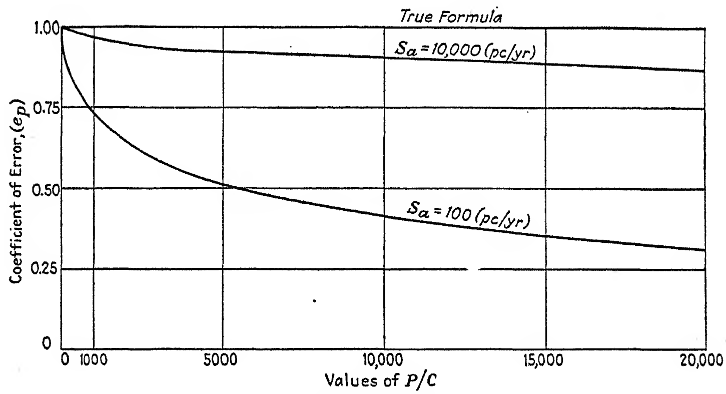

FIG. 16.—Deviations in the lot size due to the use of prime cost $c + P/Q$ in place of the direct unit production cost c, for varying rates of consumption S_a.

of production, all of which starts a vicious cycle, because if the total set-up and dismantling costs are again transformed to unit costs on the basis of the new quantity, a still smaller production quantity will be obtained, and the error will be cumulative (Fig. 16). Should the method of costing each unit of production be so arranged that these total charges on a unit basis are included in the cost of each piece, however, an analysis will show that in no case where the total preparation charges P are greater than twice the direct-operating cost c will it be permissible to include these charges with this item in the denominator because the final quantity will be reduced by more than 5 per cent. When the ratio of P/c and the annual rate of consumption are large, it will be possible, with due precaution, to make use of the accounting methods as they are at present and

avoid any need of change. The extent of the error thus incurred has been illustrated by Fig. 16, where the various curves for the coefficient of error e_p expressed in terms of the ratio P/c have been drawn for limiting values of S_a. The basis for these curves has been derived from the expressions

$$Q''_{m_s} = \sqrt{\frac{2 \cdot P \cdot S_a}{\left(c + \frac{P}{Q''_{m_s}}\right) \cdot i}}$$

for the case where the unit allotment of the preparation costs P/Q is included with the unit production cost c and

$$Q_{m_s} = \sqrt{\frac{2 \cdot P \cdot S_a}{c \cdot i}}$$

for the normal case, whereupon

$$e_p = \frac{Q''_{m_s}}{Q_{m_s}} = \sqrt{\alpha}\left[-1 \pm \sqrt{1 + \frac{1}{\alpha}}\right]$$

and

$$\alpha = \frac{P}{c} \cdot Z$$

where

$$Z = \frac{1}{8 \cdot S_a} \quad \text{(a constant).}$$

Errors Due to a Fixed Basis for Allocating the Total Preparation Charges.—A more serious situation arises when the total cost of machine changeover is reduced to a unit basis by dividing these charges by an arbitrary quantity, which has no relation to the economic lot size. This may be illustrated by the case where all such total costs were divided by 100 when the actual production quantity was over 100,000 pieces per lot. The error under these circumstances multiplies much more rapidly, and it is imperative, if any practical advantage is to be gained from the formulae, that such a practice be discontinued at once. If such a method cannot be avoided, the actual quantity produced should be used for costing purposes, provided that the limit imposed in the previous paragraph is made to hold. The deviation from the true quantity due to this ill-advised practice may be shown by comparing the simple relation for the economic-production quantity

$$Q_{m_s} = \sqrt{\frac{2 \cdot P \cdot S_a}{c \cdot i}}$$

with the equivalent relation where the unit allotment of the machine changeover cost has been made, on an arbitrary basis, q'

$$Q'''_{m_s} = \sqrt{\frac{2 \cdot P \cdot S_a}{\left(c + \frac{P}{q'}\right) \cdot i}}$$

in which q' is the quantity used for accounting purposes. The index or measure of the error in the economic quantity will be

$$e'_q = \frac{Q_{m_s}}{Q'''_{m_s}} = \sqrt{1 + \frac{P}{cq'}}.$$

The curve shown in Fig. 17, calculated from this expression, demonstrates the variation in the values of the error e'_q for values of the total preparation cost and varying fixed values of

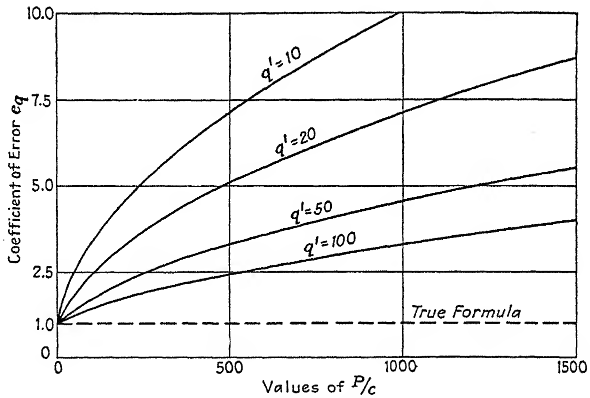

FIG. 17.—Deviations in the lot size due to an arbitrary basis q' for the unit allotment of the preparation cost P.

the arbitrary quantity q' taken as a percentage of Q'''_{m_s}, so that $q' = f'_e \cdot Q'''_{m_s}$ where the unit cost item c remains the same, as it naturally would for a given unit of production. It will be seen that when the values of q' are radically smaller than the actual quantity produced, the resulting economic quantity will be markedly decreased. The effect of this will be even greater when the value α' for the ratio of the preparation charges P to the actual direct unit production cost c is taken progressively smaller. The ratio $\alpha' = P/c$ for cases of low-cost production in large volume will be quite small, and here it is that any deviation from the best accounting practice will be most misleading.

Errors Introduced by Duplication in the Distribution of Overhead.—The current method of compiling the burden rates may be productive of errors when the usual values are employed in calculating economic quantities, if proper foresight has not been used to eliminate any chance of duplicating items in the various elements having different major characteristics. The most common source of such trouble arises when the total production-control costs for the year are included in the preparation costs P, on the basis of the average number of production or job orders issued in the year, and when, at the same time, they have not been removed from the overhead used for calculating the burden rates. Of course if it has been found that the cost of issuing and controlling each order is very small in comparison with the total machine changeover costs, then these charges can well be left in the general overhead, as they are of little significance with regard to the size of the lot. In some instances the actual machine changeover costs are charged to overhead, and, if this be so, the same precaution should apply to this item as well, because this case would be equivalent to that just described in the preceding paragraphs.

A similar situation may arise with regard to the charges for the space occupied by articles in stores normally included in the element K_v. When it is shown by the method of selection that the element K_v is of little importance, these charges may be retained in the general overhead, where they belong; however, if the index ratio R_v shows that the element K_v has considerable influence upon the production quantity, all items which go to make up the unit space charge v must be omitted when computing the burden rates for this purpose.

Misinterpretation of Prime Cost.—The author has found many divergent definitions for "prime cost" and for that reason has avoided any mention of it in this discussion. Basically, it is believed that prime cost for a unit of production should be that cost which represents all charges incurred during fabrication or assembly, including overhead distribution, as well as the cost of material at the time it enters the process, which have accumulated and should be credited to that article. In other words, it should be the value of the article at the instant it enters stores or has been transported to another department having just been removed from the process, regardless of its final destina-

tion. Some executives have argued that this value for inventory purposes should not include overhead, because the inventories should not be inflated with charges which are of a general administrative nature, and that they should be applied to the article only at the time when it is shipped from the plant on a sales order.

Fallacy of Omitting Overhead Expense in Evaluation of Inventories.—These executives state that they wish to be conservative and remain on the safe side. Is it a really honest policy to hide expenditures of an indirect nature in this way? What real accountant would fail to show the expenditure of funds on his balance sheet until such time as they are absorbed or offset by funds appropriated or received in return for the delivery of like value in material form? An expenditure is a debit against any account at the time it occurs and should be carried as such until the proper credits are made; how else will the books balance against facts, even if they can be made to do so numerically? An accumulation of material values in inventory represented by articles in stock is the same as the accumulation of real dollars that are stored in a bank. The only difference is that money has been transformed into materials, supplies, and labor, both direct and indirect, and its value remains, even though it does not appear as so many dollars upon the bank statement. This amount of money has gone into the product, in one way or another, and the total value of all units of production as they stand in inventory should be made to account for all expenditures whether on the payroll or in current bills and should exactly offset these payments. Any other policy is not correct, for it avoids the issue and may conceal undesirable conditions.

When it comes to economic quantities, the absurdity of this argument becomes even more apparent. The fact is that this policy is just the opposite to conservatism in its effect upon inventories, because if a smaller value is used for prime cost by leaving out overhead in evaluating inventories, the denominator of the economic-quantity formula becomes smaller than it ought to be and the quantity becomes greater, with the result that inventory values are inflated instead of being reduced according to the original intent. Therefore if reliable control is to be obtained and capital is to be conserved at the same time, the correct cost items should be used which will give a true cross-section of the financial aspect of manufacturing operations.

Fallacy of Including Storage Charges and Cost of Capital in Manufacturing Cost.—Charges arising from the storage of articles and the investment of capital in manufacturing operations are of a different nature and should not be added to the value of these articles as they enter stores inventories. As the storage charges accumulate during the period that the articles remain in stock and do not attain their full value until the storage period is ended, the logical time to add these charges to the value of the article is when they are removed from inventory for some definite purpose. The cost of capital employed in manufacturing operations is in reality a charge accruing to the cost of doing business and not directly a charge upon production no matter how it accumulates, whether through work in process or in stores, and for this reason it should be carried to the general administration expense account and not added to the article in any manner either upon entering or leaving stores. Accordingly, neither of these charges should appear in the overhead, even though they have an important bearing upon the economic quantity, except possibly in the case of the space charges, when the index ratio R_v indicates that the element K_v can be disregarded.

Errors Introduced by a Disregard of the Cost of Capital Invested in Work in Process.—Many of the earlier formulae for the economic size of lots failed to consider the cost of capital employed in work in process. As production methods become more closely coordinated with the sales policy and smaller inventories become practicable, the cost of capital in work in process assumes a greater importance, because the costs arising from storage of articles have been automatically reduced. A study of many industrial situations shows that under these conditions the economic quantity really depends on this factor as represented by the element K_w. If simplified forms are to be of advantage and are to yield reliable results where this situation exists, it may be advisable to use the forms Q_{e_w} and Q_{m_w}, as given in Tables III and IV, instead of the more general forms Q_{e_s} and Q_{m_s}, which are typical of the earlier expressions. Errors resulting from the use of an inappropriate form can be minimized if means are provided for the determination of economic quantities based upon all factors, even if some can be dispensed with after analysis of the conditions. Heretofore this has been impossible and this fact alone may go a long way toward

explaining why the use of economic quantities has not been favorably regarded in the past.

The Flow of Material Has Its Influence upon Costs.—The nature of the process as typified by the manner in which material flows from one operation to another as well as to and from the manufacturing areas has its influence as well. Again the earlier expressions failed to recognize that any difference existed except for the fact that the production period could overlap the sales period to a greater or lesser extent. The three types of processes, semicontinuous, batch, and non-continuous, have been defined in Chap. III[1] and their effect upon the economic quantity depends upon the degree of continuity in the flow of work through the process. The process factor f_p or the process constant k_p has been introduced to account for such variations. The closer manufacturing operations can be made to approximate continuous production the greater the continuity of flow will become and the smaller will be the value of all the elements, as less capital will be employed and less storage space will be required. Any other approach to the problem as provided by the earlier forms will yield inappropriate results.

Fallacy of Employing the Rate of Production.—All previous expressions for the economic quantity, where the overlapping of production and sales periods was provided for, contained a term that represented the rate of production and was used to correct the rate of consumption during the production period in order to account for those articles which, when removed from the process, could be diverted from stores and applied directly to fill current orders. The definition given for this term was very broad and gave no indication of its true purpose. The calculation of the actual rate of production for a process having a number of operations proceeding at different rates for each is not a simple task. Except in the case of continuous production it is entirely wrong to assume that the total number of pieces produced in a lot, divided by the total time consumed from the delivery of raw material at the first operation to the removal of the finished piece to stores, will give a representative figure for the production rate that can be used to offset sales. From the nature of its derivation it is absurd to expect that this total figure will equal the number of finished pieces that can be

[1] See pp. 29 and 201.

TABLE VIII.—CALCULATION SHEET FOR THE UNIT PROCESS TIME t_p FOR THE AVERAGE PIECE

Operation number	Data			Analysis							
	Process characteristics	Unit operation time t_o	Number n of batches	$\frac{1}{n}$	$\frac{t_o}{n}$	$\frac{\left(1 - \frac{1}{n}\right)}{2}$	$\frac{t_o}{2}\left(1 - \frac{1}{n}\right)$	Operation differentials	Transition differentials	Initial and final differentials	Unit process time t_p
1											
2											
3											
4											
5											
6											
7											
8											
9											
10											
Totals											

removed from the process as each is completed on the last operation. The correct figure to be used is the rate at which finished articles can be delivered to stores and not the rate of production. Only in the case where the rate of production is taken for the last operation can it be employed as the measure for the rate of delivery to stores. If this definition be used in collecting data for economic-quantity determinations in place of the one given above, one can be confident that the results will give a true picture of the conditions.

In like manner the rate of production will be misleading if used to determine the cost of capital in work in process, as it is not the total time T_p for all pieces that counts but the unit time t_p for the average piece multiplied by the lot size Q. The principles upon which the average unit time is determined will be discussed later in detail,[1] but it may be said here that it is equal to the sum of the reciprocals of the production rates for each operation properly corrected for the nature of the process and the flow of material. If this conception of process time is used, the common errors, which arise from a misinterpretation of this factor, may also be avoided. Since the data required for a determination of the correct time factor on which to base the investment charges on work in process are available in the records of the process engineers or cost accountants, and only need to be properly tabulated in a manner to permit a rapid computation of the necessary totals, a calculation sheet has been devised as shown in Table VIII which needs little explanation, as the arrangement of each column indicates the intended procedure.

Flexibility of the Economic Range Offsets Errors from Inaccurate Sales Estimates.—Much criticism of the value of economic-production quantities has arisen from the appreciation of unavoidable errors coming from the uncertainties of predicting the sales demand. Forecasting of future sales has been found in many industries to be of practical value and productive of surprisingly accurate data. In itself it has contributed a great deal toward the elimination of these errors and to a greater confidence in predetermined production schedules. In determining economic quantities it is absolutely necessary to know the probable rate of consumption in order to predict the minimum inventories necessary to correctly offset the preparation

[1] See p. 226.

charges. This does not imply in general that an accurate curve can be plotted for monthly, weekly, or even daily sales, but it does require a reasonably close estimate of the yearly demand. This interpretation simplifies the task of determining the rate of consumption, because the occurrence of seasonal fluctuations need not be introduced except in cases where they are vital to the business policy, and that leaves only the general trend of a specific business from year to year to be determined. In all concerns there is, of course, some record of the rate at which each part or product is consumed, and if this be corrected for any expected increase or decrease in the total sales, the desired rate of consumption for the ensuing year can be readily obtained, which will be quite satisfactory.[1] The rate of consumption is the best way of defining this item, as the sales demand may be only a portion of the total manufacturing requirements. Obviously a certain part may be used in a number of assemblies in varying multiples for each, and to this must be added the service requirements and any others that may arise, so that the actual figure used for the consumption rate may be very different from the actual sales rate.

Corrective Factor for Variable Sales Demand.—In cases where seasonal fluctuations in demand seriously affect production policies, it is possible to determine the characteristics for each period and introduce this figure as the value to be used for the stock factor k_s, instead of its normal value of $\frac{1}{2}$ which is employed when the demand is fairly uniform. A special section of Chap. XIV[2] has been devoted to a discussion of the method of handling variable or seasonal demand, and the reader is referred to it for further details. Therefore, if through the use of such a factor the mathematical relation can be made to follow more closely the actual net change in inventories due to these periodic variations, a degree of accuracy can be obtained which should remove many of the previous objections and make it possible for production executives to rely with greater confidence upon

[1] Under extraordinary circumstances when a period of peak production is known to be approaching, the lot size may be arbitrarily increased, provided that the quantity for production be kept within the economic range, or if this range must be exceeded, schedules should be adjusted according to the methods outlined in pp. 79 to 84.

[2] See Appendix IV, p. 323, for graphical method of determining the appropriate value of k_s.

the production quantities which have been determined in this manner. In fact, the idea of an economic range, which has been recommended in the beginning of this book, presupposes that a certain amount of error in the collection of the data can exist without appreciably detracting from the reliance which one should normally place upon the results computed on this basis. The latitude for adjustment in production schedules provided for by this range is sufficiently broad to counteract any reasonable inaccuracies which may arise from estimates of the sales demand.

The Fallacy of Joint Economic Quantities.—One fallacy which has been pursued largely by the more progressive type of executive has developed from the seemingly logical deduction that if the lot size can be determined individually for fabricated parts and assemblies, a joint economic quantity would be preferable if they could be properly combined. Such a situation would indeed be desirable, but these executives have overlooked the fact that this would demand an association of innumerable items, each with its own ramifications, which would tend to increase greatly the complexity of actually determining a representative production quantity. The deciding factor in this case is the fact that any lot-size determination must be restricted to the fabrication of a single piece or the assembly of a final product, because in each case it must depend upon the relation of various manufacturing costs to the costs arising from the storage of the article, in the form it has assumed due to the manufacturing process. To calculate a lot size for both fabrication and assembly would confuse the issue, as it can be demonstrated[1] that the economic balance for a combination of articles cannot produce as economical a manufacturing situation as could be obtained for each separately. If inventories are to be maintained at the smallest possible amount and the preparation charges prorated over the largest quantity, the lot size must be determined for each stage of manufacture separated by an interval when storage is required, in anticipation of the next stage. In the case where a certain part of the manufacturing operations, lying thus between two storage periods, consists in a process requiring both intermittent and continuous production owing to the relative capacities of the equipment employed in the processes, the correct lot size can be determined only by comput-

[1] See typical cases illustrated in Chap. VIII.

ing the unit production cost c on the basis of all such unit costs for the particular stage, whether they be derived from one type of process or another. It should be noticed, however, that the savings from economic-quantity production will be obtained only from that portion of the process which is intermittent. As a corollary to these facts one can state that the determination of economic lots should also be employed to indicate when and where, in the sequence of manufacture, appropriate intervals for storage, if required to smooth out production, can best be located.

Errors Due to an Incorrect Use of Special Forms.—Other errors may be incurred through an ill-advised use of any one of the simplified forms. Many cases can be found where the relation of the factors which govern a specific problem in one concern should be given a totally different mode of treatment when a similar problem arises in another, even though it be in the same general classification of industry. Losses from deterioration may be charged to overhead in one case, or carried as a separate item in the ultimate unit cost in another, or even disregarded as a whole. Similarly, obsolescence will be a source of error should the method employed to compensate for its effect fail adequately to account for the nature of its origin. The sum and substance of all this is that no one should attempt to apply any formula blindly, and that all situations should be carefully reviewed in order to minimize in a suitable manner the possibility for errors. If this has been properly done no further concern need be felt over the reliability and value of economic-production quantities. The purpose of this chapter is not to make the problem more difficult but to bring to light all the pitfalls, so that by a recognition of their causes the proper degree of accuracy can be attained and the simplest possible method adopted in order to avoid any unnecessary labor of computation.

Specifications for Data Can Conform to Current Accounting Practice.—For the most part ordinary methods of accounting will be entirely adaptable to the purpose of economic quantities, even though in some instances it may be advisable to segregate certain factors which otherwise would not be required, in order that a more comprehensive picture of the manufacturing operations can be obtained. This discussion does not contemplate a condemnation of accounting practice, but if cost accounting

is to be worth while and achieve its purpose by evolving facts which will aid production and indicate when savings and improvements can be made, certain revisions of current practice will be justified, and any broad-minded accountant should be willing to incorporate these into his system of accounting. The chief points to be borne in mind in introducing or revising production control upon an economic-quantity basis are that there are three major classifications of factors. If the conditions imposed by their respective definitions are adhered to, no difficulty should be encountered in obtaining the correct data for the calculations. Otherwise it matters little how the accounting system operates or how the burden rates are applied.

CHAPTER VIII

INDUSTRIAL APPLICATIONS

No matter how much care may be expended in the assembly and presentation of the logic in support of an hypothesis of merit, its acceptance by industry on a broad scale may be deferred for an unnecessary length of time by the lack of sufficient concrete evidence of its practical utility and adaptability. Industrial executives for the most part are practically minded men who are constantly dealing with problems of utmost importance to the daily success of the enterprise and have little time to devote to a thorough comprehension of the more or less theoretical aspects of business, no matter how valuable such an understanding may be. For them it will suffice in general if the soundness of any proposition can be simply and clearly demonstrated in terms with which their minds are familiar. Accordingly, their first impulse is to ask: can tangible results or savings be quickly realized; can it be easily applied to any particular case; and how much is it liable to increase administrative expenses and complicate routine? Such questions may best be answered by a few typical but simple examples drawn at random from industries where reliance upon economic lot sizes has proved in practice to be beneficial.

Problems to Be Illustrated.—Applications of the theory of economic lot sizes divide themselves into two major classes. The first has reference to the control of production schedules, reduction of inventories, and the conservation of working capital. The second has reference to the selection of the most economical process. In order that the routine of the production-control department may be maintained in its simplest proportions and operated at minimum expense, the use of special or simplified formulae will be included as a part of the illustration for the first case. In the selection of the best process, three alternatives may present themselves, and the illustrations have been selected so as to demonstrate the utility of analyzing manufacturing methods, production by progressive sequence, and minimum process time for the average piece.

Collection of Data.—In the collection of data for all these examples it was found that the cost records of each concern provided the requisite figures, the interpretation of which has already been given in Table V.[1] Naturally, some of the data was expressed in terms which did not lend themselves to the simplest solution of the problem, but in such cases it was a relatively easy matter to make the necessary adjustment. For example, if it were evident from the nature of a problem that the rate of consumption could be more conveniently handled when it was expressed as so many units per week, certain related cost items expressed in years could be adjusted by dividing them by the number of weeks in a year. Other items may likewise have to be transposed when the application of economic lots is first undertaken. This situation should incur no hardship upon those making the calculations, as it should be recognized that the formulae can be adapted to the most prevalent form of the data by an appropriate use of constants. Once these have been introduced, no further corrections need be made, provided that the data are always consistent.

Selection of the Appropriate Formula.—To illustrate the selection of the simplest and most appropriate form and the manner in which it may contribute to a rapid determination of the best lot size, an example has been used which came from a manufacturer whose product has a relatively large size in comparison with its unit cost and can be produced from raw material in a single operation. In following through the computations which were employed, it may be convenient to refer occasionally to the calculation sheets (Tables V, VI, and VII) as the item numbers and symbols used throughout the problem correspond with those appearing in these tables. Moreover, it should be remembered, in reviewing the computations for any one of the index ratios, that they were made with the aid of a common slide rule, since two significant figures have in general proved to be sufficiently accurate, thus avoiding the actual figuring.

[1] See p. 58.

Example 1:

DATA

Items	Symbols	Units	Data
Unit material cost.	m	\$/pc	0.0521
Unit production cost.	c	\$/pc	0.0683
Total machine changeover cost . .	M	\$	6.50
Total production-control cost per lot.	O	\$	0.48
Total preparation cost $M + O$. . .	P	\$	6.98
Unit production time.	t	day/pc	0.0014
Consumption rate.	S_a	pc/day	67
Interest rate.	i	$\%/\text{yr} \times \frac{1}{30{,}000}$	0.0002
Normal rate of return.	r	$\%/\text{yr} \times \frac{1}{30{,}000}$	0.0005
Space charge.	s	\$/cu. ft./day	0.000946
Unit volume or bulk of article. . . .	b	cu. ft./pc	0.0707
Height provided for storage.	h	ft.	10.0
Number of batches.	n	Semicontinu-	1/0 ∞
Process constant.	k_p	ous production	1
Rate of delivery to stores.	D	pc/day	4,000

I. SELECTION OF FORM

A. PRELIMINARY CALCULATIONS

Item 1. $$\frac{0.000946 \times 2}{0.0002 \times 10} = 0.9466 = \phi$$

Item 2. $$1 - \frac{67}{4{,}000} \times 1 = 0.98325 = f_p$$

Since the value of item 2 is greater than 0.90, this factor may be disregarded and assigned a value of unity instead.

Item 3. $$\frac{0.0521}{0.0683} + 1 = 0.763 = f_m$$

B. COMPUTATION OF RATIOS

Item 4. $$\frac{0.0707}{0.0683} = 1.03$$

Item 5. $0.9466 \times 1.03 = \quad 0.975 = R_v$

Item 6. $67 \div 1 = 67$

Item 7. $1.763 \times 67 \times 0.0014 = \quad 0.1653 = R_w$

Item 8. (a constant) $= \quad 1.0 = R_s$

Item 9. $R_s + R_w + R_v = \quad 2.1403 = R_c$

Item 10. $\frac{2}{3}R_c = 1.4268$

By inspection of the index ratios R_s, R_w, and R_v, it is evident that the only two which, when combined, will yield a value greater than $\frac{2}{3}R_c$

are the index ratios R_s and R_v. Accordingly, the appropriate special form of the equations for the economic-production quantity and the minimum-cost quantity should contain the elements K_s and K_v, as shown by the equations for $Q_{e_{sv}}$ and $Q_{m_{sv}}$ in Tables III and IV, respectively. Since the policy of this company is to permanently reserve storage space for each article placed in stock, the bin factor k_b has been omitted, and since the process constant can be disregarded, the formula for the minimum-cost quantity may be written as

Item 11. $$Q_{m_{sv}} = \sqrt{\frac{P \cdot S_a}{\frac{c \cdot i}{2} + \frac{s \cdot b}{h}}}.$$

Determination of the Permissible Variation.—Now in the selection of this special form an arbitrary value, $\frac{2}{3}$, for the form index f_c was employed. In order to determine whether the nature of the problem required a greater or lesser degree of accuracy, the correct value for the form index was calculated to see whether a simpler form than that given above could be employed with sufficient reliability.

II. DETERMINATION OF THE FORM INDEX AND THE ALLOWABLE VARIATION IN THE LOT SIZE

Item 12. $$2.1403 \times 0.0002 \times 2 = 0.000856$$

Item 13. $$\frac{67}{0.000856} = 78{,}260$$

Item 14. $$\frac{0.0683}{6.98} = 0.00979$$

Item 15. $$78{,}260 \times 0.00979 = 766.165$$

Item 16. $$\sqrt{766.165} = 27.68$$

Item 17. $$1 + 27.68 = 28.68 = K_o$$

Item 18.

Turning now to the chart in Fig. 13,[1] the correct value for the form index may be determined by reading on the lower scale for f_c the value which lies directly below the point of intersection between the horizontal line drawn through the value of k_o just computed and the curve for λ which represents the maximum permissible percentage increase in unit costs.

The experience of this company has shown that a value of λ which equals 0.0026 (or 0.26 per cent) will provide ample latitude in the selection of special forms, and this is confirmed by the fact that in this case the actual increase in the unit cost amounts only to $0.00018 if computed upon the value of the direct-production cost c which, as a check, is permissible because this item naturally accounts for the major part

[1] See p. 76.

of the minimum ultimate unit cost U_m. For the most part executives have agreed that it is reasonable to incur this very slight increase in cost in order not to overburden the production-control clerks with unnecessary computations. Accordingly, if these values for K_o and λ be employed, the resulting value of $f_c = 0.463$ may then be multiplied to the value of R_c to obtain the limiting or minimum value for the index ratio of the appropriate element. Hence

Item 19. $$0.463 \times 2.1403 = 0.9909 = f_c R_c.$$

If this be the case, the conditions underlying this problem will permit the use of the single element K_s because its index ratio, $R_s = 1$, is just greater than the limiting value given by item 19. Therefore the simplest of all forms can be employed as illustrated by the expressions for Q_{e_s} and Q_{m_s} in Tables III and IV, where

$$Q_{m_s} = \sqrt{\frac{2 \cdot P \cdot S_a}{c \cdot i}}$$

and

$$Q_{e_s} = \sqrt{\frac{2 \cdot P \cdot S_a}{c \cdot i \cdot \left(1 + \frac{r}{i}\right)^2}}$$

since the factors f_p and k_b may be disregarded.

Determination of the Limits of the Economic Range of Production.—Having thus selected the form, it was then possible to compute[1] the lot size or the limits of the economic range.

III. Determination of the Limits of the Economic Range of Production

A. The upper limit or the minimum-cost quantity Q_m

$$Q_{m_s} = \sqrt{\frac{2 \times 6.98 \times 67}{0.0683 \times 0.0002}}$$
$$= \sqrt{68{,}471{,}400} = 8{,}274 \text{ pieces per lot.}$$

Since the error of approximation is given by the ratio of the index ratios R_s to R_c, the exact value for the minimum-cost quantity can be ob tained from

$$e = \frac{R_s}{R_c} = \frac{1}{2.1403} = 0.467$$

whereupon

$$Q_m = \sqrt{e} \cdot Q_{m_s}$$

or

$$= 8{,}274 \times 0.683 = 5{,}651 \text{ pieces per lot.}$$

[1] When all problems are found to have similar characteristics, a special slide rule may be constructed having the appearance of that shown in Fig. 1, which will greatly facilitate this portion of the calculations also.

B. THE LOWER LIMIT OR THE ECONOMIC-PRODUCTION QUANTITY Q_e

By reference to Tables III and IV in Chap. V, it will be noticed that the equations for Q_{e_s} and Q_{m_s} differ only by the presence of the parenthesis $\left(1 + \frac{r}{i}\right)^2$ in Q_{e_s}, therefore the approximate value for the economic-production quantity can be obtained from the relation

$$Q_{e_s} = Q_{m_s} \cdot \frac{1}{\sqrt{\left(1 + \frac{r}{i}\right)^2}}$$

or

$$= \frac{8{,}274}{1 + \frac{0.0005}{0.0002}} = \frac{8{,}274}{3.5}$$

$$= 2{,}362 \text{ pieces per lot.}$$

There was little need to calculate the exact value for the lower limit since it could be demonstrated[1] that any approximation in this instance would lie within the economic range, and sufficient latitude was provided by the approximate value to permit a reasonable degree of flexibility in the control of production by means of which the desired conservation of capital could be achieved.

The Utility of the Minimum-cost Quantity.—The utility of the minimum-cost quantity as a means for the selection of the best manufacturing process can be illustrated by a simple problem which arose in one industry with regard to the possible advantage to be gained from combining three distinct steps into one in the manufacture of a certain product. The final measure in this case was obtained from a comparison of the combined minimum ultimate unit costs for each step taken separately, and that for the process as a whole.

A Simplified Expression for the Minimum Ultimate Unit Cost.—In order to shorten the calculations in this type of problem, the general expression for the ultimate unit cost was simplified by the introduction of the fact that at the minimum-cost point an equality exists between the unit allotment of the preparation costs and the unit cost of capital and storage charges. As a result the expression given in Eq. (63)

$$U_m = u' + \frac{P}{Q_m} + f \cdot Q_m,$$

[1] See p. 54 or p. 296.

where F has been assumed to be unity, and Q_m has been inserted for Q, could be rewritten in the form

$$U_m = c + 2 \cdot \frac{P}{Q_m}$$

since the terms k_{v_s} and k_{v_w} have little or no influence in this case and because, if

$$Q_m = \sqrt{\frac{P}{f}}, \qquad \text{See Eq. (65).}$$

$$fQ_m = \frac{P}{Q_m}.$$

Selection of the Best Process.—The necessity for a consideration of the three steps separately in this problem was due to the fact that the piece fabricated in the first step must be combined with other distinct pieces fabricated in the second and third steps. In fact the method of manufacture required that the various parts must be fabricated and assembled at the same time, each in its proper order. It will be noticed as evidence of this fact that in the data given below the material costs for the second and third steps both contained the accumulated values from the previous operations, together with the value of the raw material which was to be fabricated and assembled with the parts coming from the preceding steps.

Example 2:

DATA[1]

Items	Units	Step 1	Step 2	Step 3	Totals
m	\$/pc	0.0038	0.0077 0.0134	0.0127 0.1255	0.0242
c	\$/pc	0.0134	0.1255	0.1699	0.1699
P	\$	14.85	2.79	0.60	18.24
T	wks/pc	0.000165	0.000168	0.000240	0.000573
S_a	pc/wk	4,050	4,050	4,050	4,050
i	$\% \times \frac{1}{5{,}200}$	0.00153	0.00153	0.00153	0.00153
f_p	non-continuous production	1	1	1	1
Working time	wks/yr	52	52	52	52

[1] Symbols only have been employed in the remaining examples as their interpretation has been given in the first one and a full description of each one can be found in Table V.

The complete data have not been given, since it was evident that the items omitted would have no influence upon the final conclusions because the index ratio R_v proved to be very small and there was no intention in this case of computing the economic-production quantity. Accordingly, for the purposes of this problem, a reasonable approximation was obtained for the minimum-cost quantity by the use of the form Q_{ms} from Table IV.

A. THE MINIMUM-COST QUANTITY FOR ALL THREE STEPS COMBINED

$$Q_{m_{sw}} = \sqrt{\frac{2 \times 18.24 \times 4{,}050}{[0.1699 + (0.0242 + 0.1699) \times 4{,}050 \times 0.000573] \times 0.00153}}$$
$$= \sqrt{192{,}124{,}800} = 13{,}860 \text{ pieces per lot.}$$

B. THE MINIMUM-COST QUANTITY FOR EACH STEP IN THE PROCESS TAKEN SEPARATELY

a. Step 1:

$$Q_{m_{sw}} = \sqrt{\frac{2 \times 14.85 \times 4{,}050}{[0.0134 + (0.0038 + 0.0134) \times 4{,}050 \times 0.000165] \times 0.00153}}$$
$$= \sqrt{3{,}157{,}086{,}600} = 56{,}200 \text{ pieces per lot.}$$

b. Step 2:

$$Q_{m_{sw}} = \sqrt{\frac{2 \times 2.79 \times 4{,}050}{[0.1255 + (0.0211 + 0.1255) \times 4{,}050 \times 0.000168] \times 0.00153}}$$
$$= \sqrt{65{,}694{,}700} = 8{,}100 \text{ pieces per lot.}$$

c. Step 3:

$$Q_{m_{sw}} = \sqrt{\frac{2 \times 0.60 \times 4{,}050}{[0.1699 + (0.1382 + 0.1699) \times 4{,}050 \times 0.000240] \times 0.00153}}$$
$$= \sqrt{6{,}768{,}800} = 2{,}601 \text{ pieces per lot.}$$

C. THE MINIMUM ULTIMATE UNIT COSTS OF EACH OF THE ABOVE QUANTITIES

a. All Steps Combined:

$$U_m = 0.1699 + 2 \times \frac{18.24}{13{,}860} = 0.1725 \text{ \$/pc}$$

b. Step 1:

$$U_m = 0.0134 + \frac{2 \times 14.85}{56{,}200} = 0.0139 \text{ \$/pc}$$

c. Step 2:

$$U_m = 0.1255 + \frac{2 \times 2.79}{8{,}100} = 0.1262 \text{ \$/pc}$$

d. Step 3:

$$U_m = 0.1699 + \frac{2 \times 0.60}{2{,}601} = 0.1703 \text{ \$/pc}$$

Now if the cumulative unit cost for the three steps taken separately was to be correctly determined, only the manufacturing costs incurred in each step could be combined with the initial raw

material values to obtain the final joint cost. The reasons for this will be more fully explained in a later chapter[1] where the basic theory has been enlarged upon. Accordingly, the actual cumulative unit cost was found to be

$$0.0139 + (0.1262 - 0.0134) + (0.1703 - 0.1255) = 0.1715 \text{ \$/pc.}$$

Upon a comparison of the minimum ultimate unit cost for the process as a whole and the cumulative unit cost for the three steps in the process, each being operated separately upon a minimum unit-cost basis and allowing for the necessary storage of parts between each step as well as that which is required at the end of the process, it became evident that production could be most economically arranged when each step was scheduled independently. The resulting annual savings in this case amounted to

$$(0.1725 - 0.1715) \times 4{,}050 \times 52 = \$210.60.$$

Joint Economic Quantities Not Economical.—It should be noticed that this problem is really one dealing with joint economic quantities and supports an earlier contention[2] that joint economic quantities do not yield so low an ultimate unit cost as might be supposed. This is due to the fact that articles are stored at a time when they have accumulated their total unit value, and not at such times as when inventory values are less, due to the storage of a majority of the parts in a semicompleted form.

Progressive Sequence in Manufacture.—Economic-production quantities open up several other avenues for improving the methods of manufacture. If the production of articles requiring similar machine set-ups can be arranged in the sequence of fewest tool changes or machine adjustments, the ultimate unit cost for each article in the group can be reduced advantageously. Moreover, if a complex process involving an interrupted flow of material can be analyzed so that the actual unit production time for the average piece is the shortest possible, further savings can be achieved as well. A simple problem can be introduced here to demonstrate these cases, where products A and B required similar set-ups and also took an almost identical time to process.

[1] See p. 165.
[2] See p. 20.

Example 3:

DATA
In part only

Items	Units	A	B
m	\$/pc	0.046	0.037
c	\$/pc	0.250	0.240
P	\$	14.00	14.00
S_a	pc/wk	1,500	2,500
t	wk/pc	0.000288	0.000288
i	$\% \times \frac{1}{5{,}000}$	0.0016	0.0016
Working time	wks/yr	50	50

Originally the process was operated as non-continuous, that is, each operation was scheduled with no relation to the preceding or succeeding ones. Studying this situation first, the savings due to a progressive sequence of manufacture can be demonstrated. The minimum-cost quantity[1] of each article produced separately was

$$Q_{m_A} = \sqrt{\frac{2 \times 14.00 \times 1{,}500}{[0.25 + (0.046 + 0.25) \times 1{,}500 \times 0.000288] \times 0.0016}}$$
$$= \sqrt{69{,}482{,}400} = 8{,}335 \text{ pieces per lot for product } A.$$
$$Q_{m_B} = \sqrt{\frac{2 \times 14.00 \times 2{,}500}{[0.24 + (0.037 + 0.24) \times 2{,}500 \times 0.000288] \times 0.0016}}$$
$$= \sqrt{100{,}000{,}000} = 10{,}000 \text{ pieces per lot for product } B.$$

Special Adaptation of the General Formulae.—To compute the minimum-cost quantity for both products fabricated in sequence, a readjustment of the general formula had to be made, because in this case the combined unit costs U_{m_c} must be a minimum and since they were to be produced in sequence the sales and production periods of each must coincide.

Hence if,

$$T_{S_A} = T_{S_B}$$

or

$$\frac{Q_{m_A}}{S_A} = \frac{Q_{m_B}}{S_B}$$

$$Q_{m_B} = \sqrt{\frac{2 \cdot P \cdot S_B^2 \cdot \frac{1}{S_A + S_B}}{[c_c + (m_A + c_A) \cdot S_A \cdot t_{p_A} + (m_B + c_B)S_B \cdot t_{p_B}]i}}$$

[1] In this case as well the special form $Q_{m_{sw}}$ was found most suitable.

where

$$c_c = c_A \frac{S_A}{S_B} + c_B,$$

and S_A and S_B represent the average daily demand S_a for the products A and B, respectively.

Accordingly, when the data was inserted into this expression:

$$Q_{m_B} = \sqrt{\frac{2 \times 14.00 \times (2{,}500)^2 \times \frac{1}{1{,}500 + 2{,}500}}{\left\{\left(0.25 \times \frac{1{,}500}{2{,}500} + 0.24\right) + [(0.046 + 0.25) \times 1{,}500 + (0.037 + 0.24) \times 2{,}500] \times 0.000288\right\} \times 0.0016}}$$

$$= \sqrt{\frac{43{,}750}{0.001147}} = 6{,}176 \text{ pieces per lot for product } B.$$

Then

$$Q_{m_A} = Q_{m_B} \frac{S_A}{S_B}$$

$$= 6{,}167 \times \frac{1{,}500}{2{,}500} = 3{,}705 \text{ pieces per lot for product } A.$$

Now the ultimate unit costs for each under the two methods of manufacture were:

a. As originally produced

$$U_{m_A} = 0.25 + \frac{2 \times 14.00}{8{,}335} = 0.2533 \text{ \$/pc.}$$

$$U_{m_B} = 0.24 \times \frac{2 \times 14.00}{10{,}000} = 0.2428 \text{ \$/pc.}$$

b. When produced in sequence following a single set-up, the minimum unit cost must be computed from the special formula, where

$$U_m = c + \frac{P}{Q_{m_A} + Q_{m_B}} + \left[\frac{c \cdot i}{2S_a} + \frac{m + c}{2} \cdot t_p \cdot i\right] Q_m$$

so that

$$U_{m_A} = 0.25 + \frac{14.00}{3{,}705 + 6{,}176} + \left[\frac{0.25}{2 \times 1{,}500} + \left(\frac{0.046 + 0.25}{2}\right) \times 0.000288\right] \times 0.0016 \times 3{,}705$$

$$= 0.2521 \text{ \$/pc.}$$

and

$$U_{m_B} = 0.24 + \frac{14.00}{3{,}705 + 6{,}176} + \left[\frac{0.24}{2 + 2{,}500} + \left(\frac{0.037 + 0.24}{2}\right) \times 0.000288\right] \times 0.0016 \times 6{,}176$$

$$= 0.2422 \text{ \$/pc.}$$

The annual savings then, due to progressive sequence of production, were

$$
\begin{array}{lll}
 & 0.2533 & \\
 & \underline{0.2521} & \\
\text{for } A & 0.0012 \times 1,500 \times 50 = & 90.00 \\
 & 0.2428 & \\
 & \underline{0.2422} & \\
\text{for } B & 0.0006 \times 2,500 \times 50 = & \underline{75.00} \\
\text{Total savings} & & 165.00 \text{ \$ per year.}
\end{array}
$$

The Unit Process Time for the Average Piece.—When the process was further analyzed, it was found that certain operations which occurred on different machines could be made to overlap. Accordingly, the procedure shown in Table VIII[1] was employed to compute the unit process time for the average piece as shown in the following table, where the operating times are expressed in terms of 10,000 pieces.

Example 4.—On this new basis of scheduling the various operations, by which the unit production time for the average piece was reduced to 0.000222 week, the production quantity for each of the articles in the preceding problem became

$$
Q_{m_B} = \sqrt{\frac{2 \times 14.00 \times (2,500)^2 \times \dfrac{1}{1,500 \times 2,500}}{\left\{0.25 \times \dfrac{1,500}{2,500} + 0.24 + [(0.046 + 0.25) \times 1,500 + (0.037 + 0.24) \times 2,500] \times 0.000222\right\} \times 0.0016}}
$$

$$
= \sqrt{\frac{43,750}{0.001027}} = 6,530 \text{ pieces per lot for product } B
$$

and

$$
Q_{m_A} = 6,530 \times \frac{1,500}{2,500} = 3,918 \text{ pieces per lot for product } A.
$$

The ultimate unit cost for these respective minimum-cost quantities was then computed by the method previously employed. Hence,

$$
U_{m_A} = 0.25 + \frac{14.00}{6,530 + 3,918} + \left[\frac{0.25}{2 \times 1,500} + \frac{(0.046 + 0.25)}{2} \times 0.000222\right] \times 0.0016 \times 3,918
$$

$$
= 0.2520 \text{ \$/pc}
$$

[1] See p. 95.

Example 4:

DETERMINATION OF THE UNIT PROCESS TIME t_p FOR THE AVERAGE PIECE

Operation number	Data			Analysis							
	Process characteristics	Unit operation time t for 10,000 pcs in wks	Batch number n	$\frac{1}{n}$	$\frac{1}{n} \cdot t$	$\frac{\left(1 - \frac{1}{n}\right)}{2}$	$\left(1 - \frac{1}{n}\right)\frac{t}{2}$	Operation differentials	Transition differentials	Initial and final differentials	Unit process time for average piece t_p for 10,000 pcs in wks
1	Non	0.520	1	1	0.5200	0	0		0.1520	0	
2	Semi	0.304	1/0	0	0	1/2	0.1520	0.0565	0.2085		
3	Semi	0.417	1/0	0	0	1/2	0.2085				
4	Non	0.500	1	1	0.5000	0	0		0.2610		
5	Semi	0.522	1/0	0	0	1/2	0.2610	0.0840	0.1770		
6	Semi	0.354	1/0	0	0	1/2	0.1770			0	
7	Non	0.263	1	1	0.2630	0	0				
Totals					1.2830			0.1405	0.7985	0	2.2220

and

$$U_{m_B} = 0.24 + \frac{14.00}{6{,}530 + 3{,}918} + \left[\frac{0.24}{2 \times 2{,}500} + \frac{(0.037 + 0.24)}{2} \times 0.000222\right] \times 0.0016 \times 6{,}530$$

$$= 0.2421 \text{ \$/pc.}$$

The annual savings due to progressive sequence and the overlapping of operations was

$$\begin{array}{lll}
 & 0.2533 & \\
 & \underline{0.2520} & \\
\text{for } A & 0.0013 \times 1{,}500 \times 50 = & 97.50 \\
 & 0.2428 & \\
 & \underline{0.2421} & \\
\text{for } B & 0.0007 \times 2{,}500 \times 50 = & \underline{87.50} \\
\text{Total savings} & \qquad\qquad\qquad = & 185.00 \text{ \$ per year.}
\end{array}$$

Thus, if savings of some reasonable proportions could be achieved in a manufacturing problem as simple as this one where the two units of production were so nearly alike, this type of analysis should indicate the manner in which much larger savings can be obtained from the adoption of progressive sequences in production and the overlapping of operations in more complex cases. Larger savings still could have been realized in this instance had it been possible to operate the whole process semicontinuously.

Total Annual Savings the Final Measure.—Throughout this study of practical applications of the theory of economic-production quantities, the purpose has been to demonstrate the use of various methods and the manner of employing certain equations in order to obtain facts which will aid executives in a more precise control of manufacturing operations. The problems chosen to illustrate the technique involved should be viewed not so much as outstanding examples of the successful application of the theory but as sample calculations employed to bring out the interplay of the various factors in the simplest possible manner, and to aid any one in approaching a particular problem in his own plant. If the annual savings be used as a basis of judging the utility of economic lot sizes, it should be remembered that the savings achieved in one instance, even though they may appear to be unreasonably small when compared with the total cost of conducting all the manufacturing operations, give no

indication of the ultimate advantage gained when the savings for the whole plant are considered. For example, in a large electrical manufacturing plant the savings per unit of product averaged around $100 a year, but the manufacturing schedules covered a total of 10,000 out of a total of 100,000 items to which economic lots could be successfully applied. On this basis the total annual savings for the plant amounted to $1,000,000, a sum the magnitude of which should demonstrate the value of a scientific approach to manufacturing problems.

PART II

THEORY OF ECONOMICAL PRODUCTION

CHAPTER IX

HISTORICAL ASPECTS

An appreciation of the fact that, in the planning of production schedules, there was one lot size which, under a given set of conditions, would be the best to manufacture dates back as far as the inception of scientific methods of management. It was not long after the time when systematic methods of production control, based upon the principles of Frederick W. Taylor, had demonstrated their utility that the early supporters of his theories realized that the final cost of any unit of production actually depended upon the size of the production lot wherever a repetitive manufacturing process was involved. Owing to the fact that such a process required the setting up of a sequence of machines and later their dismantling each time there was occasion to manufacture an additional supply of a particular unit of production, and that in the intervals between these manufacturing periods a certain number of such completed units had to be retained in stock in anticipation of future orders, the eventual cost of producing and storing each unit could be made a minimum, when a balance had been established between the unit allotment of the machine changeover cost and the charges incurred by storage including the cost of capital, through the selection of an appropriate lot size.

Types of Intermittent Processes.—The discovery of these facts was a logical outcome of a new policy for lower costs, reduction of inventories, and a more rational balance between production schedules and sales demand. In the beginning the manner in which the best lot size was determined was in most instances extremely elemental, as no one had taken the opportunity to study the underlying factors which actually govern the situation. One of the earliest instances which gives evidence of a true appreciation of these factors may be found in the records and writings of George D. Babcock, during his connection with the installation of the Taylor System in the plant of the H. H. Franklin Manufacturing Company in Syracuse,

N. Y. In analyzing the production problems of this plant he classified the various types of processes as indicated by orders and work according to:[1]

a. One order for one piece never to be reproduced.
b. One order for several pieces never to be reproduced.
c. Repeat orders at irregular intervals for one or a few pieces.
d. Repeat orders at irregular intervals for many pieces.
e. Repeat orders at uniform intervals for one or a few pieces.
f. Repeat orders at uniform intervals for many pieces.
g. Continuous orders for the same piece.

First Attempt to Evaluate the Economic Lot Size.—From the foregoing analysis it is an obvious conclusion for him then that where repeat orders are involved an economical state of production will be dependent upon the lot size in order to regulate the frequency of these recurrences in accordance with the number of pieces required. Early in 1912[2] standard lot sizes were established at the Franklin plant for all units of production which could be manufactured by means of a repetitive or intermittent process. Apparently Mr. Babcock worked out a mathematical basis for establishing the correct lot size for a given set of conditions, but decided that the use of formulae as a matter of practical utility in a planning department was of little advantage because cubic equations had to be employed in order to yield satisfactory results.

Elemental Factors in Lot Size Determinations.—Accordingly, in order to provide some simple means for guidance in the selection of the best lot size which could be profitably utilized by any production-control division, he recommended the following procedure as later summarized by him in "Management's Handbook:"[3]

Small lots are indicated by:
High cost of material.
Large unit bulk of material.
Long operation time required for a part.
Probability of design change.
Reduction in inventory.
Conservation of floor space.

[1] *Iron Age*, p. 1068, Nov. 5, 1914.

[2] See Babcock, G. D., "Taylor System in Franklin Management," p. 125, Engineering Magazine Company, New York, 1917.

[3] Published by The Ronald Press Company, see Sec. 12, p. 637.

High unit weight of material.
Use of perishable materials in process.
Early entry of part into subassembly.

Large lots are indicated by:
High average cost of machine set-ups.
High dispatching, inspection, and trucking charges.
Rapid production, even with elaborate set-up.
Reduction of spoilage.

In general all preparation charges should be satisfied with largest possible lots.

A Simplified Form of the Franklin Formula.—It was not long afterward that evidence was given, through published material, that simple mathematical formulae can be devised which will yield approximate results for the best lot size and which are quite suitable for use by any subordinate in the production-control division when supplemented by the use of graphs. A simplified form of the more complex formula of Mr. Babcock was employed by D. B. Carter[1] at a slightly later date at the plant of the Automatic Refrigeration Company in Hartford, Conn., and may be found under Mr. Carter's name in Table IX, which has been compiled to show the evolution of economic lot formulae.

The Earliest Formula for the Economic Lot Size.—Earlier, however, printed records show that in 1915 Ford W. Harris[2] was employing, at the Westinghouse Electric and Manufacturing Company, a simple formula which depended upon three variables only. In deriving this equation Mr. Harris quite correctly assumed that the best lot size was the one for which the final cost of any unit was a minimum. In justifying his conclusions he avoids higher mathematics, however, and assumes that economical production can be attained when the unit allotment of the set-up costs are equal to the carrying charges of the stock of the specific article in completed form. The type of his equations may be represented by

$$Q = \sqrt{\frac{M \cdot S}{c} \cdot k},$$

where the symbols have the same definition as employed throughout this treatise and where the constant k includes the cost of

[1] See Bibliography, p. 357, Appendix XIV.

[2] See "Operations and Cost," pp. 48–52, McGraw-Hill Book Company, Inc., 1915.

capital, interest, and depreciation as an item of 10 per cent, together with certain other constants imposed by the specific definitions of the variables employed.

Necessity for Assumptions and Corrective Factors.—The formula developed by Mr. Harris is the simplest one that has yet been devised, but, as the practice of utilizing economic lots has grown, it has been found that under certain circumstances it will be totally deficient. He, himself, points out that it must be assumed for practical purposes that the sales demand must be uniform, whereas it is obvious that it never will be. In so many cases it is so nearly uniform, however, that he recommends the introduction of a factor to account for it if need be. This is only typical of many other similar situations. The fact is that no single formula, unless it comprehends all contributory factors, can be universally applicable, and then in all probability it will be too complex for practical advantage. As Prof. George F. Mellon[1] of Pennsylvania State College so aptly puts it, no formula of this kind should be used indiscriminately without knowing its derivation and limitations. In other words, each situation should be given individual consideration, and it will be seen in the course of this chapter that this has been the general practice. Nevertheless, bearing out his contention the time has come when this mass of experience can be collected and consolidated, in order that a sound method can be established which is universal and will then lend itself to simplification on a logical basis to suit the requirements of each special case.

Growth of the Basis Theory.—The subsequent development since the inception of economic lots has shown a trend in this direction and a tendency to introduce other factors which have an important influence, either upon the basic theory or upon the final form of the expression to be used in a specific instance.

1. It became evident that the cost which should be made a minimum ought to contain certain factors which had been disregarded or included erroneously in overhead.

2. A controversy arose among accountants over the inclusion of interest as an item of cost, and that raised the question as to just what cost figure should be employed for the evaluation of inventories.

[1] See Bibliography, Appendix XIV, p. 357.

3. It became the general practice to include, with the interest rate in computing the carrying charges on inventories, factors which took into account insurance, rent, taxes, depreciation, light, heat, storage costs, financial expenses, and, later, obsolescence and deterioration.

4. It was realized that the flow of material through an intermittent process could be handled in a manner similar to that which would be employed could the processes have been operated continuously, thus permitting the manufacturing period to overlap the succeeding sales or storage period, for which a correction could be introduced with a considerable saving in the general expense as a result.

5. It was for a time believed that reserve or emergency stocks had an influence upon the lot size.

6. There arose a desire to eliminate, if possible, the necessity of employing the rate of consumption or sales demand in the formula in order to avoid errors in estimating future sales requirements.

7. Objections were raised over the fact that economic lots often indicated as economical an increase in inventory values when experience and financial facts showed conclusively that an increase in capital turnover was paramount.

8. Owing to a higher degree of accuracy required it became necessary to coordinate economic lot size determinations with data available in and conforming to current accounting practice.

9. As lot size computations multiplied it became the belief that these could be materially reduced if economic quantities could be determined jointly for products involving similar methods of manufacture or for fabrication and assembly combined, thus leading to a more comprehensive picture of all manufacturing operations.

10. With all of these complications many executives have shown an inclination to avoid formulae entirely and translate them into charts, graphs, or slide rules and introduce such mechanical devices into their methods of control.

Examples of Typical Formulae.—Thus the theory of economic lot sizes has received impetus and has been developed until there are extant some thirty-eight or more individual attempts to devise a suitable formula, as well as numerous tables, charts, and other mechanical devices, not to mention untold schemes

TABLE IX.—EQUATIONS EMPLOYED TO SHOW THE STAGES IN THE DEVELOPMENT OF FORMULAE FOR ECONOMIC LOT SIZES

First appearance	Form	Authorities	Approx. date of record
1912	Cubic equation not published	G. D. Babcock	1912
1915	$Q = \sqrt{\frac{P \cdot S}{c} \cdot k}$	F. W. Harris	1915
		D. B. Carter	
		General Electric Co.	
		J. A. Bennie	1922
		P. E. Holden	1922
		K. W. Stillman	1923
		Benning and Littlefield	1924
		J. M. Christman	1925
		G. H. Mellen	1925
1917	Special adaptation	Eli Lilly & Co.	1917
1917	$Q = \sqrt{\frac{P \cdot S}{c \cdot i} \cdot k}$	S. A. Morse	1917
		W. E. Camp	1922
		Holtzer Cabot Co.	1924
		N. R. Richardson	1927
	$Q = \sqrt{\frac{P \cdot S \cdot k}{c \cdot (i + f_i)}}$	H. T. Stock	1923
	where	R. C. Davis	1925
	f_i = allowances for insurance, storage costs, etc.	B. Cooper	1926
		G. Pennington	1927
		E. T. Phillips	1927
		W. L. Jones	1929
		J. W. Hallock	1929
1918	$Q = \sqrt{\frac{P \cdot S \cdot D'}{c \cdot i\,(D' - S)} \cdot k}$	E. W. Taft	1918
		F. H. Thompson	1923
		R. C. Davis	1925
		J. M. Christman	1925
		B. Cooper	1926
		G. Pennington	1927
		E. T. Phillips	1927
		W. L. Jones	1929
		J. W. Hallock	1929
1918	$Q = -\frac{P}{c} + \sqrt{\frac{P^2}{c^2} + \frac{P \cdot S \cdot D'}{c \cdot i (D' - S)} \cdot k}$	E. W. Taft	1918
		G. Pennington	1927
1923	$Q = \sqrt{\frac{P \cdot S \cdot k}{c \cdot i \cdot f_d + \frac{s \cdot b}{h} \cdot k'_b}}$	F. H. Thompson	1923
		R. C. Davis	1926
		C. N. Neklutin	1929
1924	$Q = \sqrt{\frac{P \cdot S \cdot k}{c \cdot i + (m + c) \cdot t_p \cdot S \cdot i}}$	A. C. Brungardt	1923
		P. N. Lehoczky	1927

NOTE.—For specific references see Bibliography, Appendix XIV, p. 357. For an interpretation of symbols see Appendix XIII, p. 349.

which have been copied from these. To show the course of the historical background it is well worth while to note the changes as they appear in the form of the formulae proposed, and henceforth both here and in Table IX the symbols[1] employed in each typical expression will conform to those employed elsewhere in this treatise, as there is little to be gained by reproducing their original interpretations. The first in its more general form, as exemplified by that developed by Mr. Harris about 1915, may be expressed by

$$Q = \sqrt{\frac{P \cdot S}{c} \cdot k},$$

where k represents a constant which includes the interest rate i normally assumed to be 6 per cent. By 1918 it has become more general practice not to include the interest rate in the constant k but to introduce the symbol i in the denominator, as shown by

$$Q = \sqrt{\frac{P \cdot S}{c \cdot i} \cdot k},$$

where a value for i can be selected by executive decision to conform to the policy of the specific concern.

Changes in Attitude toward the Interest Rate.—Normally the value assigned to the term i was that which was employed in determining the cost of capital even though Mr. Harris earlier than 1915 proposed to include in it an allowance for depreciation. Not until after 1923 did other items such as allowances for insurance, rent, taxes, and storage costs appear. About 1926 B. F. Cooper[2] at the West Lynn works of the General Electric Company, not content with these items alone, added to them an allowance for obsolescence and deterioration. The ultimate effect of all these extra allowances is in reality a recognition that interest alone is not the sole factor in determining the cost of capital. If this line of reasoning be carried too far for the sake of conservatism, fallacious conclusions may be drawn from the results, because if an allowance is made for items of rent, heat, light, storage, etc., which are purely of factory origin and should for that fact be included in the burden distribution rate, and if they are not previously removed from overhead, a double charge will be incurred without justification. Actually, the interest

[1] For an interpretation of symbols see Appendix XIII, p. 349.

[2] See Bibliography, Appendix XIV, p. 357.

rate should be increased only by those items which relate to general business costs and financial obligations, and which can be prorated to the product as it proceeds through the various phases of manufacture in this manner alone.

Correction for Semicontinuous Production.—In the meantime two marked changes in form took place, both of which were introduced by E. W. Taft[1] of the Winchester Repeating Arms Company of New Haven, Conn., about 1918. The first was the result of the realization that some recognition should be made in the formula of the savings which resulted from the overlapping of the manufacturing period and the subsequent sales period. This could be done by correcting the quantity produced for those units which were diverted to current orders so that the investment charges would be computed only on the basis of that quantity which actually reached stores. Under these circumstances the simple formulae previously employed became

$$Q = \sqrt{\frac{P \cdot S \cdot D'}{c \cdot i \cdot (D' - S)}},$$

where in this case D' was erroneously considered as the rate of production and not the rate of delivery to stores D as is now believed to be more accurate.

Changes in the Basic Theory.—The second change introduced by Mr. Taft relates to the basic theory. Heretofore the unit cost which was to be made a minimum was composed of the unit manufacturing cost and the unit investment charges as illustrated by

$$U'' = \left[c + \frac{P}{Q}\right] + \left[\left(c + \frac{P}{Q}\right) \cdot \frac{Q}{2} \cdot \frac{1}{S} \cdot \left(\frac{D' - S}{D'}\right)\right].^{2}$$

Mr. Taft believed that if a true economic lot size was to be determined it was not sufficient to base the cost of capital upon an evaluation of inventories which depended upon the unit manufacturing cost $c + \frac{P}{Q}$ alone, but that the total unit cost U'' should be used in preference, so that

$$U'' = \left[c + \frac{P}{Q}\right] + U'' \cdot \frac{Q}{2} \cdot \frac{1}{S} \cdot \frac{(D' - S)}{D'}.^{2}$$

[1] See Bibliography, Appendix XIV, p. 357.

[2] For an interpretation of symbols see Appendix XIII, p. 349.

This is the equivalent of saying that the unit cost of capital should be employed with the unit manufacturing cost in evaluating inventories, in the same manner that C. H. Scovell[1] has recommended its use in actual accounting practice. The result of this method of treatment is to compound the cost of capital in each step of the manufacturing process and in so doing the formula for the economic lot size becomes

$$Q = -\frac{P}{c} \pm \sqrt{\frac{P^2}{c^2} + \frac{P \cdot S \cdot D'}{c \cdot i \cdot (D' - S)}},$$

which Mr. Taft has termed the exact method of solution.[2] Owing to the complexity of this equation he develops an approximate solution, identical with that previously employed which in this study has been styled the practical solution, from the above equation by a process of deduction based upon the fact that the portion of U'' which really governs the lot size is for the most part the definite unit production cost c, because the remainder representing the influence of the investment charges is more appropriately introduced when it is employed to offset the preparation charges. As production executives dislike complex formulae the exact method of solution has been rarely if ever adopted by others.

Introduction of the Space Charge Element.—From this point up to 1923 or 1924 little change in the common practice of computing the lot size appeared, even though much had been written on this subject in that period. About 1923 the Dennison Manufacturing Company[3] developed a formula under the direction of F. H. Thompson into which a new and distinct cost factor had been introduced in addition to that for the investment charges on articles in stores. This factor recognized the fact that the cost of the storage spaces should depend upon the bulk of the product and not upon its value. Previously it had been believed reasonable to include such charges in overhead, but in the case of the Dennison Company where they manufacture paper boxes and other articles, the unit value of which is quite small in proportion to the space required to store them, the

[1] See "Interest as a Cost," published by The Ronald Press Company, 1924.

[2] Plus sign only can be used as the lot size can never be negative.

[3] See Thesis by H. T. Stock, Harvard Graduate School of Business Administration, 1924.

space charges were found to be of almost equal importance to the investment charge in determining the lot size. Accordingly, their formula for the lot size includes a new element in the denominator as shown by

$$Q = \sqrt{\frac{P \cdot S \cdot k}{c \cdot i + \frac{s \cdot b'}{h} \cdot k_b}},$$

which for convenience was transposed so that certain constant factors in k and a correction for the overlapping of the manufacturing and sales periods, as described in a previous paragraph, would appear outside the radical. Following this, R. C. Davies,[1] then of the Ohio State University, similarly recognized in 1925 that these space charges should not be included in overhead, but at that time he preferred merely to treat them as a unit-cost item of the same order as the unit production cost c. Later, in 1926, he proposed a formula in which the denominator was substantially the same as that given above, and gave excellent arguments in support of the method first introduced at the Dennison Company, though apparently having no direct knowledge of their practice.

Introduction of the Cost of Capital Employed in Work in Process.—At about the same time that the Dennison formula was developed A. O. Brungardt[2] of the Walworth Company evolved a method of determining the economic lot size which included the cost of capital invested in work in process as well as that derived from articles in stores. In this case it was evident that, if inventories of work in process were as large as inventories of finished parts or products, the equality between the preparation costs and the total cost of capital must take into consideration the investment charges on both classes of inventories. In giving expression to this relationship Mr. Brungardt assumed that, since the inventory of work in process was almost equal to the inventory of finished articles in stores, the time over which the investment charges accrued to any unit of production was the same whether that article was undergoing production or awaiting in stores consumption on some future order. As a result

[1] See Bibliography, Appendix XIV, p. 357.

[2] See Thesis by H. T. Stock, Harvard Graduate School of Business Administration, 1924.

the formula expressing this situation has an appearance similar to those developed in the earlier stages, even though it actually depends upon two distinct elements[1] of the problem. Shortly thereafter Paul N. Lehoczky,[2] of Ohio State University, developed a formula which was based upon the total cost of capital, except that he neglected to consider that portion of the capital invested in manufacturing operations which appears as the accumulation of value to any unit of production through the application of labor and the distribution of overhead on direct operating time. In 1926 Mr. Cooper[2] also based his formula upon the total cost of capital, but in computing that portion incurred by investment in work in process he did not appreciate that the time which elapses between the beginning and the end of the manufacturing process is actually a function of the lot size. These formulae can be represented by the composite expression

$$Q = \sqrt{\frac{P \cdot S \cdot k}{c \cdot i + (c + m) \cdot t_p \cdot S \cdot i}}.$$

An Attempt to Develop a Universal Formula.—In 1927 the author proposed a formula[3] for economic lot sizes which was based upon the contention that if it was to be universal in its application, and since the Walworth Company and the Dennison Manufacturing Company had reason to introduce, in the first case, the investment charges on work in process and, in the second, a charge for the space occupied by an article in stores, it is logical to combine these with the investment charge on articles in stores and equate their total to the preparation charges in order to obtain the true economic balance. In so doing it was found that the time factor for the investment charges on work in process could not be justifiably assumed to be equal to the time factor for those charges on articles in stores, as it depended primarily upon the rate of production and not upon the rate of consumption, except by coincidence. Even this basis has now proved to be incorrect owing to an inability to specify a representative rate of production for a process composed of many distinct operations. Accordingly, it became evident that a general formula would have to be composed of three groups of

[1] See Chaps. XIV and XV.

[2] See Bibliography, Appendix XIV, p. 357.

[3] See "Economic Production Quantities," A.S.M.E. *Trans.*, vol. 50-MAN, No. 10, p. 65, 1928.

terms in the denominator and that the work in process element must depend upon the unit process time for the average piece.

Adaptations to Suit Specific Requirements.—In the last few years a number of other formulae have appeared but in no way have they altered the basic theory. For the most part, recent endeavor along these lines has been to give a better interpretation by a revision of the data required, so that it may conform to the actual conditions arising in a specific plant. The form of such formulae may be different, but no new features have been developed since then until the present approach to the subject got under way. However, during the last decade, some interesting variations to the general theory appeared which are well worth a moment's digression to note. Early in this period Eli Lilly of Eli Lilly and Company[1] of Indianapolis, Ind., developed formulae similar to those in use about 1917, except for the fact that special consideration had to be given to the direct labor applied to the lot as a whole in distinction to that which could be attributed to each unit of production. Accordingly, the labor cost had to be expressed by the equation

$$\frac{L}{Q} = \frac{\alpha' \cdot L'}{Q} + \beta' \cdot l',$$

and then that portion of $\frac{L}{Q}$ represented by $\frac{\alpha' \cdot L'}{Q}$, as it could not be made a constant unit charge, naturally had to be included with the unit allotment of the preparation costs P/Q. Owing to this fact separate expressions for each unit of production were evolved.

Corrections for Changes in the Sales Demand.—Another interesting phase of the subject appeared in an expression used by N. R. Richardson at the Electric Controller and Manufacturing Company in Cleveland, Ohio, to adjust for abnormal changes in the sales demand when standard lot sizes had been established by the usual means. This formula can be illustrated by

$$Y = \frac{P}{\dfrac{(1+x)P}{Q} + x \cdot (m + l)},$$

[1] See Bibliography, Appendix XIV, p. 357.

where

Y = the actual quantity to be processed,

x = the increase in cost in percentage due to the processing of a lot Y varying from the standard lot Q owing to a change in the sales demand,

and when the other symbols conform to their respective definitions as employed elsewhere in this treatise.

Another Basis for the Minimum-cost Quantity.—Still another approach to the determination of the point of minimum cost has been proposed by Dean D. S. Kimball[1] of Cornell University which does not contemplate the equality between preparation costs and investment and other similar charges. In this case the minimum cost is determined by finding the minimum point of the locus of the point of intersection between the general total work curve $y = \alpha x^2 + \beta x + \gamma$ and the curve of unit costs $y = \dfrac{\alpha' x + \beta'}{x}$. Here the point of minimum cost is expressed in terms of the number of men who can be most economically set to work to accomplish a specific task. Other variations have appeared such as the recent tendency not to express the point of minimum cost as the basis of the best lot size in terms of the quantity to produce, but in terms of either the equivalent monthly requirements or the number of set-ups or capital turnover periods required for economical production.

The Reserve Stock Factor.—In 1922 J. A. Bennie[1] introduced a factor to account for stock on hand when the new lot is placed in stores, which by an assumption that it is normally equal to one half of the monthly requirements disappears from his final formula. In 1925, however, R. C. Davis[1] actually introduced a factor which is carried through even into his final formula as a correction for whatever articles are maintained as a reserve against emergencies. The logic of this device is questioned by the author in his discussion of this factor in Chap. XVII[2] because an item of this nature presupposes a constant charge which in reality is not a factor of the unit cost so much as it is a penalty against the management for a lack of proper coordination between sales and production.

[1] See Bibliography Appendix XIV, p. 357.

[2] See p. 262.

Summary of the General Development.—In looking back over this period of development during which the value of economic lot-size determinations has become recognized, it should be noticed that a general change has taken place in the basic theory. Originally it was believed that it was sufficient merely to obtain an equality between the unit allotment of the preparation cost P/Q and the unit allotment of the investment charges accruing to articles in stock. It was recognized not long afterward, however, that this situation could be more adequately expressed if the minimum point of the curve of total unit costs representing the summation of all charges which were incurred in the manufacture and storage of a given article was determined by differentiating the equation for this curve and equating it to zero; a procedure quite commonly employed in a mathematical analysis of any engineering problem. Since the adoption of this method of approach, the chief divergence in the theory has come from a difference in opinion as to the factors which should be employed in the expression to be differentiated mathematically. In some instances it has been believed that it is only necessary to obtain in this manner the minimum point of the summation of the total preparation cost and investment charges, whereas in other instances it was believed to be better to evaluate the investment charges in terms of the total unit cost which was to be made a minimum. Similar formulae have been derived by a lengthy process of reasoning which depended upon the introduction of infinitesimal changes in various factors, and then, by making them infinitely small, a limiting value can be obtained from the original expression which is equivalent to that for the lot size which can be produced at a minimum cost. No matter how acceptable some of these methods may at first appear, however, it will become evident in later chapters, where this portion of the general problem is given a more thorough treatment, that the only sound basis for the best lot size will depend upon the minimum point of the cost curve which includes all cost items accruing to any product from the time that the raw material enters the process to the time that the finished article is removed from stores for some specific purpose.

Inadequacy of Existing Formulae.—A number of years ago it became obvious to the author that a better understanding of manufacturing operations could be achieved through a more

comprehensive study of economic lots, as by this means it seems probable that the very fiber and structure of the purpose of all manufacture could be more intelligibly exposed, so that any unappreciated or hidden cost factors could be revealed to the benefit of industry. With this purpose in mind a fundamental formula[1] was devised in 1927 which incorporated all phases of the economic lot-size problem. Even then it did not seem to be complete, as many errors were continuously appearing which canceled the desired results, due to a general lack of appreciation of the scope of the problem when applied in industry, a misinterpretation of data, or a disagreement between accounting practice and the methods previously employed in computing the production quantity.

Necessity for Conserving Capital Resources.—Chief among these difficulties seemed to be the irreconcilable situation which arose when economic lots demanded an increase in inventory values and financial experience dictated the reverse. In other words, could production at minimum cost be justified along with a proper conservation of capital resources? No consideration has been given to the financial aspect of the problem heretofore, as it relates to the interest of the owners of the business or the margin of profit which must exist between sales price and factory cost of goods sold. An important article[2] prepared by Prof. Warren K. Lewis of the Massachusetts Institute of Technology appeared in June, 1923, covering this phase of the subject, but it has been apparently overlooked by all who have approached this problem in the meantime. Unfortunately, Professor Lewis did not translate his theories on a mathematical basis into a form which could be employed in a formula for determining the economic lot size. In the reconstruction of the basic theory of minimum cost, however, means were discovered, as described in the next chapter, for evaluating the unit margin of profit so that the financial policy of a concern can be definitely linked up with the manufacturing and sales policies in order that conservation of capital can be achieved, which has led to the surprising revelation that production at minimum cost is not economical.

[1] See "Economic Production Quantities," A.S.M.E. *Trans.*, vol. 50-MAN, No. 10, p. 65, 1928.

[2] See Bibliography, Appendix XIV, p. 357.

CHAPTER X

PRICE, PROFIT, AND PRODUCTION

The various expressions which have been developed up to the present time for determining the economic size of production lots have for the most part been based upon the simple economic balance between the preparation cost and some combination of the investment and storage charges. At best the production quantities obtained from them can be only an approximation to the true minimum-cost quantity. None of these expressions just reviewed can be justifiably adopted as a general formula that can be universally employed, because in no way do any of them provide for all the possible contingencies which might arise in any industry. This is not surprising, when one stops to consider the circumstances under which these expressions were probably evolved. The originators of the various formulae were only interested in the development of some simple means which would be readily adaptable to the production problems of a specific company, and were naturally little concerned with their broader aspects.

Need for a Basic Formula for the Economic Lot Size.—The situation in the case of the present treatment of this subject is quite the reverse, because it is the intention of the author to make an exhaustive study and reveal if possible all of the factors which conceivably might, at one time or another, influence the determination of the correct lot size so as to insure that the manufacturing operations may be carried on in the most economical manner. Even though abbreviated forms of the general expression can be employed to advantage in a given manufacturing plant, the relationship of all factors must first be unerringly set forth in order to avoid any misconception of the fundamentals. Only after this has been satisfactorily accomplished should one attempt to apply any simplified forms, for then only can it be definitely assured that the form selected will contain the appropriate elements. Accordingly, the remainder of this treatise will be devoted to the derivation of a fundamental formula which

can be reliably employed in solving production problems on an economic basis.

Reconciliation of Financial, Sales, and Manufacturing Policies. If the demands of the financial executive for conservation of capital and a reasonable return on the investment in manufacturing operations and those of the sales executive for an adequate supply of attractive and diversified lines of products are to be combined with the objectives of the production executive as the basis for a truly progressive business policy, it can be shown that production at minimum cost will not be the most economical. Minimum-cost quantities are of utmost importance in the determination of the best lot size, but they should not be used indiscriminately because in many cases it will be found that they will not contribute to the success of the business enterprise in so large a measure as might be expected, especially when net profits are the ultimate goal of the investor and business manager. The true economic-production quantity will be that quantity, as earlier defined in this book, which can be produced at the lowest ultimate unit cost consistent with an economical use of capital, taking into consideration all factors which together constitute an economic balance.

The Point of Minimum Cost Illustrated.—The difference between the minimum-cost quantity and the economic-production quantity can be demonstrated graphically by reference to Fig. 18 where the curve U represents the ultimate unit cost[1] for varying lot sizes Q and attains its minimum value U_m at the point Q_m. Now if a horizontal line be drawn through some point on the curve where the unit cost U_u will be naturally greater than that at the minimum point, the difference between U_u and U_m will represent the increase in cost which has been incurred by the manufacture of a lot Q_u either greater or lesser than that for which a minimum unit cost can be obtained. By reversing the point of view for a moment, it will be seen that this increase in the unit cost is in reality a loss, because the unit margin of profit which could have been realized from the sale of an article whose component parts have been produced at minimum cost will be reduced by an equal amount.

Relation of Price, Cost, and Unit Margin of Profit.—To illustrate this further, if that portion of the minimum unit selling

[1] See definition Chap. II, p. 16, or text, Chap. XII.

price p'_s which can be attributed to one of the component parts be introduced by drawing another horizontal line across the chart from that point on the scale of ordinates having a corresponding value, the maximum margin of profit for the unit of production will apparently be represented by the spread R_m/S_y[1] between the unit sales price p'_s and the minimum ultimate unit cost U_m for a desired rate of return r. Similarly, the unit margin

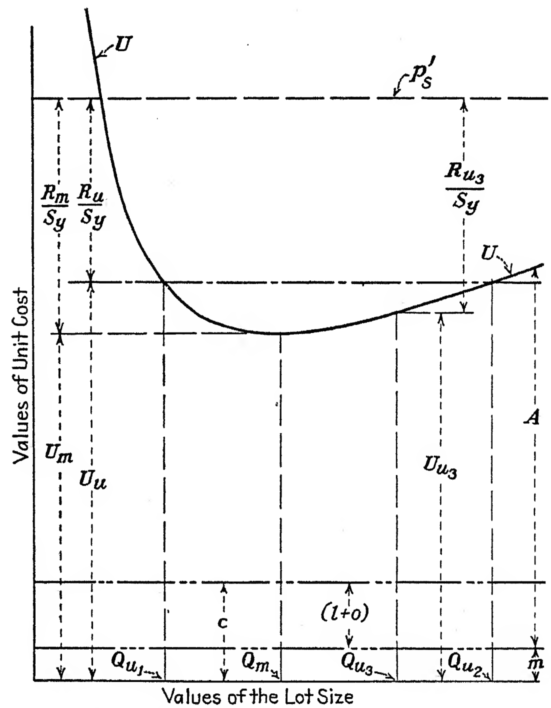

FIG. 18.—The relation of price, margin of profit, and the ultimate unit cost for the minimum-cost quantity Q_m and any other production quantity Q_u.

of profit that can be realized for the part when produced in any quantity Q_u other than that for which the cost is a minimum will be R_u/S_y.[1] Owing to the fact that the ultimate unit cost curve U rises more rapidly for lot sizes smaller than that for minimum cost, than it does for those which are larger, it is quite evident, if Q_u be assumed less than Q_m, that the expected margin of profit would be more seriously impaired should the difference between

[1] For a detailed interpretation of these symbols see text, p. 139.

these two quantities be of any sizeable amount.[1] If capital is to be conserved, the true economic quantity ought to lie in the portion of the chart to the left of Q_m. If such a quantity is to be employed as a basis for production schedules, however, the advantages to be gained from a smaller investment of capital cannot be lost by the selection of a quantity for which the margin of profit will be insufficient to yield the rate of return r which is normally expected as a reward for undertaking the risks of business.

The Economic-production Quantity Illustrated.—Now if a quantity smaller than the minimum-cost quantity is going to be the most economical one to produce, it must be a quantity of such proportions that the rate of capital turnover can be so increased as to yield the same gross return over an equal period of time, and for which the manufacturing advantage will be the same as that realized for the minimum-cost quantity. If this can be done it will be found that the portion of the gross return which must be earned by each unit of production in the smaller lot can be less than that which must be earned by a unit produced at minimum cost, owing to the fact that less capital need be employed in the first instance. Accordingly, the economical quantity which will properly conserve capital will be that one where the legitimate reduction in the unit margin of profit from that at minimum cost will just offset the increase in the ultimate unit cost which has caused the apparent loss.

Economy in Production Must Not Impair Earnings.—If this were not true it would be obvious that capital invested in manufacturing operations would be best employed when production is scheduled so as to attain a minimum ultimate unit cost. Fortunately, this is not the case because when the unit margin of profit itself becomes undesirably small, one can utilize the fact that the gross return for the year upon the capital employed can be maintained at a satisfactory amount if it be possible to increase, at the same time, the rate of capital turnover. This situation can be well illustrated by the following exaggerated example in order to show more clearly the interplay of the various factors.

[1] Compare the cases in Fig. 18 where $Q_m - Q_{u_1}$ is equal to $Q_{u_3} - Q_m$, where it will be evident that $R_{u_1}/S_y < R_{u_3}/S_y$.

Assume that \$220,000 worth of business is done in a year on an investment of \$100,000 upon which a return of 10 per cent is earned. The gross profit then will be \$20,000 as the capital is turned over twice a year. Now let the lot size be reduced to a point where the same amount of yearly business can be done on an investment of only \$25,000. Again assuming that a greater return than 10 per cent cannot be earned on account of the severity of competition, the profit in this latter case on each turnover of capital will naturally be only \$2,500. The size of this lot will permit an inventory turnover of eight times a year instead, however, and this will bring the gross profit for the period back to the original \$20,000. Obviously the smaller quantity will be the best lot size because capital can be conserved without diminishing the gross return through higher production costs. If these facts can be introduced into the problem so as to achieve a similar situation, one will be able to determine accurately the economic-production quantity.

Minimum Unit Sales Price.—This will require a considerable amount of analysis of the contributory factors on a mathematical basis, in order to obtain an expression for the economic-production quantity which will be comparable with the previous expressions for the minimum-cost quantity, aside from any analysis which should be made of the elements entering into the determination of the actual quantity for minimum-cost production. To evaluate the relations which have been expressed graphically in Fig. 18, some means must be devised for splitting up the minimum unit-sales price p_s for each item of production into its components which will apply either to the assembly as a whole or to each of the fabricated parts entering into its construction. This is made necessary by the fact that the economic lot size for an assembly process will in all probability not be the one that is the most economical for the fabrication of each component part or the purchase of raw material, and that if it be so it is only a coincidence. Accordingly, in the final solution for the economic-production quantity each phase of manufacture must be treated separately.

Elements of the Sales Price.—If all the items which are a part of the commercial and financial costs of doing business are

grouped into a single term B, a general expression may be written where

$$p_s = \frac{B + R_t}{S_y} + U_f \qquad \text{See Eq. (14)}$$

if

R_t = the gross return[1] on the capital employed during the year in the manufacture of a specific product, which is in contrast to

R_t/S_y = the unit margin of profit.

U_f = the ultimate unit factory cost for the product and is equal to the summation of the ultimate unit costs arising from assembly U_a, fabrication of parts U_p, and the purchase of raw materials U_r, less those charges incurred in each manufacturing phase which must be carried over into the cost of doing business B and a correction for items which otherwise would be erroneously duplicated, and

S_y = the total annual consumption or sales demand.

The Commercial Cost of Doing Business.—Before proceeding further it must be recognized that the cost of doing business B is composed of two distinct items: first, those which can be placed in a common group B', because they are derived from sources which have no direct bearing upon the manufacturing operations; and second, those which represent the total investment charges I_t or the cost of capital invested in inventories and work in process applicable to a specific product. It becomes apparent, therefore, that those factors of general business expense which have been included in the term B' will have no influence upon the return R_t to be earned on the manufacturing operations and can be disregarded henceforth in this discussion. Accordingly, that portion of the minimum sales price[2] which depends upon production may be expressed in general by p'_s in Eq. (16), Table X, and it is this value which has been used in drawing the upper horizontal line in Fig. 18. If the terms U_f and I_t be combined into a single term U_t [see Eq. (17)] which will represent the total ultimate unit cost of all phases of manufacturing a specific product, the relation between the unit margin

[1] See text, p. 140.

[2] See text, p. 143.

of profit and the unit costs for both the minimum-cost quantity Q_m and any other quantity Q_u can be similarly expressed by

$$p'_s = \frac{R_{t_m}}{S_y} + U_{t_m}$$

$$p'_s = \frac{R_{t_u}}{S_y} + U_{t_u},$$

respectively, where R_t and U_t have been assigned appropriate subscripts[1] to show their identity.

Sources of Profit.—It should be noted that the logic in support of the derivation of this latter relation follows a principle, which has been long accepted by business executives, to the effect that the gross profits derived from the sale of a corporation's products depend entirely upon the capital invested in the manufacturing operations and that capital invested in non-productive operations, such as sales, general administration, etc., is employed in a manner that will in no way contribute to the earnings of the corporation. Naturally, other sources of profit derived from strictly financial operations, which do affect the final earnings of a corporation, as shown on the balance sheet, will have no bearing whatsoever on the problem of economic lot sizes.

The Gross Return on Invested Capital.—For the most part Eqs. (14), (16), and (18) will have little significance without a more complete interpretation of what is meant by gross margin of profits or the gross return R_t upon the capital employed in the manufacture of any unit of production. For the purposes of this discussion these will be considered as synonymous. According to the usual interpretation, the gross margin of profit represents the total number of dollars which remain clear of any expenses incidental to the manufacture and sale of any article and which are available to the corporation after all other charges have been deducted, including the commercial cost of doing business, the payment of dividends, advances to surplus account, federal taxes, and other obligations of a strictly corporate nature, all of which may be considered as logical appropriations of the gross return which is earned by the capital

[1] These two equations are special adaptations of Eq. (18) Table X, p. 141, where the subscripts m and u indicate values for the minimum-cost point and any other point on the curve U, respectively.

TABLE X.—EQUATIONS EMPLOYED IN THE ANALYSIS OF THE MINIMUM-SALES PRICE

$$p_s = \frac{B + R_t}{S_y} + U_f \tag{14}$$

$$= \frac{B'}{S_y} + I_t + \frac{R_t}{S_y} + U_f \tag{15}$$

$$p'_s = \frac{R_t}{S_y} + U_f + I_t \tag{16}$$

but

$$U_t = U_f + I_t \tag{17}$$

$$p'_s = \frac{R_t}{S_y} + U_t \tag{18}$$

$$= \frac{r \cdot C_e}{S_y} + U_t \tag{19}$$

where

$$R_t = r \cdot C_e \tag{20}$$

$$C_e = C_a + \sum_1^n (C_{p_n} + C_{r_n}) \tag{21}$$

$$U_t = A_a + \sum_1^n (A_{p_n} + A_{r_n} + m_{r_n}) \tag{22}$$

and

$$p'_s = \frac{r}{S_y}\left[C_a + \sum_1^n (C_{p_n} + C_{r_n})\right] + \left[A_a + \sum_1^n (A_{p_n} + A_{r_n} + m_{r_n})\right] \tag{23}$$

$$= \left(\frac{r}{S_y}C_a + A_a\right) + \sum_1^n\left(\frac{rC_{p_n}}{S_y} + A_{p_n}\right) + \sum_1^n\left(\frac{r \cdot C_{r_n}}{S_y} + A_{r_n} + m_{r_n}\right) \tag{24}$$

$$= p'_a + \sum_1^n p'_{p_n} + \sum_1^n p'_{m_n} \tag{25}$$

where

$$p'_a = \frac{r}{S_y}C_a + A_a \tag{26}$$

$$p'_p = \frac{r}{S_y}C_p + A_p \tag{27}$$

$$p'_r = \frac{r}{S_y}C_r + A_r + m_r \tag{28}$$

For an interpretation of symbols see text, Chap. X or Appendix XIII, p. 349.

invested in the specific enterprise, and accrues to the owners in the form of a reward for assuming the risks of business.

The Normal Margin of Profit.—Numerically the gross margin of profit can be evaluated by finding the difference between the value of the total sales for a specific article and the total of all costs or charges which may be assessed to it over a given period, normally a year. The figure thus obtained will have little practical value in any mathematical approach, because the total sales are a composite of many transactions, each of which is affected by the particular discounts granted in its case. It would be hopeless to attempt to forecast the actual value of these total sales, as no one could possibly estimate the percentage of sales that could be obtained for each classification of discount. Accordingly, if one is to be assured that in the face of competition an adequate return is to be earned, regardless of the discount granted, one should base all such computations upon the minimum unit-sales price p_s, which would be represented by the greatest discount that any salesman is permitted to grant. If this be done, the expected return which is computed on this basis can be safely taken as the minimum return which will satisfy all interests connected with the business. Then any profit which is earned over and above this amount will be just so much more "velvet," which will still further enhance the ultimate earnings of the business.

Evaluation of the Expected Rate of Return.—The total amount of this gross return R_t, which will be satisfactory to all, can be expressed as a percentage of the capital invested, and the determination of the rate which is normally expected in excess of the interest rate is a matter of executive policy and should be fixed by the directors in accordance with the expectations of the stockholders and the recognized risks of business. Ordinarily any going concern ought to earn a return in excess of 9 per cent per annum upon its capital, because if it were any less, the capital undoubtedly could be more profitably invested in reliable listed securities and thereby relieve its owners of the responsibility of sucessful management. Normally, a return of 15 to 18 per cent per annum should be earned when no undue risks are involved, and if the risks are serious a return of 30 per cent or more is quite justifiable. With this understanding of gross profits and if the normal expected rate of return is represented by

the symbol r, the gross return R_t for the year may be expressed by

$$R_t = r \cdot C_e, \qquad \text{See Eqs. (19) and (20)}$$

where

C_e = the total average amount of capital employed over the same period in the manufacture and storage of any product.

The Total Average Capital Employed.—The total average amount of capital C_e consumed in this case is equal to the summation of all the direct and indirect charges accruing to the total number of articles of the same kind, sold in the period, from the payroll, working men and executives included, and from current bills for all materials, supplies, etc., which have contributed their share to the total value of the product, as well as from general expenses which are borne by all such articles alike with due regard for the time that each amount of capital is employed for its specific purpose. As previously stated, that portion of capital which is not directly employed in manufacturing operations, has no influence upon the problem and will accordingly be omitted from further consideration.

Evaluation of the Minimum-sales Price Applicable to Manufacture.—Now if the minimum unit-sales price p'_s for a given product is to be separated into those portions which apply to each phase of manufacture, the total average capital must be similarly subdivided so that the gross margin of profit can be allotted appropriately to the particular phase from which it would naturally be derived. Accordingly, that portion of the minimum-sales price which is applicable to any phase will be represented by the sum of the gross return earned upon the capital consumed in the specific operations, and storage of the completed unit, and that portion[1] A of the total ultimate unit cost U_t which equals the value added to the material m introduced into the process by the operations performed in any particular phase, where the full expression for U_t is given by Eq. (55).[2] This situation may be expressed mathematically by Eqs. (21) to (25) in Table X, if

[1] See Eq. (56), Table XIII, p. 161.

[2] See Table XIII, p. 161.

C_a = the average capital employed in the assembly operations and storage.

ΣC_p = the total of the average capital employed in the fabricating operations and storage of each part entering into the final assembly.

ΣC_r = the total of the average capital invested in the purchase and storage of the raw materials from which the various component parts are fabricated.

A_a = accumulated value derived from the operations performed in the assembly phase.

A_p = accumulated value derived from the operations performed in the fabricating phase for any part.

A_r = accumulated value resulting from the effort to procure and store raw materials in the purchasing phase.

m_r = the value of raw material as purchased.

p'_a = that portion of the minimum-sales price applicable to the assembly phase.

$\Sigma p'_p$ = the total of the portion of the minimum-sales price applicable to the fabricating phase.

$\Sigma p'_r$ = the total of the portion of the minimum-sales price applicable to the purchasing phase.

Allocation of the Selling Price to Each Phase of Production.—As a result the subdivisions of the minimum unit-sales price for the product can be expressed by Eqs. (26), (27), and (28), where the subscripts a, p, and r represent the various phases of assembly, fabrication, and purchase, respectively. Owing to the fact that each of these expressions has the same general form as that given in Eq. (29), Table XI, where p' replaces p'_s as the former refers to any phase, and C equals the average capital employed in the same phase, the discussion henceforth will deal only with this expression, and the conclusions can then be applied at will to whichever one they may properly belong.

Introduction of the Ultimate Unit Cost.—If from now on this problem is to be considered only upon the basis of a particular phase, regardless of the nature of the operations performed, it might be misleading to employ only that part of the ultimate unit cost for any phase which is represented by the value accumulated to the unit of production, and omit the value of the material which must in all cases be introduced at the beginning of the process. In all probability it will be more natural for executives

TABLE XI.—EQUATIONS EMPLOYED IN THE EVALUATION OF THE MINIMUM-SALES PRICE

In general

$$p' = \frac{R_o}{S_y} + A = \frac{r \cdot C_e}{S_y} + A \tag{29}$$

or

$$(m + p') = \frac{R_o}{S_y} + A + m. \tag{30}$$

But if

$$p'' = (m + p')$$

$$p'' = \frac{R_o}{S_y} + U, \tag{31}$$

$$= \frac{r \cdot C_e}{S_y} + U; \tag{32}$$

or when

$$C_e = N \cdot C_i, \tag{33}$$

$$p'' = \frac{r \cdot N \cdot C_i}{S_y} + U;$$

and when

$$C_i = (C_s \cdot T_s + C_w \cdot T_w + C_f \cdot T_f), \tag{34}$$

$$p'' = \frac{r \cdot N}{S_y}(C_s \cdot T_s + C_w \cdot T_w + C_f \cdot T_f) + U; \tag{35}$$

but

$$C_s \cdot T_s = Q^2 \cdot v_s \cdot t_s + Q \cdot k_{v_s}, \tag{36}$$

$$C_w \cdot T_w = Q^2 \cdot v_w \cdot t_w + Q \cdot k_{v_w}, \tag{37}$$

$$C_f \cdot T_f = (C_{f_s} \cdot t_s + C_{f_w} \cdot t_w) \cdot Q, \tag{38}$$

$$N = \frac{S_y}{Q}$$

Therefore

$$p'' = \frac{r \cdot S}{S_y \cdot Q} \cdot [Q^2 \cdot (v_s \cdot t_s + v_w \cdot t_w) + Q \cdot (k_{v_s} + k_{v_w}) + Q \cdot (C_{f_s} \cdot t_s + C_{f_w} \cdot t_w)] + U. \tag{39}$$

$$= r \cdot [Q \cdot (v_s \cdot t_s + v_w \cdot t_w) + k_{v_s} + k_{v_w} + C_{f_s} \cdot t_s + C_{f_w} \cdot t_w] + U. \tag{40}$$

For an interpretation of symbols see text, Chap. X, or Appendix XIII, p. 349.

to think in terms of the total of all values accumulated or introduced when combined in a single item for the ultimate unit cost U, and so in order to avoid confusion the unit value m of the material entering the process will be added to both sides of Eq. (29), as illustrated in Eq. (30). This device will in no way impair the reliability of the ultimate conclusions and may prove to be quite valuable in providing a clearer method of visualizing the elements involved. The net result will be only to elevate the relations of R_o, A, and p' by the value of m which can be readily appreciated by inspection of the diagram in Fig. 18. Accordingly, the term p'' will be used to represent the situation when p' is increased by an amount m, and its value will be found in Eq. (32).

The Effect of Capital Turnover.—The capital which is invested in manufacturing operations should never be allowed to accumulate and remain idle over any extended period of time. It is this fact which long ago was recognized by executives and led to the adoption of lot production so that capital could be constantly maintained in circulation. Accordingly, the total expenditure of capital during any year is in most cases made up of only that which is required for the production of any single lot, because as soon as it has been converted back into dollars, owing to the sale of the finished product in the manufacture of which it was originally employed, it immediately becomes available for reinvestment in the production and storage of another lot. Upon this basis, then, the average capital employed in any phase of manufacture can be expressed by Eq. (33) where

N = the number of times the initial investment of capital can be turned over in any year,

and

C_i = the initial average investment required to produce and store the total number of units of production manufactured in a lot, the size of which is Q.

The Initial Average Investment per Lot.—In determining the numerical value for this initial average investment, one must take into account not only the actual dollars expended in the production and storage of any lot but also the time during which these dollars are represented by material values, in accordance

with the particular form that the unit of production has assumed. This is of utmost importance because in no other way can one distinguish between the returns earned upon the capital when at one time it is employed in the production of a certain article and at another time in the same year it may be applied to the manufacture of a totally different article. Beside the capital invested in work in process and articles in stores, a return must be earned upon the capital invested in the manufacturing equipment, including tools, machines, and even that portion of the building and land which is utilized by the process. Therefore if

C_w = the actual capital consumed in the manufacture of the lot Q,
C_s = the actual capital invested in the storage of the lot,

and

C_f = the portion of the fixed investment in equipment utilized,
T_w = the time the capital C_w is employed,
T_s = the time the capital C_s is employed,

and

T_f = the time the capital C_f is employed,

Eq. (34) in Table XI can be written to express the relation of factors entering into the determination of the initial average expenditure of capital.

Elements of Investment in Manufacture.—In order to complete this relation and at the same time provide an opportunity for simplifying the computations later on, the value of each unit of production, whatever its form may be, must be introduced, as well as the time which is required to produce or store each single unit. Naturally, the total capital expended in work in process will equal the value v'_w which has accumulated to each unit multiplied by the number of units produced in each lot.

$$C_w = v'_w \cdot Q.$$

Likewise, the total capital invested in inventories will equal the value v'_s of each unit at the time it is placed in stores multiplied by the total number in the lot

$$C_s = v'_s \cdot Q.$$

Now if the average time for producing any one unit be t_w, and the average time that any one piece remains in stores be t_s, then

$$T_w = t_w \cdot Q,$$

and

$$T_s = t_s \cdot Q.$$

Two Sources of Capital Values.—Further inquiry for the moment into the composition of any one of these items would add little to a better interpretation of the average initial investment of capital in the manufacture of the lot. Each one of these items will be carefully analyzed in the later chapters[1] on the cost of capital invested in work in process and in articles in stores, and it should suffice for the present to recognize that the values of v'_s, v'_w, t_s, and t_w, as employed here, will be identical with those employed hereafter in the evaluation of the capital investment in either of these cases, for the reason that the same amounts of capital should be used in determining the gross return as used in computing the cost of capital. It develops, however, in the analysis[2] of the capital invested in work in process and in articles in stores, that in each case the capital value of a unit of production is made up of two items, one of which v_w or v_s,[3] is derived from the direct-production cost c and the other from the preparation charges applicable to the lot as a whole. Since the former is dependent upon the lot size and the latter independent of it, the final expression for the total average capital expenditure must take cognizance of these facts and be subdivided accordingly. Hence,

$$C_wT_w = Q^2 \cdot v_w \cdot t_w + Q \cdot k_{v_w}$$
$$C_sT_s = Q^2 \cdot v_s \cdot t_s + Q \cdot k_{v_s}$$

where[4] k_{v_w} and k_{v_s} represent that portion of the unit capital derived from the allotment of the preparation charges.

The Fixed Property Investment.—An extended analysis along similar lines might be applied to the investment of fixed capital in land, buildings, and machines. As will be seen later

[1] See pp. 215 and 237.
[2] See Chapters XIV and XV.
[3] See Table XXVII, p. 272.
[4] See pp. 215, and 237.

on,[1] however, little advantage can be gained because the amount of such capital must always be expressed as the total employed in the manufacture and storage of the lot, and will remain the same regardless of the number of lots produced in order to meet the total yearly production required by the sales demand. The time element is the only one which needs further interpretation. As long as the number of times that the facilities represented by this amount of capital are employed during the year has been accounted for in the derivation of the initial capital expenditure for the lot, it is evident that they will be in use only with respect to any lot Q over the time that the process of manufacture or storage period is in effect. Accordingly, that portion of the fixed capital applicable to manufacture C_{f_w} will be employed for a time,

$$T_{f_w} = T_w = Q \cdot t_w,$$

and that portion which may be utilized in the storage of articles in inventory C_{f_s} will be made use of over the time

$$T_{f_s} = T_s = Q \cdot t_s.$$

So that

$$C_f \cdot T_f = C_{f_s} \cdot t_s \cdot Q + C_{f_w} \cdot t_w \cdot Q. \quad \text{See Eq. (38).}$$

Investment of Capital through the Purchase of Raw Materials.—Moreover, the expenditure of capital in the procurement and storage of raw materials C_r could be separated into its controlling elements, but as this relates entirely to the determination of an economic-purchase quantity,[2] further discussion of this phase will be deferred until such time as it becomes feasible to enlarge upon it individually. It will suffice for the present merely to show that a certain amount of the total capital upon which a return must be earned is actually expended in this function of the business, and that when so provided for it can be totally disregarded when computing the lot size for either assemblies or fabricated parts.

The Basic Relation of Price to the Ultimate Unit Cost of Manufacture.—Now that it has been possible to attain a more

[1] Note in Eq. (47), Table XII, p. 156, that the terms $(k_{v_s} + k_{v_w})$ will cancel out.

[2] *Cf.* DAVIS, R. C., Minimum Cost Purchase Quantities, A.S.M.E. *Trans.*, January–April, vol. 50-MAN, No. 6, p. 41, 1928.

perfect interpretation of the average capital employed in any phase of manufacture, the various expressions given in Eqs. (36), (37), and (38) can be inserted in the general expression, Eq. (35), in accordance with their respective symbols, so that a final equation (39) can be written for that portion of the minimum sales price which can be employed in the determination of an economic production quantity. In so doing the rate N at which capital can be turned over will be represented by the number of lots of a given size Q which must be produced to meet the sales demand S_y or the rate of consumption for the ensuing year, whereupon

$$N = \frac{S_y}{Q}.$$

It will be noticed that by the introduction of this ratio certain items will cancel out and Eq. (39) will eventually take the form shown in Eq. (40). It is obvious from the foregoing discussion that the summation of these unit prices will naturally equal the minimum unit-sales price which would be charged for the finished product when the maximum discount has been granted, and this fact should give ample proof that this method may be justly employed in the determination of the economic lot size with reference to the conservation of capital, as all elements of the final price have been fully accounted for.

CHAPTER XI

LIMITS OF THE ECONOMIC RANGE

If it now be assumed that the sole objective of the production executive be to manufacture each unit of production so that the lowest unit selling price can be placed upon the manufactured article in order to meet competition and yet obtain the return normally expected upon the capital invested, the lot size will have to be that which will yield a minimum ultimate cost for each unit produced. Accordingly, if Q_m and U_m, respectively, be introduced into Eq. (40), Table XI, to represent these items, the minimum-sales price under ideal conditions can be expressed by p''_m in Eq. (41), Table XII.

The Effect of Uneconomical Production on Price.—The question arises, however, whether this same minimum unit price can be applied with equal satisfaction to any other production quantity Q_u which has not been produced at minimum cost, and, if Q_u be made smaller than Q_m, can capital be conserved as well? There would seem to be some opportunity to accomplish this, because it is evident that the amount of capital invested in work in process and articles in stores depends upon the lot size,[1] and if this can be made small enough it should be possible to offset any increase in the ultimate unit cost due to a smaller production quantity.

The Maximum Permissible Ultimate Unit Cost.—To show what quantity less than the minimum-cost quantity Q_m can be produced without a reduction in the expected rate of return r, a curve can be drawn on the diagram given in Fig. (19), which will represent the maximum ultimate unit cost U' which is permissible under the condition that the minimum unit-sales price will remain unchanged and yet yield the desired return. The expression for this curve is given by Eq. (43). As long as the only variable in this relation is the lot size Q_u, it is evident

[1] See Eq. (40), Table XI, p. 145.

that this curve will be a straight line which must pass through the minimum-cost point on the curve U when

$$Q_u = Q_m.$$

As under these conditions only two points are required to locate the line representing the curve U', the other point can be deter-

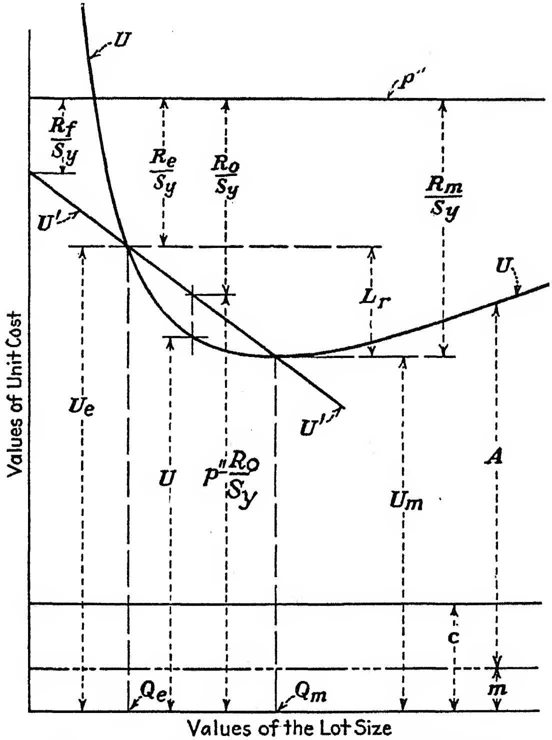

FIG. 19.—The relation of price, margin of profit, and the ultimate unit cost within the economic range of production.

mined when Q is made equal to zero, because then the terms associated with Q_u will disappear from the expression and the curve U' will meet the ordinate at a point where

$$U' = U'_o = p''_m - C_f \cdot T_f \cdot r.$$

The Economic Range of Production Illustrated.—It will be seen now that the curve U' intersects the curve U at the point U_e for a quantity Q_e, as well as at the point of minimum cost U_m. Between the points U_e and U_m the curve of actual unit

costs U lies below the curve of permissible unit costs U', so that for any point along the curve U, for quantities ranging from Q_e to Q_m, the actual unit cost will be less than the permissible unit cost. This means that any quantity between these two limits may be produced at a unit cost, even though greater than the minimum unit cost, which will yield a gross margin of profit equal to or greater than the expected return upon the capital employed. As any such quantity will have the same or greater manufacturing advantage than that obtainable for the minimum-cost quantity, this range will be termed the "economic range" of manufacture.

Advantages of the Economic Range.—Owing to the fact that in the majority of cases the curve U is comparatively quite flat for a considerable distance either side of the minimum-cost point, this range will provide a relatively wide variation in quantities to select from, in order to adjust production schedules to suit fluctuations in the sales demand. These facts substantiated the earlier contention[1] that an economic range will be of greater value in the control of production than a single inflexible quantity ever could be. The values of Q_e and Q_m can be determined in advance, at the time the process for the particular unit of production is laid out, and can be recorded once for all in the production department for future reference without any liability of repeated revision for every change that may occur. The limits for this range will need correction only at those times when the process is improved and the unit cost lowered thereby.

Conservation of Capital and the Economic Quantity.—To achieve the greatest saving in capital employed without impairing the expected return, the smallest possible quantity must be produced. This quantity will naturally be the lower limit of the economic range Q_e, as any smaller quantity will not fulfill the basic requirements. Therefore the quantity Q_e will be designated as the economic-production quantity, because it not only can be produced for the same manufacturing advantage that can be attained for the minimum-cost quantity Q_m, but will also satisfactorily conserve capital to the greatest degree. This economic-production quantity will serve as a measure of ideal manufacturing practice and should be adhered to as closely as the conditions will permit. Variations from it will indicate an

[1] See p. 17.

undesirable increase in working capital, and, if they should exceed the economic range, a warning is immediately given that the expected return upon the capital employed is impaired. How much farther these variations can go depends upon executive decision and business policy, but in any case the value for r, which can be determined for any quantity, will be an index of the situation and will be a great aid to executive judgment.

Realization of Savings.—The capital thus freed from manufacturing operations for each unit of production can be withdrawn from those funds set aside for working capital and can be applied to other channels where an equal or greater return or manufacturing advantage exists. There is little need for speculation here as to the manner in which an alert executive will invest this capital, as an expansion or improvement program will already be a part of any aggressive business policy.

The Interrelation of Cost and Margin of Profit.—As there is an ideal quantity for the economical size of a production lot, some means must be provided for determining its amount. This may be accomplished through the fact that the economic-production quantity is the smallest quantity which has the same manufacturing advantage that can be attained by the production of a lot the size of which is equal to the minimum-cost quantity. If this was not true it would be necessary to produce the minimum-cost quantity in all instances. At this point it should be remembered that the effect of the increase in unit cost due to the production of the quantity Q_e smaller than Q_m upon the gross margin of profit must be offset by a decrease in the average amount of capital required to finance the manufacture and storage of a unit of production, without any change in the normal expected rate of return. By referring again to Fig. 19 it will be seen that this increase in cost is equal to the difference between the unit cost U_e for the economic-production quantity and the unit cost U_m for the minimum-cost quantity. Similarly this difference in unit costs is equal to the difference between the expected return R_m/S_y from the sale of an article manufactured at minimum cost, and the return R_e/S_y from the sale of the same article when produced in an economic quantity. In both cases the same minimum selling price will, of course, apply. If the increase in unit cost—or, really, the loss in gross margin of profit, in this case only theoretical—is represented by

L_r, this relation may be expressed mathematically by Eq. (46) where

$$U_e - U_m = \frac{R_m}{S_y} - \frac{R_e}{S_y} = L_r.$$

Evaluation of the Loss L_r.—The loss L_r in the gross margin of profit which will be incurred by the production of a quantity Q_l in the economic range varying from the minimum-cost quantity Q_m, which still will have the same manufacturing advantage, may be evaluated from Eqs. (47) or (48) by substituting in Eq. (46) the expressions for R_m/S_y and R_e/S_y as given by Eqs. (44) and (45), respectively. Similarly, the other half of the relation given in Eq. (46) can be evaluated if an expression for the unit costs U_m and U_e, respectively, is available. At the end of the next chapter[1] it will be shown that the ultimate cost for any unit of production is in general for the practical solution:

$$U_c = u' + \frac{P \cdot F}{Q} + f \cdot Q$$

in which

$P \cdot F$ = all total charges which are independent of the production quantity Q,

$f \cdot Q$ = all unit charges which are dependent upon this quantity,

F = a coefficient for the adjustment of the charges P with respect to certain time elements involved,

and

u' = a group of charges which only enter into the determination of the ultimate unit cost and have no bearing upon the minimum-cost quantity.

Accordingly, if the appropriate subscripts are applied to the various terms in this equation to indicate their position in either U_m or U_e, L_r can be again expressed by Eq. (49). These two halves may now be combined as they both equal L_r, and the resulting expression, Eq. (50), solved for Q_l in terms of Q_m. [See Eq. (51).] It is of little importance at this point to inquire into the derivation of the minimum-cost quantity, as it is simply a matter of mathematical technique to determine the minimum point for the curve U representing the values of the ultimate

[1] See Eq. (58), p. 161.

TABLE XII.—EQUATIONS EMPLOYED IN THE DERIVATION OF THE LIMITS OF THE ECONOMIC RANGE

$$p''_m = U_m + r \cdot [Q_m \cdot (v_s \cdot t_s + v_w \cdot t_w) + k_{v_s} + k_{v_w} + C_{f_s} \cdot t_s + C_{f_w} \cdot t_w] \quad (41)$$

$$U_m = p''_m - r \cdot [Q_m \cdot (v_s \cdot t_s + v_w \cdot t_w) + k_{v_s} + k_{v_w} + C_{f_s} \cdot t_s + C_{f_w} \cdot t_w] \quad (42)$$

$$U' = p''_m - r \cdot [Q_u \cdot (v_s \cdot t_s + v_w \cdot t_w) + k_{v_s} + k_{v_w} + C_{f_s} \cdot t_s + C_{f_w} \cdot t_w] \quad (43)$$

$$\frac{R_m}{S_y} = r \cdot [Q_m \cdot (v_s \cdot t_s + v_w \cdot t_w) + k_{v_s} + k_{v_w} + C_{f_s} \cdot t_s + C_{f_w} \cdot t_w] \quad (44)$$

$$\frac{R_e}{S_y} = r \cdot [Q_l \cdot (v_s \cdot t_s + v_w \cdot t_w) + k_{v_s} + k_{v_w} + C_{f_s} \cdot t_s + C_{f_w} \cdot t_w] \quad (45)$$

For the quantity Q_l

$$U_e = U'_e,$$

and then

$$U_e - U_m = \frac{R_m}{S_y} - \frac{R_e}{S_y} = L_r \quad (46)$$

or

$$\begin{aligned} L_r &= r \cdot [Q_m \cdot (v_s \cdot t_s + v_w \cdot t_w) + k_{v_s} + k_{v_w} + C_{f_s} \cdot t_s + C_{f_w} \cdot t_w] - \\ &\qquad r \cdot [Q_l \cdot (v_s \cdot t_s + v_w \cdot t_w) + k_{v_s} + k_{v_w} + C_{f_s} \cdot t_s + C_{f_w} \cdot t_w], \quad (47) \\ &= r \cdot (Q_m - Q_l) \cdot (v_s \cdot t_s + v_w \cdot t_w) \quad (48) \end{aligned}$$

if

$$U_m = u' + \frac{P \cdot F}{Q_m} + f \cdot Q_m,$$

and

$$U_e = u' + \frac{P \cdot F}{Q_l} + f \cdot Q_l,$$

then

$$L_r = (U_e - U_m) = P \cdot F \cdot \left(\frac{1}{Q_l} - \frac{1}{Q_m}\right) + f \cdot (Q_l - Q_m). \quad (49)$$

Combining Eqs. (48) and (49),

$$L_r = r \cdot (Q_m - Q_l) \cdot (v_s \cdot t_s + v_w \cdot t_w) = P \cdot F\left(\frac{1}{Q_l} - \frac{1}{Q_m}\right) + f \cdot (Q_l - Q_m). \quad (50)$$

Whereupon (see Appendix I for intervening steps),

$$Q_l = Q_m \left[\frac{\frac{2}{Q_m{}^2} + \frac{r}{P \cdot F} \cdot (v_s \cdot t_s + v_w \cdot t_w) \pm \frac{r}{P \cdot F} \cdot (v_s \cdot t_s + v_w \cdot t_w)}{2\left\{\frac{1}{Q^2{}_m} + \frac{r}{P \cdot F} \cdot (v_s \cdot t_s + v_w \cdot t_w)\right\}} \right]. \quad (51)$$

If the plus sign be used

$$Q_l = Q_m.$$

If the minus sign be used

$$Q_l = Q_e,$$

or

$$Q_l = \frac{Q_m \cdot P \cdot F}{P \cdot F + Q_m{}^2 \cdot r \cdot (v_s \cdot t_s + v_w \cdot t_w)} \quad (52)$$

For an interpretation of symbols see text, Chap. XI, or Appendix XIII, p. 349.

unit cost, as defined at the beginning of this book, for any production quantity. The various mathematical steps involved in this transition have been enlarged upon in Table XIII.[1]

The Limits of the Economic Range.—It is of importance to note that a plus or minus sign appears in the numerator of Eq. (51), which indicates that two possible values can be obtained for the quantity Q_l. If this be so, then each of them must have the same manufacturing advantage as that derived from the production of the minimum-cost quantity. This is quite in line with the basic theory, because if the larger of these two values can be shown to be equal to Q_m the smaller will be that for the economic production quantity, and then Eq. (51) for Q_l can be designated as that which represents the limits of the economic range.

The Upper Limit: The Minimum-cost Quantity.—Since the upper limit $Q_{l\ \max}$ will naturally be the larger, its value may be found by using the positive sign, whereupon

$$\begin{aligned} Q_{l\ \max} &= Q_m \cdot \left[\frac{\dfrac{2}{Q_m{}^2} + \dfrac{2 \cdot r \cdot (v_s \cdot t_s + v_w \cdot t_w)}{P \cdot F}}{2\left\{ \dfrac{1}{Q_m{}^2} + \dfrac{r \cdot (v_s \cdot t_s + v_w \cdot t_w)}{P \cdot F} \right\}} \right], \\ &= Q_m, \end{aligned}$$

and this provides an excellent check upon the original assumption.

The Lower Limit: The Economic Production Quantity.—Now, if the minus sign is used, the value of the lower limit will be that for the economic production quantity Q_e, which should be employed in the control of production in order properly to conserve capital. Hence if

$$\begin{aligned} Q_{l\ \min} &= Q_m \cdot \left[\frac{\dfrac{2}{Q_m{}^2} + \dfrac{r \cdot (v_s \cdot t_s + v_w \cdot t_w)}{P \cdot F} - \dfrac{r \cdot (v_s \cdot t_s + v_w \cdot t_w)}{P \cdot F}}{2\left\{ \dfrac{1}{Q_m{}^2} + \dfrac{r \cdot (v_s \cdot t_s + v_w \cdot t_w)}{P \cdot F} \right\}} \right], \\ &= Q_m \cdot \left[\frac{1}{1 + \dfrac{Q_m{}^2 \cdot r \cdot (v_s \cdot t_s + v_w \cdot t_w)}{P \cdot F}} \right], \\ Q_{l\ \min} &= Q_e, \end{aligned}$$

[1] See also Table XXV, p. 268.

and should the previous expression be rewritten in a more convenient form

$$Q_e = \frac{Q_m \cdot P \cdot F}{P \cdot F + Q^2{}_m \cdot r \cdot (v_s \cdot t_s + v_w \cdot t_w)}. \quad \text{See Eq. (52)}$$

From this fundamental formula for the economic-production quantity, simpler forms may be derived depending upon the elements employed in the expression for the minimum-cost quantity, when the particular form, which seems to be most appropriate, is substituted for its symbol Q_m, wherever it may appear. Moreover, this expression may be employed indiscriminately for the determination of an economic-assembly quantity as well as an economic-production quantity, provided that the data be properly selected in accordance with the conditions which govern each specific case.

CHAPTER XII

THE ULTIMATE UNIT COST AND ITS MINIMUM POINT

Since in the last chapter it is apparent that the limits of the economic range are dependent upon the minimum-cost point for the processes employed in the manufacture of any article, it will be necessary to inquire into the composition of the unit cost which is to be made a minimum. Unit cost may be defined in a variety of ways, each of which differs from the others merely in the matter of terminology, depending upon the particular purpose it supposedly fulfills. Terms such as prime cost, factory cost, manufacturing cost, unit cost of production, and many others are familiar to every one. None of these common ways of designating unit costs can be utilized in defining the total unit cost, however, which must be employed in the determination of an economic-production quantity, as each one fails to comprehend all of the cost items which actually enter into this problem, and if any one of these terms were used confusion might arise as to its precise intent.

Total Unit Cost Defined.—Accordingly, the total unit cost to be used in this case must be defined as the summation of all costs which accrue to a unit of production from the time the raw material used in its manufacture becomes the property of the concern to the time that it is removed from the books of the company as a saleable article, whatever its form or relation to other similar articles may be. As in most cases the finished product is an assembly or combination of a number of units of varying composition or shape, the total unit cost can be subdivided[1] into the unit cost of purchased raw material U_r, the unit cost of each fabricated part U_p, and the unit cost of the assembled article U_a, with due consideration for any repetition of material values which may be carried over from one phase of manufacture to another.

[1] See Table XIII, p. 161, Eqs. (53) to (54).

Its Relation to Other Cost Items.—Each one of these unit costs respectively represents the total money value accruing to the unit of production in its final form, whether as raw material purchased, a fabricated part, or the finished product, at the time it is removed from inventory to be fabricated, assembled with other parts, or sold. In ordinary accounting practice each one of these unit costs exceeds the preceding unit cost, in the order they are listed above, by the value of the direct labor applied during the intervening operations, in order to bring that unit of production into the form which it attains subsequent to its transfer into the next logical stage toward its inclusion in the final product, together with the appropriate value for overhead of all kinds which is distributed in the customary manner to that unit as it progresses. Such practice fails to comprehend another cost item which is equal in importance to the unit cost of labor and overhead when it comes to the selection of the best manufacturing method, and that is the cost of carrying the capital employed in the manufacturing operations upon this unit or in its storage after processing while awaiting future consumption.

Total Unit Cost and the Cost of Capital.—In order to obtain the correct value for the total unit cost as defined above,[1] the cost of capital I, however employed in the production or storage of articles irrespective of their form, must be added to the combined unit costs of labor and overhead.[2] This situation brings to the front again a long-standing controversy among leading accountants over the justification of including the cost of capital as a part of the total cost of any unit of production. A very interesting treatise on this subject has been published by Clinton H. Scovell in his book on "Interest as a Cost,"[3] in which many excellent reasons are given in support of his theory. The author has no desire to comment upon this subject, because it is purely one of accounting practice,[4] but he does wish to support the contention that the cost of capital is an important factor in the determination of the ultimate unit cost. The very necessity[5] of an economic balance in lot production is sufficient proof that

[1] See p. 159.
[2] See p. 189.
[3] Published by The Ronald Press Company, 1924; in which see Chap. IV, pp. 21, etc.
[4] See p. 165.
[5] See p. 15.

TABLE XIII.—EQUATIONS EMPLOYED TO SHOW THE RELATION OF THE ULTIMATE UNIT COST IN EACH MANUFACTURING PHASE

The total ultimate unit cost of the product is

$$U_t = U_f + I_t. \tag{53}$$

The total unit-factory cost of the product is

$$U_f = U_a - I_a.$$

The ultimate unit-assembly cost is

$$U_a = A_a + m_a.$$

The cost of material consumed by the assembly process is

$$m_a = \sum_1^n (U_p - I_p)_n.$$

The ultimate unit-production cost is

$$U_p = A_p + m_p.$$

The cost of material consumed in the fabricating process is

$$m_p = U_r - I_r.$$

The ultimate unit-purchase cost is

$$U_r = A_r + m_r.$$

The original price paid for raw materials is

$$m_r = p_p.$$

Now, if that portion of the ultimate unit cost for each manufacturing phase which must be transferred to the cost of doing business through the term I_t is I_a, I_p, I_r, respectively, then in general

$$U_f = A_a - I_a + A_p - I_p + (A_r + m_r) - I_r,$$

but

$$\begin{aligned} I_t &= I_a + I_p + I_r \text{ (by definition)} \\ U_f &= A_a + A_p + A_r + m_r - I_t, \\ &= (U_a - m_a) + (U_p - m_p) + (U_r - m_r) + m_r - I_t, \\ &= U_a + U_p + U_r - (m_a + m_p) - I_t. \end{aligned}$$

Then

$$U_t = U_a + U_p + U_r - (m_a + m_p). \qquad \text{From Eq. (53)} \tag{54}$$

Then for any phase of manufacture

$$U = m + A, \tag{55}$$

where

$$A = (u_m - m) + \frac{I_s + I_w + V_s + V_w + L_t}{Q}, \tag{56}$$

$$(u_m - m) = l + o + \frac{P}{Q},$$

and

$$I_t = \frac{I_s + I_w}{Q}, \qquad \text{See also Eqs. (61) and (62), p. 168.} \tag{57}$$

or if the terms in U be regrouped so that

$$U = u' + \frac{P \cdot F}{Q} + f \cdot Q, \tag{58}$$

the minimum-cost point can be expressed by

$$\frac{dU}{dQ} = 0 = \frac{-P \cdot F}{Q^2} + f, \tag{59}$$

$$Q^2 = \frac{P \cdot F}{f},$$

$$Q_m = \sqrt{\frac{P \cdot F}{f}}. \tag{60}$$

For an interpretation of symbols see text, Chap. XII or Appendix XIII, p. 348.

it must be taken into account in the management of industrial manufacturing operations. The manner in which the cost of capital is carried through the accounting method upon the books of the company, however, and to what accounts this cost should be charged, has no bearing whatsoever upon the present problem.

Effect of Manufacturing Methods on Cost.—In the first part of this book attention was called to the fact that in production of an intermittent nature, it is necessary to process at specified intervals a lot of a given quantity of articles which, because of the rapidity of production in comparison with the rate of consumption or the sales demand, have to be placed in stores for a certain period in anticipation of their eventual withdrawal upon prospective orders. During this storage time, production of the article will have ceased, and this situation will continue until the stock has been reduced to that amount which will just be sufficient to cover the expected withdrawals while additional articles are being processed. If too large a number of articles are produced in each lot in proportion to the demand, the time of storage for the average number will be excessive and the cost of the capital I_s invested in stores inventories[1] will be high. On the other hand, if this cost of capital is to be kept low, a greater number of lots should be processed during the year, each one of which will necessarily be made up of a smaller number of articles.

An Economic Balance between Cost Factors.—In this latter case the total preparation costs,[2] which are repeated each time a new lot is processed, will be greater for the year, and the unit cost of the article will be increased. Somewhere between the extremes of these two conditions there will be a point of economic balance where the unit allotment of the preparation costs for a lot will equal the unit allotment of the cost of capital thus employed. If it is the desire of production executives to manufacture their product at a reasonable cost, in no way unduly inflated by excessive charges for preparation or investment of capital, the effect of the cost of capital must be introduced in order so to proportion the lot size that this economic balance may be attained

[1] See Chap. XIV, p. 194.
[2] See p. 184.

The Total Cost of Capital Must Be Employed.—If this be true, with regard to the cost of capital invested in articles in stock, it will be equally true if the cost of all the capital invested in inventories pertaining to this article is considered as well. This means that the cost of capital I_w invested in work in process[1] must also be taken into account. No confusion should arise at this point in the mind of the reader with regard to that capital invested in manufacturing operations which is represented by the value of lands, buildings, machines, equipment, tools, etc., and may directly or indirectly enter into the manufacturing operations for this article. The cost of this capital is independent of both the size of the lot and the number of lots per year, and therefore need not be considered in connection with the economic balance which only depends upon those factors having some definite relation to either the lot size or the number of lots required to meet the annual sales demand.

Classification of Cost Factors by Characteristics.—Herein lies the whole essence of the principle underlying the choice of the best quantity to produce in any lot. It will be seen upon closer analysis that the total unit cost from which the economic balance[2] is derived depends upon three types of cost items (see Eq. (58), Table XIII), which accrue to each lot. The first $P \cdot F$[3] is a group of total charges which are incurred by the production of a lot irrespective of its size and which cannot be reduced for this reason to a constant unit value. The second $f \cdot Q$[3] will be a group of charges which can be directly prorated as unit values because they depend upon the quantity produced in the lot. The third u' will be a group of general or constant cost items, which are independent of either the lot or its size and, even though they appear in the make-up of the ultimate unit cost, will have no effect upon the economic balance which depends upon the first two groups. Another group of terms which will only appear in the final expression for the eventual lot size might be referred to here, except for the fact that none of them has any relation to cost. For the most part they will be constants or else some factor which will represent the goal toward which the specific endeavor is directed, such as the total anticipated sales for the year.

[1] See Chap. XV, p. 217.

[2] See p. 15.

[3] See p. 267.

Other Cost Factors.—Bearing these facts in mind it is well to inspect all charges which contribute to the final value of any unit of production. If all total charges which arise from the preparation and control of a production order, from its issue through to its completion, are classified in the first group $P \cdot F$, the second group $f \cdot Q$ will be composed of those unit charges which are derived from the cost of capital together with two others which have a very similar bearing upon the problem though quite different in nature. These are the cost of the space V_s[1] occupied by any article in storage and the loss L_d[2] incurred by the deterioration of the article if held too long in stock. The first unquestionably depends upon the quantity produced at any one time, as sufficient and suitable storage space must be provided to hold these articles and safeguard them as much as possible during the time any one of them remains in stores. The second likewise depends upon the lot size, because if a smaller quantity is produced the loss will be less not only for the reason that there is a smaller quantity stored but also for the reason that the storage period will be shorter, as it will take less time to consume this smaller number of articles. In fact, it might so happen that if the quantity was small enough no deterioration would occur, which of course, would depend this time upon the nature of the article.

A Survey of Typical Cost Factors Advisable.—In extraordinary cases an exhaustive analysis might bring to light other factors[2] which would be of sufficient importance to warrant their being included in either one or the other of these two groups. In all probability it will be found that they can be classed with some factor already evolved which has similar characteristics, and in the end change in no vital way the arrangement of terms which would be contained in a mathematical expression for the economic balance. A review of the structure of a typical accounting system as given in any good textbook on that subject may aid in such a study. For all practical purposes those factors just enumerated will suffice.

The Ultimate Unit Cost Defined.—Owing to the fact that the total unit cost which contemplates an economic balance contains certain factors which need not be carried on in the usual account-

[1] See Chap. XVI, p. 240.

[2] See Chap. XVII.

ing method, a specific term will be introduced at this point to distinguish it from any other and avoid confusion. Accordingly, the "ultimate unit cost" U will be that total unit cost for any phase which takes into consideration beside the unit costs of material m, labor l, overhead o, and the unit allotment of the preparation charges P/Q, the unit cost of capital invested in inventories I_s and I_w including that from work in process, the unit cost of the space occupied in stores V_s, and the loss L_t, if any, due to deterioration L_d or any other similar cause.[1]

The Relation of the Ultimate Unit Costs for Each Phase of Manufacture.—The ultimate unit cost may be determined separately for any unit of production, whether it be a unit of raw material, a fabricated part, or the assembled product. In each case the ultimate unit cost will be distinct and will represent the accumulation of value A accruing to that unit from all sources, and will include the initial value m of the material introduced to the process. For the reason that ordinary accounting practice does not include the cost of capital, however, the sum of the ultimate unit cost for each change in form or grouping of parts into an assembly will not equal the ultimate unit cost of the finished product. These facts may be more fully appreciated by reference to Table XIII, where the relation of the ultimate unit cost for each phase has been carefully outlined,[2] and certain items can be described as

I_t = the total cost of capital.
I_a = the cost of capital employed in the assembly process.
I_p = the cost of capital employed in a fabricating process.
I_r = the cost of capital employed in the purchase of raw material.

Only that part of the ultimate unit cost which does not contain the cost of capital can be carried on as an item to be included in the ultimate unit cost for the next phase of the process, in which case it will appear as the cost of the materials consumed. In other words, the cost of capital cannot be cumulative. When it has once come into existence it cannot be carried further, and must be diverted until it reappears as one of the costs of doing business, in which form it may rightly be assessed to the finished

[1] See Eqs. (55) and (56) in Table XIII.
[2] See Eqs. (53) to (54).

product at the time its sale is consummated. Only in this manner can it become a part of the total unit cost as understood by accountants today.

Changes in the Ultimate Unit Cost between Phases.—Whenever a unit of production is changed in form or is permanently grouped with others in a subassembly or a final assembly, a distinct change occurs in the economic balance which prohibits any extension of the ultimate unit cost beyond this point on account of the new factors that have been introduced. This may be readily appreciated when one realizes that a variety of parts may enter into such an assembly, each varying in quantity from the other, and that the method employed in their manufacture may be also widely divergent in nature. This situation assumes such proportions in actual practice that any mathematical expression which might be devised to handle this problem as a whole would become so complex and unwieldy that it would be entirely worthless. Similarly, raw material may enter into the construction of a number of different parts, and these in turn may never be brought together again into the same final assembly.

Determination of the Ultimate Unit Cost for Each Phase.—Accordingly, an ultimate unit cost must be individually determined for raw materials, fabricated parts, and assemblies. If this be true then the economic-production quantity for each of these must be separate. As a result we have an economic-purchase quantity Q_r for the first, an economic-production quantity Q_p for the second, and an economic-assembly quantity Q_a for the third. The last two may be determined in exactly the same manner, the data in one instance being that which applies to a fabricated part and the data in the other being that for an assembly. The economic-purchase quantity[1] will be determined in a slightly different manner because certain items are of a different nature due to the purchasing function involved, but, in the last analysis, the basic principles in the make-up of its ultimate unit cost will be the same.

Relation of Cost Factors When the Cost of Capital Is Retained as a Part of the Value of a Product.—On the contrary, if it were

[1] See Davis, R. C., Minimum Cost Purchase Quantities, A.S.M.E. *Trans.*, January–April, vol. 50-MAN, No. 6, p. 41, 1928.

common accounting practice[1] to include the cost of capital in the cost of a unit of production which would be used to evaluate the article for inventory purposes when finally placed in stock, the ultimate unit cost on this basis could then be used directly as the material cost for the next stage in the process to which the article would be transferred. In this case, then, the cost of capital would not have to be removed from the total cost and carried, as before, to the cost of doing business. This means that the ultimate unit cost of purchased raw material, whatever its form, would be the material cost for the fabricated part, the sum of the ultimate unit costs for all parts would be the material cost for the assembly, and the ultimate unit cost for the assembly would be the factory cost of the final product to which the unit cost of doing business would be added in order to obtain the cost of the article at the time of sale. Under these conditions all the ultimate unit costs can be added together to find the total unit cost accruing to the product up to the time it is ready to leave the factory.

Joint Costs.—In this case it would not be difficult to set up a mathematical expression for the joint economic-production quantity, that could be of practical advantage, even though it might be somewhat more complex in form. The chief objection to such a formula would be the complication which arises when raw material or fabricated parts are used in more than one assembly. Therefore in general a joint economic quantity[2] can be successfully applied only where there is but a single product manufactured in any given plant. Cases of this sort might be found in the automotive industry or the textile industry, but in all probability, where they exist, continuous production is in operation and lot production is no factor, giving little cause for the determination of a joint economic quantity.

Evaluation of the Ultimate Unit Cost.—The two methods of handling the cost of capital as supported by these two points of view make it necessary to set up two expressions[3] for the ultimate unit cost U_c and U_i or any article manufactured on a lot basis. The first, which conforms to the common practice of accounting, assumes that the inventory value of any article in

[1] See p. 160.
[2] See p. 109.
[3] See Eqs. (61) and (62), p. 168.

stock is the unit cost of material m, labor l, and overhead o together with the unit allotment of the preparation charges P/Q on the basis of the quantity produced in the lot. This particular group of cost items will henceforth be termed the "unit-manufacturing cost" u_m. The second, which contemplates including the unit cost of capital with the unit-manufacturing and storage costs, according to Mr. Scovell's theory, assumes that the ultimate unit cost U is the value of any article in inventory. Accordingly, these two methods of evaluating the ultimate unit cost may be expressed mathematically as follows:

$$(1) \qquad U_c = c + \frac{P}{Q} + \frac{I_s}{Q} + \frac{I_w}{Q} + \frac{V_s}{Q} + \frac{V_w}{Q} + \frac{L_d}{Q}, \qquad (61)$$

$$(2) \qquad U_i = c + \frac{P}{Q} + \frac{I'_s}{Q} + \frac{I_w}{Q} + \frac{V_s}{Q} + \frac{V_w}{Q} + \frac{L'_d}{Q}, \qquad (62)$$

where

U_c = the ultimate unit cost excluding the cost of capital for inventory purposes.

U_i = the ultimate unit cost including the cost of capital for inventory purposes.

Q = the quantity produced in the lot.

c = the unit cost of material, labor, and overhead.

P = the total preparation costs.

I_s = the total cost of capital employed in storing the finished article valued upon the manufacturing cost $c + \frac{P}{Q}$ alone.

I'_s = the total cost of capital employed in storing the finished article, valued upon the ultimate unit cost U_i.

I_w = the total cost of capital employed in the manufacture of the lot.

V_s = the total space charges accruing to the lot during the time of storage of any article manufactured in the lot.

V_w = the total space charges (if any) accruing to the lot during its manufacture.

L_d = the total loss due to deterioration of articles in stock valued at their manufacturing cost.

L'_d = the total loss due to deterioration of articles in stock, valued at their ultimate unit cost.

Furthermore, these expressions can be made to conform with the preceding analysis for any phase of manufacture as shown in Eqs. (55) and (56) in Table XIII, if all items of cost over and above the material m introduced at the beginning of the process, which accrue to the unit of production during that phase and its subsequent storage period, be combined into one term A representing the accumulation of value, and if the relation $u_m - m$ be inserted to show that portion of the manufacturing cost u_m which is added by the direct application of labor.

Expressions for the Practical and Exact Solutions.—These two expressions for the ultimate unit cost will yield different formulae for the economic-production quantity, for the reason that in the second one, the ultimate unit cost U_i is expressed in terms of itself. This arises from the fact that the factors I'_s and L'_d contain the term U_i, as will be more clearly shown in later chapters.[1] The method for determining an economic quantity from the first expression will be called the "practical solution," because it conforms to the actual practice most commonly found in cost-accounting systems of the present day; that from the second expression will be termed the "exact solution," because from a theoretical standpoint it more accurately represents the financial situation by including the cost of capital with the manufacturing cost of an article as it accumulates. These expressions will be developed at greater length[2] after an investigation has been made into the composition of each factor. It is presupposed that the first expression which yields the practical solution will be of most general value, as the second expression, unfortunately, demands the compounding of the cost of capital in each successive process. It is undeniably admitted by accountants on both sides that there can be no cost upon the cost of capital as would be the case in this instance and this would preclude any arguments in favor of the latter method.

Basic Formulae for the Ultimate Unit Cost.—Returning for a moment to study these expressions from the basis of the type or nature of each factor, it will be seen that the term P for the preparation cost, as before, contains all items which are independent of the quantity but are dependent upon the lot itself. Similarly, the factors I_s, I'_s, I_w, V_s, V_w, L_d, and L'_d may be

[1] See Chaps. XIV and XVII, pp. 215 and 257.

[2] See Chap. XVIII.

classified in the group of terms which are dependent upon the quantity produced in the lot, except for certain constant items in their composition which will eventually drop out, as will be demonstrated[1] when it comes to the final derivation of the minimum-cost quantity. The third group will contain the unit-production cost c, together with these other constants. With this grouping in mind the basic expressions for the ultimate unit cost may be written as:

$$U_c = u' + \frac{P \cdot F}{Q} + f \cdot Q, \text{ (practical solution)} \tag{63}$$

$$U_i = u' + \frac{P \cdot F'}{Q} + (f' \cdot U_i + f'') \cdot Q + f''' \cdot U_i, \text{ (exact solution)} \tag{64}$$

where

u' = all constant or general items independent of either lot or quantity.

P = all items which are dependent upon the lot.

f = all items which are dependent upon the quantity.

f' = those items contained in f which depend as well upon the ultimate unit cost.

f'' = all other items contained in f which are independent of the ultimate unit cost.

f''' = all constant items dependent upon the ultimate unit cost.

F or F' = a coefficient employed to adjust item P for the passage of time.

Graphical Analysis of the Ultimate Unit Cost.—It may be instructive to digress for a moment and study the shapes of the curves U_c and U_i for the ultimate unit cost as derived for each of these types of solution. Referring to Fig. 20 it will be seen that for the practical solution the ultimate unit cost curve U_c has a minimum value for positive quantities Q, and that in this range it approaches the vertical line $Q = O$ for one asymptote and the sloping line[2] $U_k = u' + Q \cdot f$ for the other, which crosses the vertical ordinate at $U = u'$. It is an unsymmetrical curve which for small quantities has a relatively high unit cost and for large quantities one which approaches the unit cost of capital and storage space as a limit. In like manner the curve U_i for the exact solution (Fig. 21) has a minimum value for positive

[1] See pp. 172–173.

[2] In Figs. 20 and 21, the general expressions for these asymptotes have been replaced by more specific ones.

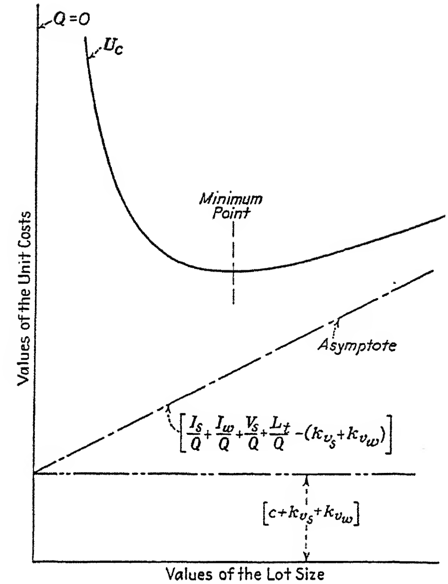

FIG. 20.—The ultimate unit cost and its minimum point as derived from the practical solution.

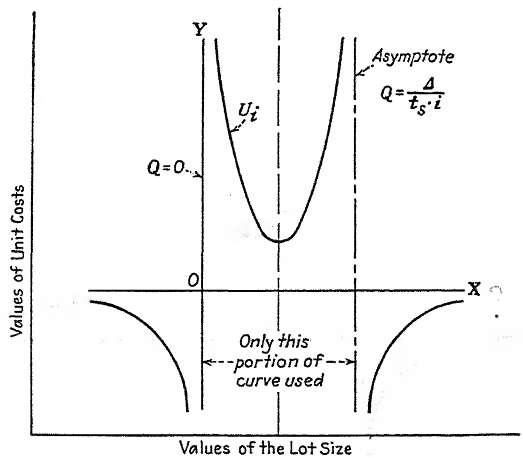

FIG. 21.—The ultimate unit cost and its minimum point as derived from the exact solution.

quantities, but is symmetrical around the minimum-cost point instead. Even though it approaches the same vertical line $Q = O$ for small quantities, its asymptote for large ones is the vertical line[1] $Q = \frac{1 - f'''}{f'}$. Accordingly, it is evident in this case that there is a positive production quantity which should never be exceeded, because, by the compounding of capital costs, the unit cost will become infinite. Fortunately, however, if one should compare the numerical values of U for identical values of Q along either of these curves between the limits $Q = O$ and $Q = Q_m$, the minimum-cost point, the variation between U_c and U_i will be insignificant except for very small values of Q. As the economic range will lie in the portion of the diagram where there is the least divergence, there is little need for apprehension over any discrepancy between the two results in determining an economic quantity, and the formulae hereafter derived from the practical mode of solution may be employed with little regard to the theories underlying accounting practice.

The Minimum-cost Point.—As in general the purpose[2] of all manufacturing operations is to produce at the lowest possible unit cost, the quantity which will achieve this ideal will be that one for which a minimum value U_m for the ultimate unit cost exists. This will be known as the "minimum-cost quantity" Q_m and may be found by differentiating either expression for the ultimate unit cost and equating the result to zero. This mathematical procedure is often used for such purposes in engineering problems and needs no explanation here. Accordingly, the minimum-cost quantity Q_m may be found as follows: From Eq. (63),

$$\frac{dU}{dQ} = O + \left(\frac{-P \cdot F}{Q^2}\right) + f = O,$$

$$Q_m{}^2 = \frac{P \cdot F}{f},$$

$$Q_m = \sqrt{\frac{P \cdot F}{f}}. \qquad \text{(practical solution)} \qquad (65)$$

[1] In Figs. 20 and 21, the general expressions for these asymptotes have been replaced by more specific ones.

[2] For the application of the minimum-cost point to the selection of the best manufacturing process see p. 106.

From Eq. (64),

$$U \cdot (1 - f' \cdot Q - f''') = u'' + \frac{P \cdot F'}{Q} + f'' \cdot Q,$$

$$\frac{dU}{dQ} = \frac{\left(O + \frac{-P \cdot F'}{Q^2} + f''\right) \cdot (1 - f' \cdot Q - f''') - \left(u + \frac{P \cdot F'}{Q} + Q \cdot f\right)(-f')}{(1 - f' \cdot Q - f''')^2} = O,$$

whereupon the expression for Q from the exact solution may be obtained by using the formula (Appendix XII)

$$Q_m = \frac{-P \cdot F' \cdot f'}{f} \pm \sqrt{\frac{P^2 \cdot F'^2 \cdot f'^2}{f^2} + \frac{P \cdot F'}{f}}. \quad \text{(exact solution)} \quad (66)$$

The actual procedure for this determination will be found in Table XXIX, where it also may be seen, when the factors Δ and θ are unity, that

$$Q \cdot f \cong Q(U \cdot f' + f'') + U \cdot f''',$$

which was substituted in the next to the last equation for the denominator of the last term under the radical in order to present here an expression which might be comparable with that for the practical solution. Furthermore, as the ultimate unit cost curves shown in Figs. 20 and 21 have but one minimum point for a positive value of Q, the plus sign before the radical will be used at all times.

The Utility of Each Type of Solution.—Either of these expressions for the minimum-cost quantity Q_m can be substituted into Eq. (52)[1] for the economic-production quantity Q_e in order that its true value may be obtained. This is evident from the fact that either of the two curves shown in Figs. 20 and 21 is almost identical for quantities less than the minimum-cost quantity, and, by definition, the economic-production quantity lies in this range. The first expression, as given in Eq. (65) for the actual solution, will be found most practical, first, because it is based upon common accounting practice, and, second, because its form is so simple. The second, as given in Eq. (66), is not recommended for general use from the reverse reasoning, and

[1] See p. 156.

henceforth will be omitted from further consideration,[1] except where it may be used for discussion on a theoretical basis or its derivation indicated in the course of establishing the composition of the various ultimate unit-cost factors. These will be taken up individually in the next chapters and explained in full. In order that this work will cover the whole field and show the derivation of all factors that at any time have been considered as influencing the problem, certain ones will be discussed and eventually discarded. Of course, in the practical application of economic quantities to the control of manufacturing operations, some of the items will be found to be extraneous in specific cases, and a method[2] has been devised whereby the most appropriate value for the factors included in the term f can be selected to insure reliable results.

[1] See PENNINGTON, GORDON, Simple Formulae for Inventory Control, *Manufacturing Industries*, vol. 13, No. 3, p. 199, March, 1927; also TAFT, E. W., The Most Economical Production Lot, *Iron Age*, vol. 101, pp. 1410–1412, May 30, 1918.

[2] See Chap. XIX.

CHAPTER XIII

MANUFACTURING COST

The most important item in the ultimate unit cost of a unit of production whether it be the finished product, a subassembly, or a fabricated part is the manufacturing cost. This cost represents the value which in addition to that for raw materials has accumulated to the unit of production in its finished form at the time it enters stores, or proceeds, if storage can be dispensed with, either as the material to be utilized in another process or as the final product on its way to a specific customer. It should account for all expenses incurred through payrolls or current bills for materials or supplies used in or contributing toward its manufacture, except, under certain circumstances to be described later,[1] those expenses which are incurred by the storing and safeguarding of such articles when held in stock. Ordinarily this cost would be considered the same as prime cost, but, owing to the various interpretations[2] given to this latter term, there exists too great an opportunity for a misunderstanding of its true meaning, and for this reason its use in this connection will be avoided throughout the rest of this discussion. Moreover, it should be recognized that the manufacturing cost does not include the cost of capital, as that charge accumulates, as does the cost of the storage space, during the time the article remains in stores. Both of these are distinct items in themselves and will have to be considered separately. Thus it may be said that the unit-manufacturing cost u_m is made up of the unit-production cost c and the unit allotment P/Q of the preparation charges on the basis of the actual quantity produced in any given lot. The fallacy of allotting these preparation charges on any other basis has been enlarged upon in a previous chapter.[3]

The Unit-production Cost.—The unit-production cost represents the actual money values incurred by the application of

[1] See Chap. XVI.

[2] See p. 91.

[3] See p. 89.

TABLE XIV.—EQUATIONS EMPLOYED IN THE DERIVATION OF THE MANUFACTURING COST u_m

$$c = m + l + o + a, \tag{67}$$

where

$$l = d_{p_1} + d_{p_2} + d_{p_3} \cdots + d_{p_n}, \tag{68}$$

or

$$= t_{o_1}d_{l_1} + t_{o_2}d_{l_2} + t_{o_3}d_{l_3} \cdots + t_{o_n}d_{l_n}. \tag{69}$$

$$o = t_{o_1}d_{o_1} + t_{o_2}d_{o_2} + t_{o_3}d_{o_3} \cdots + t_{o_n}d_{o_n}, \tag{70}$$

and for assemblies

$$m_a = U_p - (I_{s_p} + I_{w_p}). \tag{71}$$

For fabricated parts

$$m_p = U_r - I_{s_r} + c_x. \tag{72}$$

For purchases

$$m_r = p_p. \tag{73}$$

$$P = M + J + O + E_g + G, \tag{74}$$

where

$$M = m_M + l_M + o_M \tag{75}$$

$$= m_M + (d_{l_M} + d_{o_M})(t_{M_s} + t_{M_d}).$$

Hence

$$u_m = m + l + o + a + \frac{P}{Q},$$

or

$$= c + \frac{P}{Q}. \tag{76}$$

For interpretation of symbols see Chap. XIII or Appendix XIII, p. 349.

labor directly upon an article to accomplish either its fabrication or assembly into a finished product, together with the usual distribution of expenses arising from indirect labor and other departmental or general overhead items and the unit cost of material used as the basis of fabrication or assembly. All such items which enter into its total value for the lot are obviously dependent upon the quantity produced. This may be expressed generally by the relation given in Eq. (67), Table XIV,

where

c = the unit-production cost.
m = the unit-material cost at the time it enters the process.
l = the unit direct-labor cost.
o = the overhead distributed directly upon the actual time of manufacture.
a = an allowance for spoiled articles and any credits that can be returned from their salvage to offset this loss.

Evaluation of Direct-labor Costs and Overhead Distribution. Where a series of operations occur the unit direct-labor cost l is naturally equal to the sum of the basic piece rates d_p for each operation, as shown in Table XIV by Eq. (68), or, in the case of hourly wage rates, the sum of the product of the hourly rates d_l and the unit-operating time t_o for each operation. [See Eq. (69).] Similarly, in a series of operations, the overhead o borne by each piece is equal to the sum of the product of the unit-operating time t_o and the burden rate d_o for each operation regardless of the manner of distribution. [See Eq. (70).]

If the overhead is distributed on a labor-hour rate, the time factor t will be the direct-operating time per piece t_o, the same as used in the case of hourly wage rates. If the overhead is distributed on a machine-hour rate, the time factor t will be the direct-machining time per piece t_m. A third method may be employed for distributing overhead which is based upon the simple unit-manufacturing cost. This implies that every dollar of direct cost, whether derived from material, labor, or preparation allotment, must bear the burden of a certain number of dollars or fraction of a dollar of the overhead charges. In this special case no overhead will be carried by either the simple production cost $m + l + a$ or the preparation cost P separately, so that

$$o = \left(m + l + a + \frac{P}{Q}\right) \cdot d_u,$$

where

d_u = the burden rate on each dollar of simple manufacturing cost.

Evaluation of the Unit-production Cost.—Accordingly, a more comprehensive expression can be written for the production cost; namely:

(a) For piece rates and machine rates

$$c = m + \Sigma(d_p) + \Sigma(t_m \cdot d_o) + a.$$

(b) For labor-hour rates and machine rates

$$c = m + \Sigma(t_o \cdot d_l) + \Sigma(t_m \cdot d_o) + a.$$

(c) For labor-hour rates for wages and distribution or in the case where $t_o = t_m$,

$$c = m + \Sigma[t_o \cdot (d_l + d_o)] + a.$$

Evaluation of the Unit Cost of Materials Entering a Process.—Whatever the process may be, the material cost m will always equal the value of the article at the time it enters the process. If it be an assembly, it will equal the sum of all such costs m_a for each part entering into it, so that

$$m_a = m_{a_1} + m_{a_2} + m_{a_3} \ldots m_{a_n},$$

where the value for any one of these parts in general is

$$m_{a_n} = U_p - (I_{s_p} + I_{w_p}), \quad \text{See Eq. (71).[1]}$$

or

$$= c_p + \frac{P_p}{Q_p} + \frac{V_{s_p}}{Q_p} + \frac{V_{w_p}}{Q_p} + \frac{L_{d_p}}{Q_p}.$$

If it be a fabricated part, the material cost will equal the value of the raw material plus any cutting off charges, if these are not considered, as the first operation, or the value of the metal as it flows from the spout of the furnace in the foundry. In either case

$$m_p = U_r - I_{s_r} + c_x, \quad \text{See Eq. (72).[1]}$$

$$= c_r + \frac{P_r}{Q_r} + \frac{V_{s_r}}{Q_r} + \frac{L_{d_r}}{Q_r} + c_x,$$

[1] For an interpretation of symbols see pp. 165 and 168, or Appendix XIII, p. 349.

when

c_x = the extra cost of cutting off or the melting cost of the metal.

If it be a purchased article in the form of raw material, the unit cost c_r in this case will be the purchase price p_p. [See Eq. (73).] As this item enters only into the determination of an economic-purchase quantity, which involves factors of a slightly different nature than those used in an economic-production quantity, a further discussion of it will be left for a separate analysis of this specific phase of the general problem.

The Use of Standard Predetermined Costs.—As an economic quantity is primarily a measure for the control of production, its value must be determined from standard data. The unit-production cost used in its calculation will be the basic or predetermined unit cost obtained from time studies made during the process of selecting the best method for its production. These same time studies will naturally be the basis for rate setting and the payment of labor involved in the actual production of an article, and so it is reasonable to expect that the unit cost, when thus determined, will closely agree with the values obtained from operating records. Accordingly, the basic piece rate should be used in figuring the unit-production cost, as the bonus[1] paid to labor for increased production is earned out of savings due to the resulting improvement in the methods and is in fact only a measure of the divergence from the standard in a desirable direction. The bonus, therefore, is not a part of the standard cost and will in no way affect the best quantity to produce. Moreover, savings from any source are of an advantage and will result in a greater gross margin of profit, so that as long as the standard time allowance on each operation is unchanged, these particular savings will merely cause a greater return on the capital invested than was originally allowed; a situation which should be gladly welcomed.

Adjustment of Standard Costs for Improvements in the Method of Manufacture.—Of course, if conditions are sufficiently changed to warrant new time studies, in so doing one is merely capitalizing the improved methods, either to permit the use of

[1] See LYTLE, C. W., "Wage Incentive Methods," The Ronald Press Company, 1919.

a larger value for the expected rate of return in the economic-quantity formula or to achieve a reduction in the working capital by obtaining a still lower limit for the economic range. Only when the latter circumstance is desirable is there any need for calculating a new unit-production cost upon the new and appar-

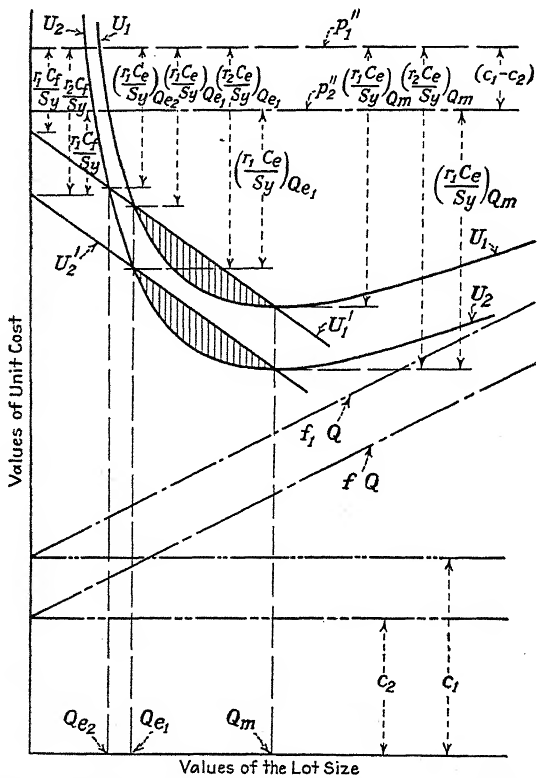

FIG. 22.—The effect of reducing the direct production cost c upon the limits of the economic range of production.

ently lower base rate. The effect of this may be demonstrated by reference to Fig. 22, where it will be seen that a reduction in the value of c will shift the position of the ultimate unit-cost curve U_1 downward in relation to the minimum-sales price p'_1 or the maximum allowable ultimate unit-cost curve U'_1, and extend the economic range to the left to a new point of intersection Q_{e_2} between the curves U_2 and U'_1. If there be no real advantage in conserving capital to a greater degree, the curve

U'_1 can also be moved downward to a position indicated by U'_2, parallel to U'_1, employing a larger value for the expected rate of return r_2, so that the intersection of the new curves for U_2 and U'_2 will still fall upon the former lower limit of the economic range Q_{e_1}. The savings under these circumstances will be represented by a larger gross profit[1] or can be passed on to the consumer by a reduction in the base or minimum unit-sales price as indicated by p'_2. To account for these savings in any other way, particularly if one attempts to introduce them into the problem through the inclusion of an allowance for the bonus in each case, would involve great complication that would lead to no real gain except that of having the satisfaction of being able to represent mathematically each minute detail of the manufacturing operations. Therefore the bonus had best be left entirely out of the problem.

Allocation of Overhead Accruing on Idle Machine Time.—The distribution of overhead upon the basis of actual machine-operating time implies the separate consideration of idle time, which is not the case if overhead is distributed on a man-hour basis. In the latter instance direct labor bears the general overhead and the labor incidental to machine changeover maintenance and repair may or may not carry overhead, as in the opinion of the executive seems most desirable. Then idle time can be disregarded entirely. In the former instance overhead is distributed on a machine-hour rate, calculated on the basis of available machine-operating time, and so if the machine is ever out of production or idle for any cause, a certain amount of overhead will accumulate, which cannot be prorated to any units of production. Executive opinion in recent years has seemed to favor this method because it provides a means of controlling idle time,[2] and so it has become customary to charge the resulting amount of undistributed overhead to the production division account as a penalty for allowing the machine to be idle owing to poor production control or to their failure to prevent the breakdowns. Furthermore, the overhead which accumulates on set-up and dismantling time will naturally appear in the preparation cost as it should as a logical part of that expense. To complete this

[1] Compare the gross return $\frac{r_1 C_e}{S_y} Q_{e_1}$ with $\frac{r_2 C_e}{S_y} Q_{e_1}$

[2] See GANTT, H. L., Productive Capacity, a Measure of the Value of Industrial Property, A.S.M.E. *Trans.*, vol. 38, p. 876.

picture, however, overhead, accumulating from idle time caused by a lack of sales orders, should, on the other hand, be charged to the sales division, as a penalty for its failure to fulfill its responsibility of keeping the plant supplied with orders; or, if the idle time is due to an over amount of machine capacity, the overhead from this cause should be charged to the executive division as a penalty against too great an expansion program or a lack of foresight. This method of treating overhead on a machine-hour basis has been enlarged upon even though it is but a side issue to the problem of economic quantities, because these penalties for idle time provide an additional factor which can be used as a valuable measure of management and a check upon the production-control methods.

An Allowance for Spoilage and Salvage.—The allowance a for spoilage and salvage in the unit-production cost may or may not be required, depending upon the accounting practice. The piece-rate method of wage payment takes care automatically of spoilage, as imperfect articles are not counted in compiling wage figures; however, some means must be provided for including the material loss and any loss due to the previous accumulation of labor values and overhead which are always present. Salvage can account for some of it but not all. In other wage plans spoilage and salvage will probable have to be specially accounted for. In any event, it must be remembered that an economic quantity must depend upon standard conditions, and so an allowance for spoilage and salvage can be introduced only when experience shows that a uniform amount of waste is bound to arise from this source, which can justifiably be assessed to each perfect unit produced. An excellent method of determining this average allowance or loss per lot or per year has been given in a very interesting treatise[1] on this particular subject by W. A. Shewhart, which has been found to be quite practical even though it involves the theory of probability.

Mathematical Procedure Adaptable to Current Accounting Practice.—The foregoing discussion is primarily intended to assist in any analysis of manufacturing operations which may lead to an improvement in conditions. It was presupposed that any ordinary cost-accounting system could supply the necessary

[1] See Shewhart, W. A., "Quality Control," Bell Telephone Laboratories, Reprint B-277, November, 1927.

MANUFACTURING COST CARD

	Operation	Machine changeover				Size of lot	Operation time	Labor cost		Overhead		Spoilage	
	No.	Total time	Labor rate	B'd'n rate	Total cost			Wage rate	Unit cost	B'd'n rate	Unit B'd'n	% loss per op.	Unit cost
	n_o		d_{lM}	d_{oM}			t_{on}	d_l		d_o			
	Totals	T_M	—	—	M	Q	t	—	l	—	o	—	$+a$

Total unit material cost	=	m	Total brought forward	=	c
Total unit labor cost	=	l	Unit allotment of		
Total unit overhead cost	=	o	Machine changeover cost	=	M/Q
Total unit spoilage cost	=	$\underline{+a}$	Production control cost	=	O/Q
			Tool, etc. preparation cost	=	$(J+E_g)Q$
Total unit salvage value	=	$\underline{-a}$	General expense	=	G/Q
Unit production cost	=	c	Unit manufacturing cost	=	u_m

FIG. 23.—A typical cost card arranged for the collection of data required in the computation of the unit manufacturing cost.

data directly from its current records for use in accurately determining an economic quantity. The manner in which the system is laid out or how it is worked in detail is of little importance here, except that it must always be emphasized that the least amount of effort in obtaining accurate data is always desirable. To show how this data can be directly withdrawn from the accounts, a typical cost card is shown in Fig. 23. Here again its arrangement is of the least importance. The symbols for each item of data are given in the columns or spaces to which they refer, so that it will be readily seen where the appropriate values are obtained. In fact any system of cost accounting which provides means for segregating those items of cost which are independent of the quantity in any lot from those which are dependent upon the quantity will suffice.

Segregation of Cost Factors to Avoid Duplication.—Similarly, if an analysis shows that the bulk or size of the article demands special consideration, owing to the relatively large space any unit occupies when placed in stores, it should be possible to remove those items which enter into the cost of this storage space from the burden rate applied to direct-operating time whether on a man-hour or machine-hour basis. In extreme cases losses from deterioration should likewise be removed from the overhead. The reason for this is to prevent duplication of items in the various factors controlling the economic quantity. Their elimination from the method of overhead distribution in no way alters the fundamentals of the cost system; it merely means that there are just one or two less items to be added into the total overhead. In ordinary circumstances these two factors will not have to be considered separately, and therefore in general any method of overhead distribution will be satisfactory. In no case should overhead be omitted from the value of the unit-production cost, as such practice is most inadvisable and ultimately tends to increase rather than decrease the actual values in inventories, as pointed out in the chapter on errors.[1]

The Preparation Charges.—The preparation cost P is the one factor which contains all total charges incurred by the processing of a lot including the cost of issuing and controlling the production or job order, all of which are independent of the quantity produced in the lot. As such charges are incurred in

[1] See p. 92.

advance of the manufacturing of the unit of production, except possibly certain items which arise prior to specific operations or the movement of material due to features of the production control system, they represent the cost of preparing for the processing of that unit and can be legitimately termed "preparation costs." It is impossible to reduce these charges to unit costs, as the total charges of this nature accrue to the lot and not to the unit produced. To apportion these costs to each lot their sum for a given accounting period must be divided by the number of lots processed or the number of orders issued. In any event, they must be subdivided according to orders for assembled products, subassemblies (if used), and fabricated parts, and in the case of raw materials, purchase orders. All charges incurred by the function of purchasing must be left to a later discussion, as in some respects these items will differ from similar ones incurred by manufacture.

Machine Changeover Cost.—The chief item entering into the preparation cost P is the cost of machine changeover M which includes both the cost of setting up the machine from an inoperative condition and that for dismantling the machine to the same condition preparatory to the next lot of dissimilar articles to be processed over it. This neutral condition of the machine must be considered because if the dismantling cost of some preceding operation is included with the cost of set-up for the given operation, in prospect, a host of costs could be obtained which would be unintelligible. Therefore, in order to be consistent, the set-up and dismantling costs arising from the tooling of a machine for a given purpose only can be combined. In evaluating the machine changeover cost M, overhead may or may not be included, depending upon current accounting practice. If it is, it may be derived in any one of the three ways described for direct-operating time upon the piece. If it is distributed on a manufacturing cost basis no specific burden can be applied to the machine changeover time. If it is preferable not to include overhead in the cost M but to carry it all upon direct-operating time, the terms o_s and o_d may be omitted from Eq. (75) for the machine changeover cost, where

m_M = material and supplies consumed in setting up and dismantling the machine,

d_{l_M} = the wage rate for set-up and dismantling labor,

d_{o_M} = the burden rate for set-up and dismantling time on either basis,
t_{M_s} = the time for setting up the machine,
t_{M_d} = the time for dismantling the machine,

and the other symbols have their usual interpretation with appropriate subscripts to indicate their identity.[1] The term m_M has been introduced merely to recognize the fact that some material and supplies are consumed in this preliminary operation, and it need not be used if such items are carried to the overhead account.

Tool Costs.—The next item of similar order is the cost J of preparing the required tools, jigs, or fixtures or repairing them prior to their use. The cost of sharpening tools during the process is not a preparation cost, as it recurs at intervals depending upon the number of pieces produced. If desirable, the initial cost of sharpening tools may be included, as this is a portion of the true preparation costs. Similarly, machine maintenance is not a preparation cost, as this item occurs regularly in order to keep the machine or the manufacturing equipment up to the required degree of efficiency and is a departmental or general expense independent of both the quantity and the number of lots processed. Likewise, the cost of special tools, jigs, and fixtures or special adjustments of the equipment, due to the processing of an order for a piece other than a standard, should not be included in the regular total cost of tools, etc., which is allotted to the regular or recurring orders, as such an expense is only chargeable to the special order and should not increase the cost of the regular ones.

Production Control Costs.—The third item is the cost O of issuing production orders and supervising and controlling them throughout the process. It is simply the total operating cost of the production-control department. It should account for order writing, planning, scheduling, routing, disposition of material, and follow-ups. The value to be used will equal the sum of all charges accruing to this department when prorated on the basis of the number of orders issued for all subdivisions of the whole process for a definite period of time. In some instances, the cost of inventory control, of material handling, and internal transportation, and even that of storing and safe-

[1] See Appendix XIII, p. 349, for complete interpretation of symbols.

guarding of articles in stock may be included as well in this item, if any of these charges are apparently independent of the quantities processed on each order.

Technical and Engineering Expense.—Any technical and engineering expense E_g which is required for the layout of the process or the design of a standard unit of production, including the cost of checking and approving specifications, and any drafting-room work of an incidental and related nature, which cannot be attributed to the development expense or the cost of special articles, is likewise a part of the preparation costs. Such an item would also include the expenses of operating a control laboratory for current physical and chemical analyses, as well as the cost of quality control or inspection which depends upon the number of orders and not upon the lot size.

General and Administrative Expense.—Finally, any total items G from the general or administrative account of the manufacturing plant that relate to the whole scheme of operations and the successful production of the final products may be withdrawn from the overhead, as normally made up, and prorated in like manner to the number of orders for each type of process, and thence to each lot produced, should a more reliable method of control be thus obtained. The effect of such a practice will be negligible upon the final manufacturing cost, and is well worth the change when a clearer understanding of the cost of controlling various parts of the process is required. It is quite possible that the fabricating or the assembly process bears more than its proper burden of these general expenses under the usual accounting practice, and, if so, this provides the necessary warning so that steps may be taken to correct the situation.

Characteristics of the Preparation Charges.—Here again the detailed analysis of the preparation costs has only been introduced to permit, when desired, an analysis of manufacturing operations which will expose hidden sources of loss or errors in accounting practice and to provide means to overcome them satisfactorily. For the determination of an economic-production quantity these items may be handled in the usual accounting manner and prorated as totals to the cost of each lot. Accordingly, if these items are combined, the expression for the preparation cost will be given by Eq. (74), Table XIV, wherein the symbols refer to the various items in the order in which they have been discussed.

Dual Category of Labor Costs.—The author has found cases in industry where it is impossible to segregate those labor costs, which are independent of or dependent upon the size of the lot, out of the total costs for direct labor which should be allotted to the unit of production. In such instances it has been evident that a certain amount of direct labor is applied to the lot as a whole and a certain other amount to each unit. A typical example of this is in a textile-finishing mill where a given consignment of cotton cloth is run through the mercerizing processes in a single piece, the ends of each roll of cloth being sewn together.

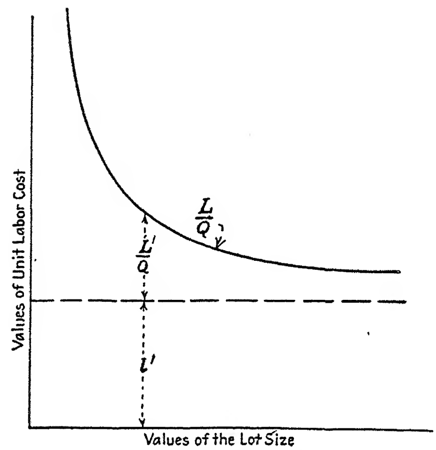

FIG. 24.—A typical chart employed in the determination of the constant and variable elements of direct-labor charges.

At the end of the process, labor is applied to cut the total lot into the required lengths, fold them and wrap them according to the wishes of the customer. The labor in the mercerizing part of the process is chargeable to the lot; the labor of cutting the cloth and wrapping each bolt separately is chargeable to the final package.

To segregate these items, a curve of the resulting total unit costs can be drawn, as shown in Fig. 24, for various sizes of lots from which the equation of the curve may be determined. In general it will have the form

$$\frac{L}{Q} = \frac{\alpha' \cdot L'}{Q} + \beta' \cdot l',$$

where

L = the total direct-labor cost,
L' = the total labor applied directly to the processing of the lot as a whole,
l' = that portion of L/Q which can be directly assessed to each unit in the lot,

and α and β are constants which represent a specific solution. The reader is referred to any authoritative book on graphical analysis for the mathematical procedure for determining this equation from the curve or a series of points for varying sizes of lots plotted on a sheet of cross-sectioned paper.[1] The usual method employed is not difficult to apply, and when the constants α and β for the equation are once found, they can be used indefinitely in a variety of cases. To avoid recalculation upon a change of conditions, a series of curves for varying wage rates may be plotted and the constants obtained for each. In all probability the mathematical method will be too complicated for practical purposes for a limited number of cases. If enough points taken from actual cost records can be plotted to show the trend of the total unit direct-labor costs, however, the fact that it will eventually, with large enough lots, approach the value of that cost which can be attributed to the labor applicable to each unit as a limit, will make it possible to estimate what this will be and to draw a line (represented by l' in Fig. 24) across the diagram. Then for any lot size the value as measured by the distance lying above this line to the curve L/Q, when multiplied by the quantity produced in the particular lot, will be that portion L' of the total direct-labor cost which must be chargeable to the lot as a whole and not prorated to each unit.

Evaluation of the Unit-manufacturing Cost.—It is now possible to summarize this chapter by writing the complete expression for the unit-manufacturing cost u_m which will be in general

$$u_m = m + l + o + a + \frac{P}{Q},$$

or

$$= c + \frac{P}{Q}, \qquad \text{See Eq. (76).}$$

[1] See LIPKA, JOSEPH, "Graphical Analysis," John Wiley & Sons, Inc., 1918.

and, in the special case just referred to, where the total labor cost L must be expressed as

$$L = \alpha' \cdot L' + \beta' \cdot l' \cdot Q,$$

or the unit labor cost as

$$\frac{L}{Q} = \frac{\alpha' \cdot L'}{Q} + \beta' \cdot l',$$

the expression becomes

$$u_m = m + \beta' \cdot l' + o + a + \frac{\alpha' \cdot L' + P}{Q}, \tag{77}$$

or

$$= c' + \frac{P'}{Q},$$

where

$$c' = m + \beta' \cdot l' + o + a,$$

and

$$P' = \alpha' \cdot L' + P.$$

When the simple unit-manufacturing cost (overhead omitted) is employed as the basis for the distribution of the burden,[1] the expression becomes

$$u_m = \left(m + l + \frac{P_o}{Q}\right) \cdot (1 + d_u)$$

or

$$= \left(c_o + \frac{P_o}{Q}\right) \cdot (1 + d_u), \tag{78}$$

where

$$c_o = (m + l),$$

and

$$P_o = P \text{ (with overhead omitted).}$$

Graphical Analysis of the Unit-manufacturing Cost.—The actual relation between the unit-production cost and the total preparation charges may be shown graphically by drawing the curve for the unit-manufacturing cost u_m as illustrated in Fig. 25. As the cost u_m is a unit value, the unit-production cost c can be represented by a horizontal straight line at a uniform distance above the X axis, because its value does not change with

[1] See p. 177.

the quantity. On the other hand, the unit allotment of the preparation costs P/Q does vary with the quantity and for large lots is extremely small, increasing rapidly for smaller lots. This admirably demonstrates the reason for segregating all total costs which are independent of the quantity from those which are dependent upon it. Accordingly, it will be seen that the unit-manufacturing cost curve u_m follows the shape of tne curve for

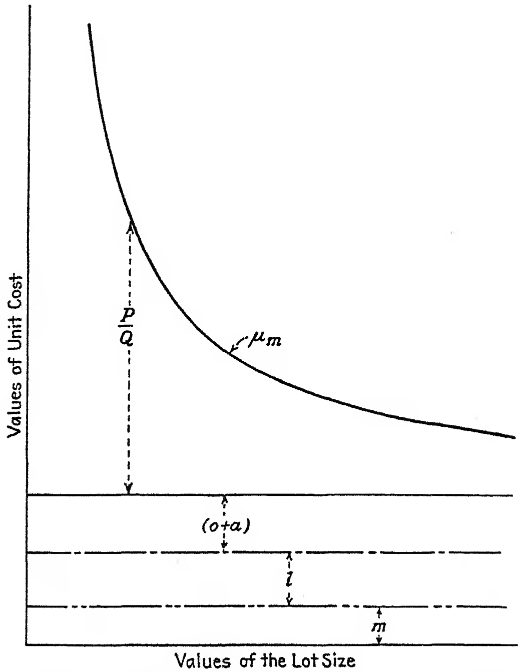

FIG. 25.—The relation of cost items entering into the composition of the unit manufacturing cost.

the unit allotment of the preparation costs but approaches the Y axis where Q equals zero for one asymptote and the horizontal line u_m equals c for the other when Q is infinite, as then the unit cost P/Q approaches zero as a limit.

Application of the Unit-manufacturing Cost.—As the value of u_m in either of the expressions referred to above is the total unit value of all items incurred by the production of any type of article up to the time that it enters stock or is removed from the process for assembly or sale, this value may be used as the

unit value of each article held in stores inventories or as a part of the material cost in assembly or the factory cost, if the article is to be sold immediately. Only when the article first remains in stores for a time, no matter how short, will there be any cost of storage that should be added in such cases to the unit-manufacturing cost, if the ultimate unit cost is to be properly determined.

CHAPTER XIV

COST OF CAPITAL

A. INVESTED IN ARTICLES IN STOCK

Much has been said in previous chapters[1] about the cost of capital invested in inventories and the expected return upon the capital thus employed. In the determination of the true minimum-cost quantity only the cost of capital can be considered and not the sum of the cost of capital and the expected gross return, where the interest rate and the normal expected rate of return are combined in one figure in order to give proper recognition to the risks assumed in the ordinary conduct of business, as the more general practice has been in recent years. The cost of capital I invested in inventories whether it be for articles in stores or work in process should be computed upon the basis of the value of the total number of articles produced in the lot, with due regard for the manner in which it has accumulated to each unit, over the average time that is required to produce or consume the entire lot, whichever the case may be, by multiplying this average time-value figure by the interest rate, expressed as a decimal, that ordinarily would be paid on borrowed capital. As the cost of capital accumulates to the lot over the period that any unit of production remains in the process or in stores, it cannot be added into the cost of any one of these articles until all have actually been removed from the manufacturing area or withdrawn from inventories on current orders. Therefore these items in the ultimate unit cost must be kept distinct and cannot be included in the unit cost of manufacture.

Interpretation of the Cost of Capital.—It makes little difference whether this capital was originally invested by the stockholders or whether it was borrowed on a short- or long-time basis. The owner of this capital, whosoever he may be, is entitled to a return for the use of his money, even if there is no express provision made for such payments in the indenture of stock ownership as

[1] See p. 86; also Chaps. II and III.

there is in the case of bonds or other forms of time or commercial loans. Any stockholder should recognize the fact that the first part of his dividend represents a return in payment for the use of capital invested in the business at the prevalent money rate, and that any other part in excess of this is the return due him as an investor for risking his funds in the business and is his share of the actual profits, whether they be real or otherwise. Few realize that a dividend rate less than the current money rate represents a distinct loss. This condition may be easily explained if one actually takes the point of view that in declaring such a dividend the full payment has been made for the use of the capital, and in so doing a net business loss has been incurred instead of a profit. Then in order to account for it the expected rate of return which represents the liability in such a case must be negative instead of positive; and the algebraic sum of this factor and the interest rate will naturally yield the reduced dividend rate.

Investment Charge on Articles in Stock Defined.—Now, if the general definition for the cost of capital be specifically applied to articles in stock, it will be found that the investment charge I_s in this case will equal the product of the total value of the average number of articles held in stock, the time during which the last one of the lot to be removed remains there, and the interest rate as defined above. Owing to the fact that there is a divergence of opinion with regard to the correct accounting method to be employed when the investment charges are included as an item of the ultimate unit cost, the value of I_s must be expressed mathematically in two ways. If the cost of capital is to be excluded from inventory evaluation, the first method of expressing the investment charge on articles in stock is represented by Eq. (79), Table XV,

where

I_s or I'_s = the total cost of capital invested in articles in stock.
Q_v = the average number of articles held in stock during the sales-turnover period.
u_m = the unit-manufacturing cost of these articles $= \left(c + \frac{P}{Q}\right)$

TABLE XV.—EQUATIONS EMPLOYED IN THE DERIVATION OF THE INVESTMENT CHARGE ON ARTICLES IN STOCK

For the practical solution

$$I_s = Q_v \cdot u_m \cdot T_s \cdot i, \tag{79}$$

For the exact solution

$$I'_s = Q_v \cdot U \cdot T_s \cdot i, \tag{80}$$

if

$$S_a = \frac{S_y}{T_y} \tag{81}$$

and

$$S_s = Q = T_s \cdot S_a. \tag{82}$$

For uniform demand

$$T_s = \frac{Q}{S_a}, \tag{83}$$

For variable demand

$$T_s = \frac{Q}{S_{a_s}}. \quad \text{See Appendix II.} \tag{84}$$

For non-continuous production

$$Q_{r_N} = Q \cdot k_s, \quad \text{See Table XVI.} \tag{87}$$

For semicontinuous production

$$Q_{r_C} = Q \cdot \left(k_s - \frac{S_a}{2 \cdot D}\right), \quad \text{See Table XVI.} \tag{93}$$

For batch production

$$Q_{r_B} = Q \cdot \left(k_s - k_p \cdot \frac{S_a}{2 \cdot D}\right), \quad \text{See Table XVI.} \tag{98}$$

where for uniform demand

$$k_s = \tfrac{1}{2}$$

and

$$k_p = \left(1 - \frac{1}{n}\right). \tag{99}$$

For variable demand

$$k_s = 1 - \frac{1}{Q \cdot T_s} \cdot \int_0^{T_s} S_c \cdot dT, \quad \text{See Appendix III.} \tag{100}$$

when

$$S_c = \int_0^{T_s} S \cdot dT. \tag{101}$$

So, in general,

$$I_s = \frac{Q^2 \cdot i}{S_{a_s}} \cdot \left(c + \frac{P}{Q}\right) \cdot \left(k_s - k_p \cdot \frac{S_{a_s}}{2 \cdot D}\right), \tag{102}$$

and

$$I'_s = \frac{Q^2 \cdot i}{S_{a_s}} \cdot U \cdot \left(k_s - k_p \cdot \frac{S_{a_s}}{2 \cdot D}\right). \tag{103}$$

For an interpretation of symbols, see text, Chap. XIV, or Appendix XIII, p. 349.

T_s = the extent of time in days that the sales-turnover period lasts.

i = the interest rate usually expected on borrowed capital, as a decimal in terms of days.

The second method, which depends upon the theory that the cost of capital should be included with the manufacturing cost in evaluating inventories is illustrated by Eq. (80) where U equals

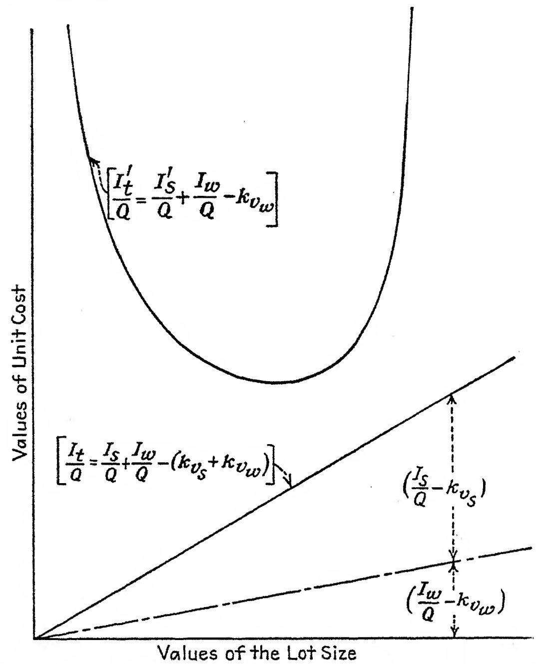

FIG. 26.—The investment charges I_t for the practical solution and I'_t for the exact solution.

the ultimate unit cost of the article, as previously defined, and all other items are the same as in the first expression. It will develop that when the investment charge I_s is allotted to each unit produced, the resulting expression[1] will be a straight-line function of the production quantity Q and the unit cost u_m in terms of dollars per piece, which, when plotted (Fig. 26) with Q as the abscissa and u_m or U as the ordinate, will show that the unit-investment charge increases steadily as the quantity produced increases.

[1] See Eqs. (104) and (105), p. 215.

The Length of the Sales-turnover Period.—Before attempting to evaluate an expression for the average number of articles in stock, it will be necessary to consider the length of time consumed in the sales-turnover period, as both depend upon the coordination of production schedules and the sales demand. Under ordinary conditions of business the sales demand can be assumed to be uniform even though it may only approximate the actual conditions, because whatever variations occur can be provided for by the choice[1] of the most suitable quantity to produce from those which constitute the economic range. It is absurd to expect that ordinary daily fluctuations can be sufficiently well foreseen to warrant their introduction into the problem. On the other hand, extraordinary situations must be considered as an emergency and treated as a special case. In any event it should be remembered that an economic quantity must primarily be founded on standard conditions, because its chief value lies in its usefulness as a measure of management and a supplementary guide in the determination of any production policy. When such an emergency arises no concern need be had over the additional requirements for capital, as the situation has probably developed from the receipt of an abnormally large order for that particular time of year, and the additional capital thus expended will again be available for other purposes as soon as the goods on that order have been shipped and billed. The charges which arise from the storage of the increased number of articles required by this order will be of little consequence, because in all probability shipments can be made as rapidly as the final product is completed, and no large quantity will have to be held in stores for any length of time. When it is anticipated that the effect of an emergency of this sort will be prolonged, owing to the necessity of storing such articles while awaiting a release from the customer, the method[2] of determining the maximum-production quantity Q_m may be used in order to select the most desirable size for the production lot, if the apparent quantity required at any one time for production exceeds the minimum-cost quantity or the upper limit of the economic range.

The Influence of Variable Demand upon Production Schedules.—In cases where the demand is decidedly variable following

[1] See p. 73.
[2] See p. 78.

a fairly consistent cycle of seasonal fluctuations, special provision must be made for the production of a quantity which will be the most economical for the particular part of the cycle that is approaching. In this way production schedules and inventories can be controlled so that a more effective use of working capital and manufacturing equipment can be realized at the time of greatest need and not employed unnecessarily at a time which would only inflate inventories for no useful purpose. This stands to reason because too many articles may be produced at a time when the demand is slack, and the investment and storage charges on the finished articles in stock will be increased, as they will remain in inventory for a longer time and the additional charge cannot be absorbed elsewhere. The converse will be true if the demand in a given period increases, so that an unnecessary additional process lot may be required which will involve a further expenditure of working capital sooner than ordinarily expected. In the first instance, sales records of previous years, when referred to, will give ample data for determining the total annual sales or the average yearly rate of consumption S_y. Any general trend of business upward or downward for an article, or the product into which it is assembled, may be found by comparing these rates for a number of years and accounted for by a correction factor showing the percentage increase or decrease in the demand for each period. The application[1] of this factor has been discussed in Chap. VI and, as there pointed out, results chiefly in an increase in the number of sales turnover periods per year. Its ultimate effect was even more fully illustrated by the discussion[2] of the maximum-earning power which can be attained for a stipulated minimum-sales price and the corresponding unit cost with reference to the economic-turnover quantity where it was shown that the rate of consumption did not enter into the determination of the maximum return on all the capital invested, as changes in this rate are entirely absorbed by a proportionate increase in the number of turnover periods. In this manner capital may be conserved even under conditions of business expansion, as the total average amount of capital employed in the manufacture of a unit of production depends upon the size of each lot and not upon the total consumption or sales in the year.

[1] See p. 79.

[2] See p. 82.

Special Treatment for Variable Sales Demand.—Seasonal fluctuations of a violent nature cannot be provided for in as simple a manner. Recourse must then be had to sales analysis by means of which the volume of business for the immediate future can be forecast with reasonable accuracy. Considerable study has been made along these lines, and it has been proved that reliable information and data can be obtained for determining the prospective demand for any period. The methods of making such an analysis do not concern us here, as many valuable

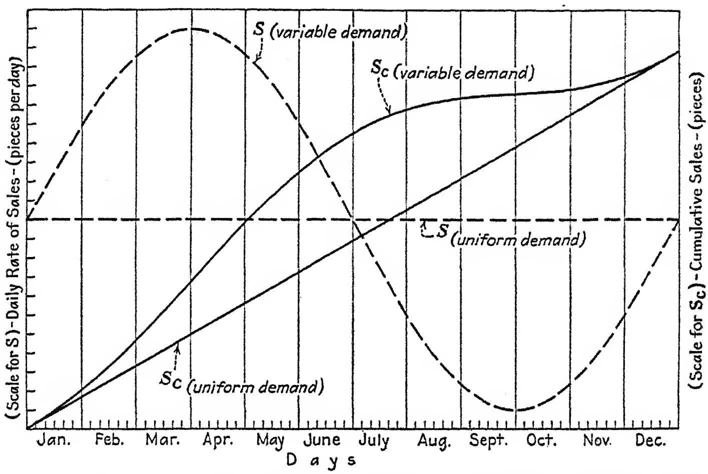

FIG. 27.—Typical curves for the daily rate of sales S and the cumulative sales S_c under conditions of variable demand compared with those for uniform demand.

articles and books have been written on this subject, and the reader is referred particularly to two books by Joseph H. Barber of the Walworth Company, "Budgeting to the Business Cycle,"[1] and "Economic Control of Inventory."[2] As a result of this method of attack, a typical curve, as shown in Fig. 27[3], may be obtained, from which appropriate corrective factors can be derived and introduced into the expression for the average-stock quantity Q_v. The actual determination of these quantities involves independent calculations.[4]

[1] The Ronald Press Company, 1925.

[2] Codex Book Company, 1925.

[3] See p. 212.

[4] See graphical method, p. 323, Appendix IV.

Evaluation of the Sales Period: Uniform Demand.—Turning for the moment to the simpler case for uniform demand, it will be realized that the length of the sales-turnover period T_s is quite dependent upon the quantity produced. If the yearly consumption rate S_y is reduced to terms of days by dividing it by the total number of working days per year T_y, the average daily rate of consumption S_a can be expressed by Eq. (81). Now it is obvious, if the quantity Q produced in a lot is to be sufficient to meet the sales demand which is anticipated over the time, T_s, that the total sales

$$S_s = T_s \cdot S_a \quad \text{See Eq. (82).}$$

for this period must be equal to the quantity Q. Accordingly, the length of the sales-turnover period in days will be equal to the ratio of the lot size Q to the average daily rate of consumption S_a as given in Eq. (83).

Evaluation of the Sales Period: Variable Demand.—If variable demand exists, the average quantity held in stock must be determined by finding the area under the typical curve plotted for the expected daily rate of sales S for each day over the sales period and dividing the value for this area by the duration of the period. An example of this curve is shown in Fig. 27. For ordinary purposes this curve may be obtained by plotting the sales that have occurred upon each day in question since the beginning of the period on the ordinate representing that day and repeating this operation for the whole sales period. The expected values for each day's sales thus required, which ought to have been prepared in advance by some suitable method of forecasting, may be obtained from the sales department. The approximate daily rate of sales at any point on this curve can then be obtained by summing the sales for an equal number of days either side of the day in question and dividing the total by the number of consecutive days for which the sales were totaled. Since the procedure for determining the average daily sales S_{a_s}, for a period where the demand is variable, implies the use of calculus, the mathematical basis for its derivation has been placed in Appendix II[1] in order not to burden the reader at this point with too much detail. It will be sufficient to state that for all practical purposes a graphical method has been

[1] See p. 317.

devised,[1] which will permit a rapid determination of the appropriate value of S_{a_s} for any period T_s without recourse to higher mathematics.

The Mathematical Expression for the Duration of the Sales Period.—As a result of the mathematical analysis referred to in the previous paragraph, it becomes evident that the average daily sales either for uniform demand S_a or for variable demand S_{a_s} can be used alike[2] in expressing the relation between the quantity produced and the duration of the sales period. The only difference between them lies in their method of derivation. It is important to note, however, in all cases where the average rate of consumption is expressed by the average daily demand S_{a_s} instead of S_a, that the value of S_{a_s} must be of necessity expressed in terms of days, owing to the nature of the mathematical procedure involved in determining its value. This will also require that the interest rate, when expressed decimally, be reduced to terms of days. If the value S_a is used, however, it may be expressed in terms of days, weeks, months, or years, whichever is most convenient according to the manner in which data is supplied from current records, but in any case the interest rate must conform. Accordingly, the general value of T_s to be used in the equation I_s may be expressed as

$$T_s = \frac{Q}{S_a} \quad \text{See Eq. (83),}$$

or

$$= \frac{Q}{S_{as}}. \quad \text{See Eq. (84).}$$

Types of Processes: Semicontinuous Production.—Returning now to a consideration of the average stock requirements, it will be advisable, first of all, to review the characteristics of various manufacturing processes with regard to the manner in which the raw material enters into production and passes through any operation in the sequence of manufacture, as well as the manner in which it is removed from the process and either delivered to stores, diverted directly to current sales orders, or transferred to another department to be combined with other units of production in the assembly of the final product. For the purpose of

[1] See p. 323.

[2] Compare Eqs. (83) and (84). Table XV.

this study industrial processes of an intermittent nature[1] will be divided into three types, even though G. D. Babcock employed six types in his classification.[2] To the first type belong those processes which are found in the more fortunate industries where continuous methods of production can be approximated, even though the manufacture of an article must be interrupted at intervals in order not to enhance inventories unnecessarily, owing to the fact that the rate of production in all intermittent processes is in excess of the rate of consumption. Under these conditions the process may be termed "semicontinuous," because articles can be manufactured at a continuous rate for a stated period and withdrawals from work in process can be regularly maintained to meet current sales during the entire production period. Only those articles which were produced in excess of current needs will have to be placed in stock to supply further orders received during the period when production of these articles has ceased; however, the stock required in this case will be less than in any other except where continuous production can be maintained.

Types of Processes: Non-continuous and Batch Production. To the second type belong those processes of directly opposite characteristics, where all raw material in the lot must be brought to each machine in the manufacturing sequence before any single piece can enter into production, and where all parts or assemblies must remain at each machine until all units in the lot have been completed on that operation. This is due to the fact that the manufacturing equipment cannot be arranged in logical sequence and operated as a group or that the control expense to achieve a better situation is not warranted. Under these conditions the processes will be considered as "non-continuous." Thus no finished parts or assemblies can be withdrawn from work in process, as in the first case, until the final operation on the last piece has been finished. Accordingly, the inventory problem has obviously become more complex, as the entire number of articles in the lot must be placed in stock before any can be made available for actual consumption. This situation can be modified, however, if the lot can be conveniently subdivided evenly into a number of batches and each batch processed separately, even though all

[1] See p. 14.

[2] See p. 120.

units in a batch must proceed as in the case for non-continuous production. Thus, in the third type of process to be known as "batch production," it will be possible to overlap operations. That is, the first batch can be started on the second operation at the same time the second batch enters production at the first operation, and so on for each batch and each succeeding operation. As a result groups of articles may be withdrawn from the process and delivered to stores as each batch is completed, and at the same time a certain number can be withdrawn from the batch, as released, to meet current orders. By this means the inventory problem can be improved over that in the second case to such an extent that it can be made to approximate closely that which could be obtained in the first case, as the manufacturing period can then be made to overlap the sales period, except for the time required to produce the first batch.

Average Stock under Non-continuous Production.—With this clearer interpretation of industrial processes, it will be possible now to derive an expression for the average stock Q_v

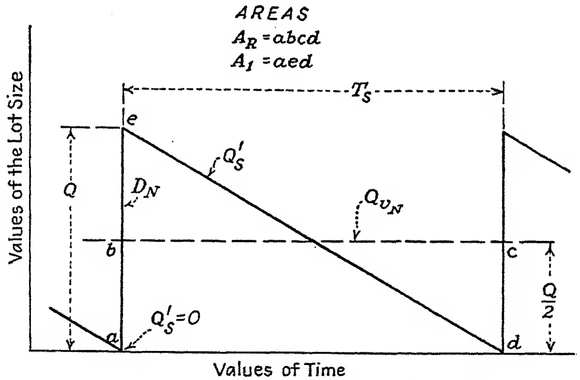

FIG. 28.—Stock curve, non-continuous production, uniform demand.

which can be used in the determination of the investment charge I_s in order to properly account for the average inventory, which must be maintained in accordance with the type of process employed. Turning to the simplest of all cases, that for non-continuous production will be first considered where the entire lot of articles produced Q are delivered at the same moment to stores ($D = \infty$).[1] The reason for this is due to the same fact

[1] See p. 207.

TABLE XVI.—EQUATIONS EMPLOYED IN THE DERIVATION OF THE AVERAGE STOCK

In general

$$Q_v = \frac{A_n}{T_s}. \tag{85}$$

For non-continuous production

$$A_1 = \tfrac{1}{2} \cdot Q \cdot T_s, \tag{86}$$
$$Q_{v_N} = k_s \cdot Q, \tag{87}$$

where

$$k_s = \tfrac{1}{2}.$$

For semicontinuous production

$$Q_{v_C} = \frac{A_2 + A_3}{T_s} = \frac{A_1}{T_s}$$
$$A_2 = \tfrac{1}{2} \cdot (Q - q) \cdot T_d, \tag{88}$$
$$A_3 = \tfrac{1}{2} \cdot (Q - q) \cdot T_s - T_d, \tag{89}$$
$$A_1 = \tfrac{1}{2} \cdot (Q - q) \cdot T_s, \tag{90}$$

but

$$q = S_a \cdot T_d, \tag{91}$$

and

$$T_d = \frac{Q}{D}, \tag{92}$$

so

$$Q_{v_C} = Q \cdot \left(k_s - \frac{S_a}{2D}\right) \tag{93}$$

where

$$k_s = \tfrac{1}{2}.$$

For batch production

$$Q_{v_B} = \frac{A_1 - A_2}{T_s} = \frac{A_3}{T_s} \quad \text{See Appendix V.} \tag{94}$$
$$A_2 = \tfrac{1}{2} \cdot Q \cdot T'_d, \tag{95}$$
$$= \frac{1}{2} \cdot \frac{Q^2}{D} \cdot \left(1 - \frac{1}{n}\right),$$

where

$$T_d' = \frac{Q}{D} \cdot \left(1 - \frac{1}{n}\right).$$
$$A_1 = k_s \cdot Q \cdot T_s, \tag{96}$$
$$= k_s \cdot \frac{Q^2}{S_a},$$

where

$$k_s = \tfrac{1}{2} \text{ (as before)},$$

and

$$T_s = \frac{Q}{S_a} \text{ (as before)}.$$
$$Q_{v_B} = \frac{k_s \cdot \frac{Q^2}{S_a} - \frac{1}{2} \cdot \frac{Q^2}{D} \cdot \left(1 - \frac{1}{n}\right)}{\frac{Q}{S_a}}, \tag{97}$$
$$= Q \cdot \left(k_s - k_p \cdot \frac{S_a}{D \cdot 2}\right), \tag{98}$$

where

$$k_p = 1 - \frac{1}{n}.$$

For an interpretation of symbols, see text, Chap. XIV or Appendix XIII, p. 349.

that all the operations performed on each piece must be completed before any one of them can be removed from the process for any purpose whatsoever. In such an event the manufacturing period must entirely precede the sales-turnover period and there can be no possibility of their overlapping; hence the time T_d which would represent such an overlap, if it really did exist, must be zero. This situation can be illustrated by the diagram given in Fig. 28 where it can be readily seen that the average number of articles Q_v which, if they remain in stores for the full period

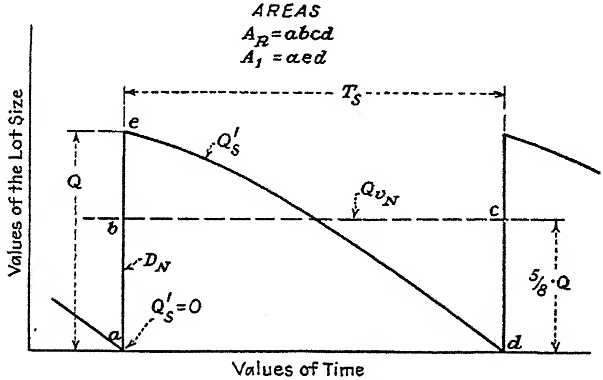

FIG. 29.—Stock curve, non-continuous production, variable demand.

T_s, will incur the same investment charge, as would be incurred by a gradually changing number of articles, will be equal to one half the actual quantity produced in the original lot.[1] This is obviously true because the area of any triangle is equal to one half the base times the altitude, and if the base be the same, as in this case it is T_s, the rectangle having an equal area must naturally have a height but half as great. Accordingly, if the fraction ½ be designated by the term k_s, which will be known otherwise as the "average stock factor," Eq. (87) can be used to represent the average stock Q_{v_N} for non-continuous production.

Average Stock under Semicontinuous Production.—In the case of the other extreme, semicontinuous production, where the process can proceed continuously throughout the manufacturing period, so that articles can be withdrawn from work in process at the instant each is removed from the last operation, it is possible to have the manufacturing period overlap the sales

[1] See Eqs. (85) and (86), Table XVI.

period to a considerable extent. Of those articles removed during this overlapping period some can be used to fill current orders while others at the same time will be placed in stores to provide for future orders. This situation has been illustrated by Fig. 30 where q equals the number of articles diverted to current orders, which never will enter the inventory, and T_d equals the time that the manufacturing period overlaps the sales period. In this case one must divide the diagram into two triangles, one having the base T_d and the other the base $T_s - T_d$. The altitude for each will be the same, as the quantity $Q - q$,

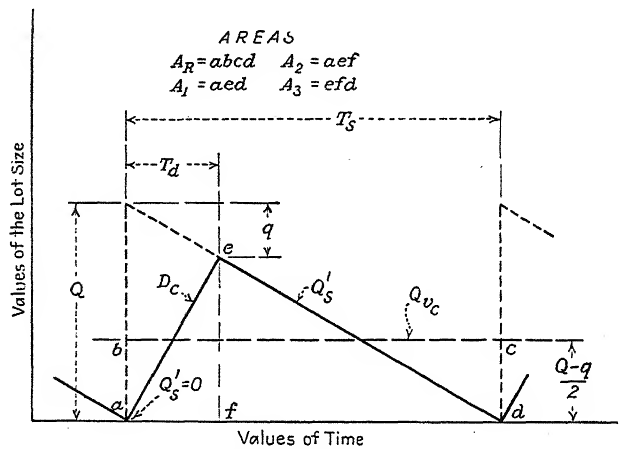

FIG. 30.—Stock curve, semicontinuous production, uniform demand.

which is in stores at the end of the manufacturing period, is common to both. This quantity will not be equal to that manufactured because of those articles which never reached stores, and so these must be subtracted from the total number produced in the lot, and the altitude of these triangles will be equal to $Q - q$.

The Influence of the Rate of Delivery to Stores.—Now if the previous method of reasoning is again applied, it is evident that the rectangle, which will have an area equal to that of the two triangles[1] and a base equal to T_s, will have an average altitude equal to one half that common to the two triangles. If the length of this vertical line can be expressed in terms of the lot size, it can be employed to evaluate the average stock. The quantity q diverted to meet the existing demand will naturally

[1] See Eqs. (88), (89), and (90), Table XVI.

be equal to the average daily demand S_a times the number of days T_d consumed in the overlapping period [Eq. (91)], and this time will be equal to the ratio of the quantity produced to the number of pieces D that are regularly removed each day from the process [Eq. (92)]. If these relations be combined as shown in Table XVI, Eq. (93) can be employed to represent the average stock Q_{vc} for semicontinuous production. As a result a considerable saving can be achieved in the investment charges over and above that incurred by a non-continuous process by the fact that articles can be withdrawn continuously. This saving is repre-

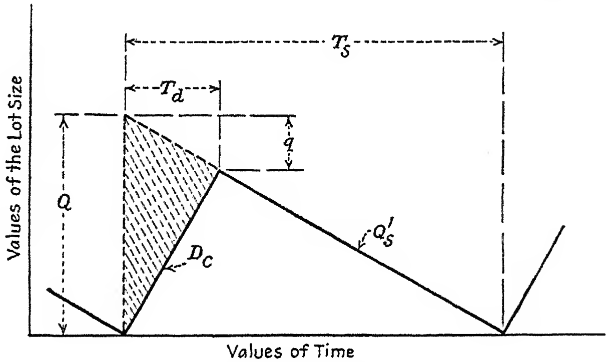

FIG. 31.—Savings by semicontinuous production over non-continuous production, uniform demand.

sented by the reduction in the cost of capital due to the fact that the articles q need not be carried in stores and that the only charge will arise from those articles $Q - q$ which do enter the inventory. This has been shown graphically by the shaded area in Fig. 31.

Average Stock under Batch Production.—In the most general case, batch production, where the lot can be subdivided into a given number n of equal groups or batches, a comparable saving in the investment charge, approaching that for semicontinuous production, can be realized as well, which has been illustrated by the shaded area in Fig. 32. This can be achieved if the size of each batch is determined by the number of pieces which can be conveniently carried in a group from one operation to another, so that each operation overlaps the other, reducing the total production time once it has been set up and can be continuously maintained. Accordingly, once production has begun, batches

will be completed at uniform intervals and can be delivered to stores at the rate D in pieces per day, and thereupon will be available to meet withdrawals from stock. In other words, the principle of batch production approximates the principle of

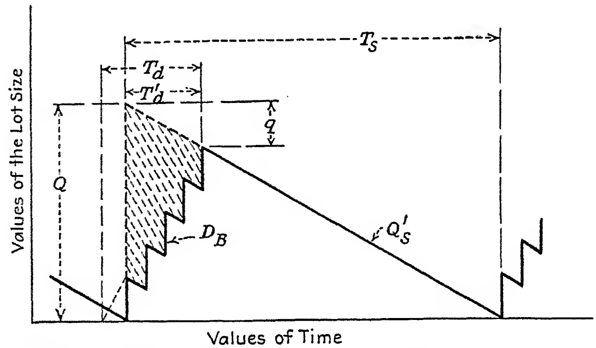

FIG. 32.—Savings by batch production over non-continuous production, uniform demand.

continuous production (Fig. 33,) and if an infinite number of batches are arranged it becomes continuous production. Batch production has the advantage of reducing: first, the time required

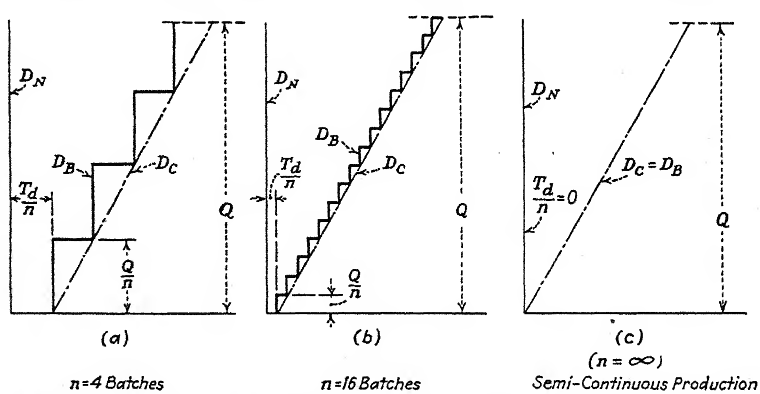

FIG. 33.—Effect of increasing the number of batches n in a production lot on the delivery of articles to stores D, where D_N is for non-continuous production, D_B for batch production, and D_C for semicontinuous or continuous production.

between the order point and the withdrawal of all stock; second, the minimum-stock required; third, the average stock in stores during the remaining part of the production period, where it overlaps the sales period; and, most important, the total invest-

ment charge. The resulting effect on the stock is shown by Fig. 32.

Evaluation of the Process Constant k_p.—The value for the average stock Q_{v_B} to be used in this case cannot be obtained by quite the same process of reasoning as employed for either of the other two, because the relations are much more complex. It can be evaluated, however, if the area A_3 in Fig. 34, bounded by a, f, d, can be found, because it will then be possible to divide its value by the total time for the sales period T_s and get the

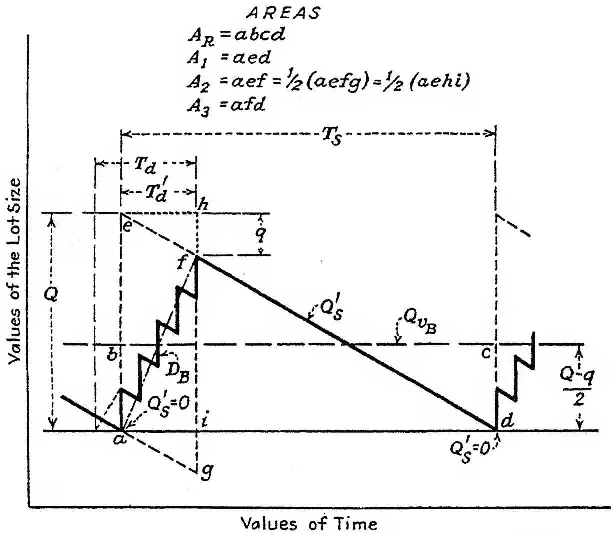

FIG. 34.—Stock curve, batch production, uniform demand.

equivalent height of the rectangle which will have the same area, expressed in terms of the average quantity, which would incur the same investment charge that accrues to the varying number of articles in stock over the same length of time. This can be done because the area A_3 is equal to the difference between the area A_1 and the area A_2 [Eq. (94)]. It can be shown that the irregular area A_2 bounded by a, e, f, as also illustrated by the shaded portion of Fig. 32, has an area equal to that of the rectangle whose base ai is measured by the length of time that the manufacturing period overlaps the sales period. In this case

$$T'_d = \frac{Q}{D}\left(1 - \frac{1}{n}\right),$$

the height of which $\frac{1}{2}$ (hi) is equal to one half the total quantity produced. At the same time, the area A_1, bounded by a, e, d, will have the same value as that used in the case of non-continuous production [Eq. (86)]. The mathematical expressions for these areas and their component factors are given by Eqs. (95) and (96) in Table XVI, and the proof for the geometric relations shown in Fig. 32 has been worked out in greater detail in Appendix V, page 328 if the reader cares to follow through the process of thought underlying the method of reasoning. Accordingly, if the expressions thus derived be now substituted in Eq. (94) and Eq. (97) simplified, it will be found that Eq. (98) can be directly employed to obtain the value for the average stock Q_{v_B}, where batch production proves to be advantageous, and where

$$k_p = \left(1 - \frac{1}{n}\right)$$

which will henceforth be designated as the "process constant" in order to account for the manner in which the lot is produced.

Application of the Process Constant.—It should be noted at this point that the expression for Q_{v_B}, as given in Eq. (98), is of universal application, because if the correct value for n is used in k_p, any of the previous expressions for Q_v may be obtained directly from it. For example, in an intermittent process where the production is non-continuous

$$n = 1,$$
$$k_p = 0,$$

or semicontinuous

$$n = \infty,$$
$$k_p = \tfrac{1}{2},$$

and if

$$n = 0,$$
$$k_p = -\infty,$$

showing that no lot has been processed and no production has occurred. Another interesting fact to observe is that if the number of batches is in any way some function of the quantity produced, which can be expressed by

$$n = \frac{Q}{q_c},$$

where q_c equals the definite number of pieces per batch, such as the number of pieces which any standard container will hold,

$$k_p = 1 - \frac{q_c}{Q},$$

and when this is carried through in the factor I_s for the minimum-cost quantity, all resulting constants can be disregarded and the expression for the average stock, as given in Eq. (93), where $k_p = \frac{1}{2}$, can be used instead of that given in Eq. (98). This should not be interpreted to mean that a consideration of batch production is valueless in any attempt to use it as a basis for improvement in manufacturing operations, for it actually can be utilized to indicate the manner in which the disadvantages of a supposedly unchangeable non-continuous process can be altered in order to approximate the advantages which can be completely realized only when the process can be made semicontinuous. In reality it is the basis for a general discussion of the average stock factor in any type of process wherein the cases for non-continuous or semicontinuous production are special.

The Average Stock Factor, k_s: Uniform Demand.—The diagrams given in Figs. 28, 31, and 32 illustrate the average stock condition under uniform demand for non-continuous, semicontinuous and batch production, respectively. The diagrams given in Figs. 29, 35*a*, and 35*b* illustrate the average stock condition for each one of these different types of processes when variable demand exists. In comparing these two sets of diagrams it will be seen that the line, which represents the number of articles held in stock at any instant Q'_s, which depends upon the rate of consumption or sales, is in the first case a straight line and in the second case a curved line, having the same general equation as given on page 318 in Appendix II. When the line Q'_s is straight, the average stock Q_v may be determined in the manner explained above for non-continuous production where the average stock factor $k_s = \frac{1}{2}$. Where the line Q'_s is curved, it cannot be so easily derived, but it is evident that the average stock factor k_s may have a value greater or less than $\frac{1}{2}$. If the demand increases during the sales period, as illustrated in Fig. 36*a*, k_s will be greater than $\frac{1}{2}$, and if the demand decreases, as illustrated in Fig. 36*b*, k_s will be less than one half.

The Average Stock Factor: Variable Demand.—To obtain a method for evaluating the average stock factor k_s under conditions of variable demand, one must return to the relation expressed in Eq. 84[1] for the average daily demand S_{a_s} and extend the mathematical analysis employed in its derivation so as to

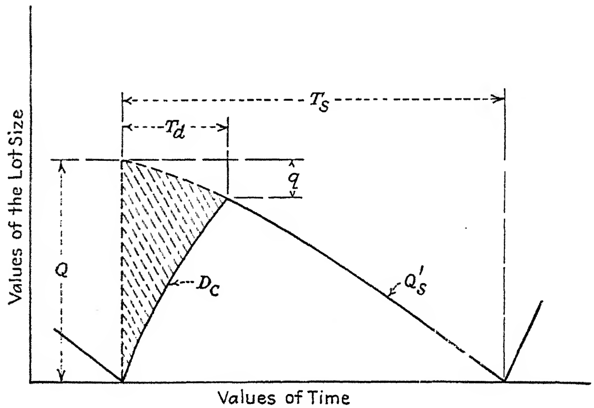

FIG. 35a.

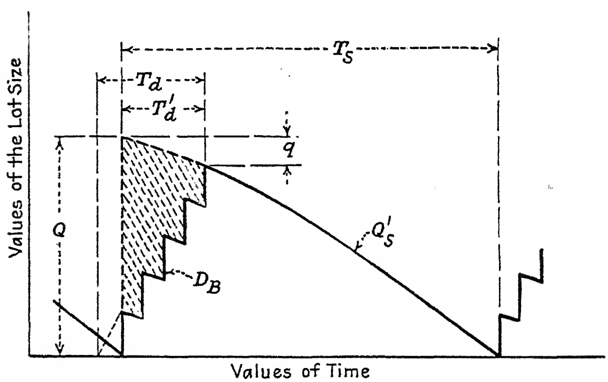

FIG. 35b.

FIGS. 35a and 35b.—Savings by semicontinuous production *a* and by batch production *b* over non-continuous production, variable demand.

obtain an equivalent expression for Q_v. When this has been done, the appropriate value for k_s will be obtained through the ratio of the average stock Q_v to the quantity Q produced in the lot. The particular treatment required in developing this phase of the subject is again quite complex and so it has also been placed in

[1] See p. 201.

Appendix III,[1] if any one is interested in following through the method of reasoning. For purposes of the present discussion, it should only be necessary to indicate that when variable demand

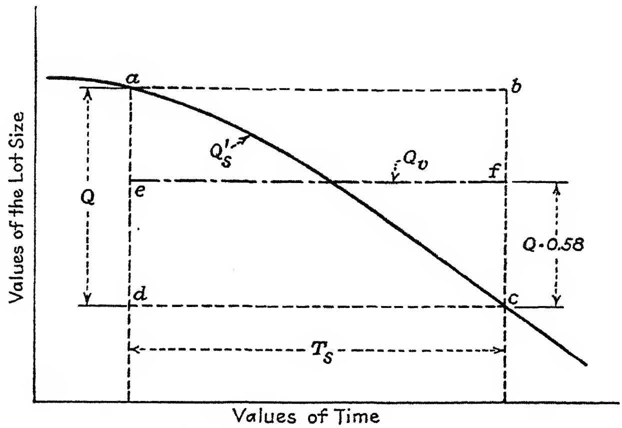

FIG. 36a.

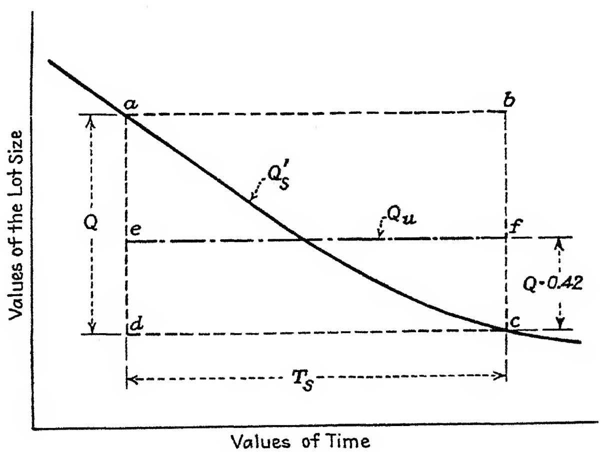

FIG. 36b.

FIGS. 36a and 36b.—Influence of increasing a or decreasing b rates of consumption upon the average stock.

enters into the problem the value for the average stock factor may be derived from the expression

$$k_s = 1 - \frac{1}{Q \cdot T_s} \cdot \int_o^{T_s} S_c \cdot dT, \text{ See Eq. (100).}$$

[1] See p. 320.

and the relation

$$T_s = \frac{Q}{S_{a_s}}.$$

In the solution of an actual problem the value for k_s should be derived by means of the graphical method presented in Appendix IV and then the determination of the investment charges I_s[1] can be made accurately to conform to the actual situation which is taking place in any period.

Application of the Average Stock Factor.—By the introduction of a factor for the average stock, the complication of having to evaluate an expression for the daily demand curve and the subsequent computations which would result may be dispensed with. Once an appropriate value has been assigned to the factor k_s, no further concern need be had for changes in the total consumption or sales, provided that the characteristics of the seasonal or business cycle remain unchanged for the period. This is due to the fact that the influence of the average stock factor is derived not from the magnitude of the lot size and the rate of consumption but from the relation of these two items in accordance with the particular business trend that may occur in a given period. It may be necessary to have available various values of k_s, each one representing characteristics of a given period, as, for instance, in the typical case of Fig. 27, if the sales period for a lot occurs between March and May, or between September and October, k_s will be $\frac{1}{2}$. If it occurs between May and September, however, k_s may be $\frac{1}{3}$ or even $\frac{1}{4}$, or between October and the following February it may be $\frac{2}{3}$ or even $\frac{3}{4}$, depending upon the rate of increase or decrease during the various periods. These values for k_s apply only when the process is intermittent. If the process is continuous, no regular stock of the article need be maintained to anticipate future sales, owing to the fact that the production rate equals the consumption rate, and this situation is recognized by the value of k_s, because it will be zero under those circumstances.

Other Factors of the Investment Charge I_s.—The other items entering into the investment charge I_s are the manufacturing cost u_m or the ultimate unit cost U and the interest rate i. All of these have been discussed in detail; the first two in previous

[1] See p. 194.

chapters[1] and the third at the beginning of this one.[2] It is now possible to write the complete expressions for I_s which will apply to either uniform or variable demand, as given in Eq. (102) for the practical solution and in Eq. (103) for the exact solution. Should it be necessary to account for the distribution of overhead on the basis of manufacturing cost, Eqs. (104) and (105) should be employed instead of Eqs. (102) and (103), where $c_o(1 + d_u)$ replaces c in the item v_s.

Evaluation of the Investment Charge on Articles in Stock.—In applying these expressions for the investment charge upon articles in stock to the general formula for the ultimate unit cost, it may be of advantage if the various groups of terms are split up into those factors which relate to the value of each unit v_s and to the average time t_s that is used as a basis for computing the cost of capital, as well as those terms grouped under k_{v_s}, which will be constant when the investment charge is reduced to a unit item. Under these conditions

$$\frac{I_s}{Q} = Q \cdot v_s \cdot t_s \cdot i + k_{v_s} \tag{104}$$

where

$$v_s = c$$

or

$$= c'$$

or

$$= c_o \cdot (1 + d_u), \text{ (as the case may be)}$$

$$k_{v_s} = \frac{P \cdot i}{S_{a_s}}\left(k_s - k_p \cdot \frac{S_{a_s}}{2D}\right),$$

and

$$t_s = \frac{1}{S_{a_s}}\left(k_s - k_p \cdot \frac{S_{a_s}}{2D}\right).$$

Similarly,

$$\frac{I'_s}{Q} = Q \cdot U \cdot t_s \cdot i, \tag{105}$$

where t_s is the same as that given above.

Either of these expressions may be used to determine the investment charges for raw material, a fabricated part, a subassembly,

[1] See Chaps. XII and XIII.

[2] See p. 193.

or a finished product which arise from its storage while awaiting some form of consumption. Owing to the fact that joint economic quantities are not feasible, except in the most remote instances,[1] each storage function must be kept distinct as the investment charges arising from them will be employed separately in the determination of the economic-purchase quantity, the economic-production quantity and the economic-assembly quantity in accordance with the manner in which each has been incurred.

Universal Application of Investment Charge Factors.—This grouping of the various terms which enter into the composition of the investment charges I_s or I'_s will be of particular importance when the time comes to evaluate the expression [Eq. (52)] for the economic-production quantity. In the analysis of the gross return on invested capital and its relation to the minimum-sales price which may be apportioned to any unit of production in accordance with its position in the process of manufacture, it was stated that the initial capital invested in the storage of articles upon which a return should be earned depended upon the value of those articles as they lay in stock and the average time during which any one of them remained in storage. Characteristic symbols were then assigned each of these items, but no attempt was made to determine their identity. Now that it has been possible to evaluate the cost of capital in inventories, it is evident that the value-time factors v_s, t_s, and k_{v_s} found in the composition of I_s should be likewise employed as the basis for the return from this phase of the manufacturing operations, as naturally the capital investment will be identical in each case. Accordingly, the relations represented by v_s, t_s, and k_{v_s} in Eq. (104) will be utilized when, in Chap. XVIII, the general expression for the economic-production quantity Q_e is derived.

[1] See p. 166.

CHAPTER XV

COST OF CAPITAL (*Continued*)

B. INVESTED IN WORK IN PROCESS

If an investment charge is incurred by articles in stock, a similar investment charge must have been incurred while these same articles were undergoing production, if the total cost of capital invested in inventories is to be considered. The cost of capital invested in work in process should be computed from (*a*) the average value of each unit of production in the lot, with due consideration for the manner in which this value accumulates; (*b*) the average unit time of production which with (*a*), must be extended to include all units; and (*c*) the same interest rate which is expected in return for the use of borrowed capital. Even though the investment charge I_w for work in process has only completely accrued to the lot at the time the last unit has been completed and removed from the production areas, it cannot be included in the manufacturing cost of each article, for the reason[1] that at present accounting practice does not recognize the theory that interest or an investment charge is a part of the manufacturing cost or the value of articles in stores. Accordingly, the amount of these charges must be carried over into the cost of doing business in a manner similar to that provided for those arising from the storage of articles.

The Investment Charge on Work in Process a Major Factor. This does not prevent the use of the cost of capital employed in work in process as one of the major factors in the determination of a minimum-cost quantity. Only one or two of the earlier formulae[2] for the economic size of lot contemplate this factor and these were developed rather late in the history of this subject. It may have been justifiable from the point of view of the production executives at that time to disregard this item, as the obvious factor in the economic balance to them was the

[1] See p. 165.

[2] See Table XI, Chap. IX.

investment charge incurred by articles held in reserve for future orders. It is really a corollary to this idea to consider the investment charge on work in process, because by a natural process of reasoning it is evident that if one is important the other must also be, as each is a part of the total cost of capital. The omission of this factor can probably be best explained by the fact that it is the simplest expression for an economic lot size which will, in the last analysis, be adopted by any executive. Every one of them abhors any apparently unnecessary increase in non-productive effort. Notwithstanding the arguments in its favor, the investment charge on work in process may still be omitted, but if it is, the reasons[1] for so doing must be justified. On the other hand, it may be the investment charge on articles in stores that is of lesser importance, and in that case the preparation charges will be counterbalanced by those on work in process instead. In other words, one or the other of these charges must be applied in the economic balance as the cost of capital is the controlling factor.

Preliminary Evaluation of the Investment Charge I_w.—Thus, if a legitimate use for the investment charge I_w on work in process exists, it will be valuable to derive a mathematical expression for it from the definition given above, which for general purposes can be written as shown in Eq. (106)
where

C_w = the total capital invested in work in process due to the manufacture of the lot Q.
T'_p = the apparent total production time in days.
i = the interest rate usually paid on borrowed capital expressed as a decimal in days.

The total capital C_w upon which the investment charge should be based is equal to the total money value as represented by expenditure through payrolls and current bills for labor, materials, and supplies that have been consumed in some manner by the process. The aggregate of this value which accrues to each unit of production has already been determined and is included in the value for the manufacturing cost u_m. Accordingly, the total capital consumed in work in process will be based upon the total manufacturing cost for the lot, as given in Eq. (107).

[1] See p. 280.

TABLE XVII.—EQUATIONS EMPLOYED IN THE DERIVATION OF THE INVESTMENT CHARGES I_w ON WORK IN PROCESS

In general it may be stated that

$$I_w = C_w \cdot T'_p \cdot i, \tag{106}$$

and approximately that

$$C_w = Q \cdot u_m, \tag{107}$$

$$= Q \cdot \left(c + \frac{P}{Q}\right).$$

However, if actually

$$C_w \cdot T'_p = Q \cdot \left[c'' \cdot T_p + \frac{P}{Q}\left(\frac{T_M}{2} + T_p\right) \right], \text{ See Appendix VII.} \tag{108}$$

and if average values are introduced for c'' and $P \cdot k_M$

$$I_w = Q \cdot \left[m + k_a \cdot \left(l + o\right) \right] \cdot T_p \cdot i + \cdot P \cdot i \left\{ \frac{T_M \cdot k_M}{2 \cdot 4} + T_p \cdot [1 - k_M \cdot (1 - k_a)] \right\}, \tag{109}$$

$$= Q \cdot k_a \cdot \left[\left(\frac{1}{k_a} - 1\right) \cdot m + c \right] \cdot T_p \cdot i + P \cdot i \cdot \left\{ \frac{T_M \cdot k_M}{2 \cdot 4} + T_p \cdot [1 - k_M \cdot (1 - k_a)] \right\},$$

but

$$k_a = \frac{A_c}{A_o}. \text{ See Appendix VI for the expressions for } A_c \text{ and } A_o. \tag{110}$$

However, if

$$k_a \propto (0.6 \text{ to } 0.4) = \tfrac{1}{2},$$

$$I_w = Q \cdot \left[m + \frac{1}{2} \cdot (l + o) \right] \cdot T_p \cdot i + P \cdot i \cdot \left[\frac{T_M \cdot k_M}{8} + T_p \cdot \left(1 - \frac{k_M}{2}\right) \right],$$

or if $\left(\frac{m}{2} - \frac{m}{2}\right)$ be added

and

$$c = (m + l + o),$$

$$I_w = Q \cdot \left[\frac{m + c}{2} \right] \cdot T_p \cdot i + P \cdot i \cdot \left[\frac{T_M \cdot k_M}{8} + T_p \cdot \left(1 - \frac{k_M}{2}\right) \right]; \tag{111}$$

but if

$$T_p = Q \cdot t_p, \tag{112}$$

where in general

$$t_p = \Sigma t_N + \Sigma t_C + \Sigma t_B + \Sigma t_t + t_d, \tag{120}$$

See Tables XVIII and XIX, or VIII for calculation sheets.

or if

$$T_p = Q \cdot k_d \cdot t_p, \tag{128}$$

then in general

$$I_w = Q^2 \cdot \left[\left(\frac{1}{k_a} - 1\right) \cdot m + c \right] \cdot k_a \cdot k_d \cdot t_p \cdot i + P \cdot i \cdot \left[\frac{T_M \cdot k_M}{8} + Q \cdot k_d \cdot t_p \cdot \left(1 - \frac{k_M}{2}\right) \right], \tag{129}$$

or if

$$k_a = \tfrac{1}{2},$$

$$I_w = Q^2 \cdot c'' \cdot k_d \cdot t_p \cdot i + P \cdot i \left[\frac{T_M \cdot k_M}{8} + Q \cdot k_d \cdot t_p \left(1 - \frac{k_M}{2}\right) \right],$$

where

$$t_p = t_N$$

and when raw materials remain full time in the process, and finished articles remain full time in the process

$$c'' = \left(\frac{m + c}{2}\right),$$

or when raw materials remain no time and finished articles remain full time

$$= \left(\frac{c}{2}\right),$$

or when raw materials remain full time and finished articles remain no time

$$= \left(\frac{m}{2}\right),$$

or when raw materials remain no time and finished articles remain no time.

$$= (o).$$

NOTE.—When the flow of material varies with the type of process, employ $\left(\frac{m + c}{2}\right)$ and calculate t_p from Table VIII.

For interpretation of symbols, see text, Chap. XV, or Appendix XIII, p. 349.

The Accumulation of Capital Values.—Unfortunately, the problem is not quite so simple as this would seem to indicate, because the period of time over which the investment charges accumulate is different for each of the subdivisions of the unit manufacturing cost u_m. For that part which depends upon the unit-production cost c, the actual process time T_p is required, which as will be shown further on[1] is equal to the unit-production time t_p for the average piece in the lot multiplied by the number of pieces being processed together. For the other part, which depends upon the preparation cost P, the appropriate time will be half the machine changeover time T_M plus the actual total time of production T_p. Thus, if each part must be treated separately, the expression for the total investment charge must be changed for the moment into that given in Eq. (108).

Accumulation of Value from Labor and Overhead.—Even in this form, however, the latter expression does not as yet fully represent the true situation with regard to that part of the unit-production cost c which accounts for the total increment in value $(l + o)$ due to the application of labor and its attendant overhead. The fact is that this portion of the unit-production cost exclusive of material costs accumulates during the process at a rate equal to that at which the labor is applied to each piece, and so the unit capital thus being invested cannot be assigned its final value $(l + o)$ until the specific piece has been completed. Therefore the accumulation of value up to any designated instant must depend upon the degree of completion that has been attained at that particular stage in the process. Accordingly, a factor k_a must be introduced in front of the parenthesis $(l + o)$ in order to properly account for the average expenditure of capital from this source in accordance with the rate at which it is being employed, and then Eq. (109) or (111) can be utilized with complete assurance that it is truly representative of the actual situation when it comes time to insert the value for the total capital C_w into the final expression for the investment charge I_w.

The Average Value Factor k_a.—In no process does the accumulation of value proceed in a uniform manner, because in all probability the piece rates or the hourly wage of the worker performing one operation will differ considerably from that of the workers at the other operations in the sequence of production.

[1] See p. 227.

Similarly, burden rates may vary with machines, and all these variations must be taken into account in order to insure the use of the correct value for k_a. A typical case of this sort is illustrated in Fig. 37. If the wage and burden rates had been uniform throughout the process the straight dotted line would represent the actual accumulation of value, which would be independent of the unit time of production for each operation, and then the factor k_a could be assigned a value of one half. Since the normal

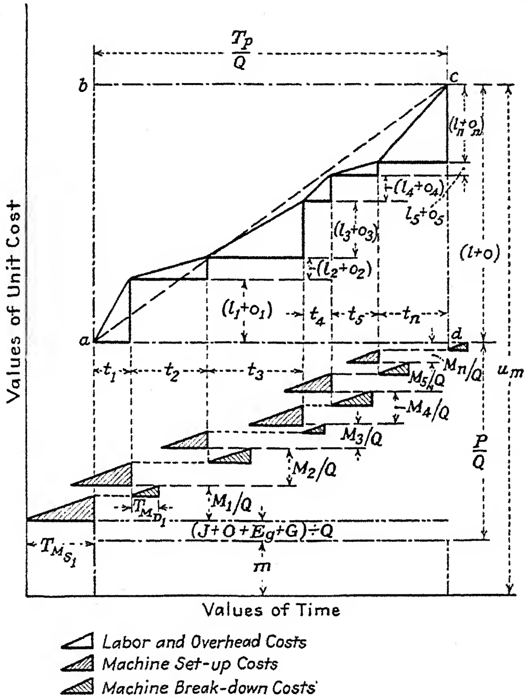

FIG. 37.—Accumulation of value to a unit of production while in process.

situation is never uniform, the actual value for k_a should be specifically determined for each case. The procedure for computing the value is too complicated for daily use, however, and a study of typical cases has shown that it need be employed only when a special analysis of a particular process is found to be advisable. Accordingly, the mathematical derivation of Eq. (110) has been placed in Appendix VI where any reader who is interested in following through the various steps required to evaluate k_a may find it on page 330.

Practical Application of the Average Value Factor.—Now, if actual values from a specific problem be introduced into Eq. (110), it will be found that in the majority of cases the resulting value for k_a will range between 0.6 and 0.4 with a mean of 0.5. If no more variation occurs than this, the value of $k_a = \frac{1}{2}$ can be used indiscriminately in all problems, as the error in no case will be greater than 1 per cent in its effect upon the final quantity. Therefore, if this value be introduced into Eq. (109) the expression for I_w can be reduced to the more practical form given in Eq. (111).

Accumulation of Value from the Preparation Charges.—A similar situation also arises with regard to the allocation of the preparation costs and the time over which an investment charge accrues to each unit of production upon each portion of these costs. Any one familiar with manufacturing processes will quickly recognize that the portion of the preparation costs derived from machine changeover is incurred throughout a process at such time, in advance of the start of any operation, as is necessary to put the machines or equipment in readiness. On the other hand, the remaining portion of the preparation costs derived principally from production control has for the most part attained a value, which, within all reasonable limits of accuracy, can be charged against the lot from the moment the first piece is put into production. Accordingly, in order to maintain the correct relationship of these various subdivisions of the preparation cost, so that the investment charges arising from their presence in the values accumulating to the unit of production will appropriately represent the actual situation, a factor k_M must be introduced which equals the ratio of the total machine changeover costs M to the total preparation costs P, as well as the average value factor k'_a, which is required, as before, to account for relative times for each operation with regard to the total process time T_p. Even though the set-up and dismantling periods occur at unrelated times during the process, it can be shown that a total time-value factor $M \cdot T_M$ can be employed instead of the sum of the individual time-value factors for each operation. Again, the mathematical analysis of this portion of the problem involves certain detailed explanations which would serve no real purpose here, and so they have likewise been placed in Appendix VII where they may be found on page 333. The

resulting complicated expression as shown in Eq. (109), however, can be materially simplified for practical purposes by the introduction of the normal value for k_a when it can be made equal to one half.

Unit Process Time: Non-continous Production.—As the interest rate has been discussed in previous chapters[1] on several occasions, the only factor still needing investigation is that for

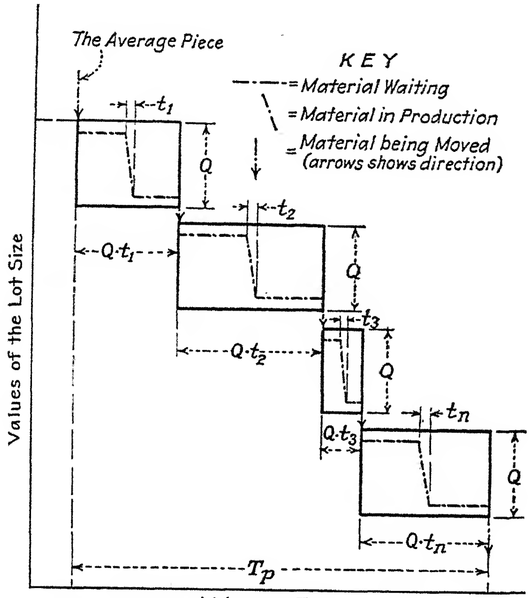

FIG. 38.—The relation of the flow of material and the total production time T_p for a non-continuous process.

the total process time T_p for the lot. In all other derivations of economic lot size formulae it was assumed[2] that T_p expressed in days equaled the reciprocal of the production rate in pieces per day for the whole process, multiplied by the lot size Q. This is true only when the process consists of one operation and occurs so seldom that it has relatively little value. In the simplest possible case of a process composed of a number of operations, each of the non-continuous type as illustrated in Fig. 38 where the whole lot remains at one operation until all

[1] See p. 193.
[2] See p. 94.

pieces have passed through it, the total production time T_p is equal to the sum of the unit-production times t_1, t_2, t_3 . . . etc., for each operation multiplied by the lot size Q, as shown by Eqs. (112) and (113) in Table XVIII, where for general purposes t_n equals the unit-production time for any operation and the subscript n denotes the serial number of the operation according to its position in the sequence. In this one special case it would be possible still to employ the rate of production in computing the

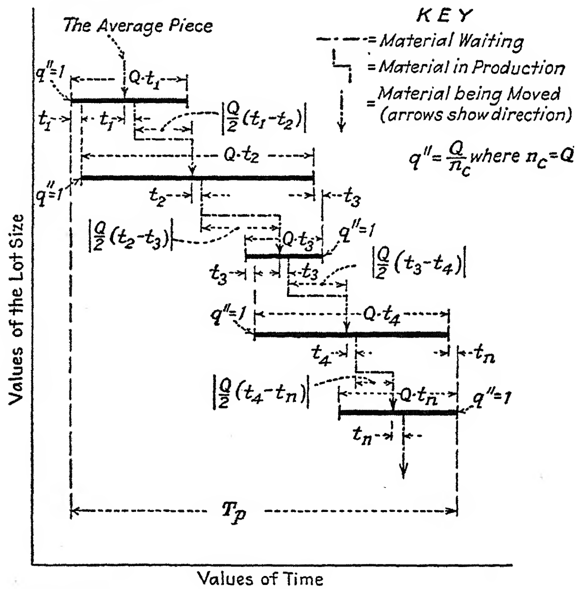

FIG. 39.—The relation of the flow of material and the total production time T_p for a semicontinuous process.

total process time, but it is entirely incorrect to use the apparent rate for the whole process, because such a figure is practically meaningless. The only legitimate procedure, if this is to be done, is to consider that the total time T_p is equal to the reciprocal of the sum of the reciprocals of the production rates for each individual operation multiplied by the lot size Q. Obviously computations of this order will be exceedingly complex, and, as they will require the conversion of data which will have to be obtained from current production-control records and could have been employed in its original form directly in any formula if its terms were so arranged as to include the time of production of any unit

in any operation with much less effort, little advantage can be gained by this method unless the rates for each operation are known in advance.

Influence of Flow of Work through a Process.—When the economic lot size is to be determined for a process, the nature[1] of which is far less simple, the sequence of operations and the

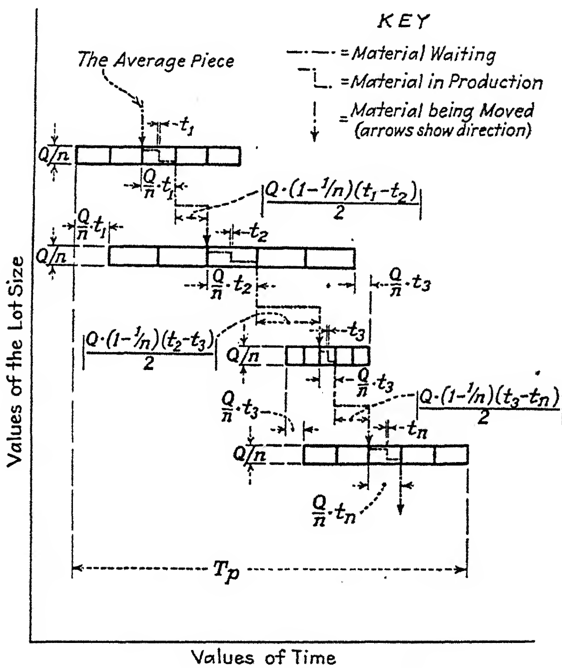

FIG. 40.—The relation of the flow of material and the total production time T_p for a batch process.

degree of continuity in the flow of material will complicate this situation to such an extent that the original interpretation of T_p must be discarded. In a semicontinuous process, where each piece can move independently of any other, the various operations in the sequence of production overlap to a marked degree, as shown in Fig. 39. Again in the case of batch production, where each batch moves independently but where the pieces in the batch cannot proceed until all have been completed on each operation, the operations will again overlap but to a lesser degree,

[1] See p. 201.

as in Fig. 40. Moreover, it is conceivable that the size of each batch may vary between operations and to complicate things still more, one type of process may be followed by another of a totally different nature, so that a situation similar to the one illustrated in Fig. 41 might easily arise.

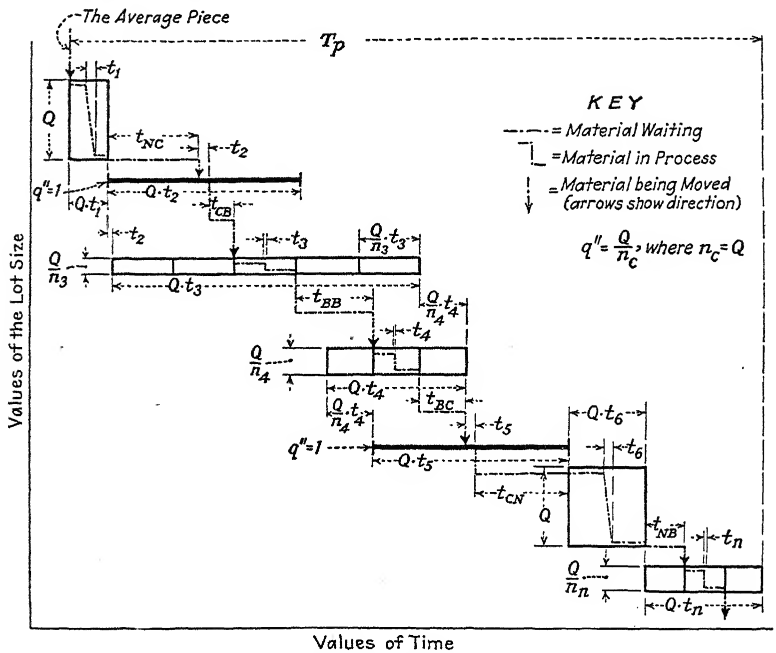

FIG. 41.—The relation of the flow of material and the total production time T_p for a complex sequence of operations. See Table XIX for evaluation of the transition times.

The Unit Process Time for the Average Piece t_p.—If some method is to be devised which can be easily employed in all such cases, it must be assumed that the total process time T_p will equal the product of the lot size Q and the total unit time of production t_p in days for the average piece [Eq. (112)]. Then a general expression must be developed for this latter item t_p, which will take into account the time of production for the average piece, as it passes through each operation, and include the necessary corrections for the transitional periods between operations, where the nature of the process or the batch size changes, as well as any allowance for the manner in which the

raw material is introduced to or the finished articles removed from the process [Eq. (120)]. It may seem to the casual observer that the implied calculations may be even more complicated and less desirable than those previously employed, even though they may be more accurate; however, as a result of an extended analysis, which is outlined in the following paragraphs, a method has been devised which can be closely related to the work of the process engineers, without much additional effort, and for which the data is already available in their records of operation time normally used for rate setting and the computation of standard costs. Naturally, any such method, if it is to be universal, must comprehend all possible contingencies, but, when it is to be used in actual practice, it will be found that it can be usually quite easily simplified, as only those factors need be employed that are characteristic of the specific process in question. The details of the computations required by this method have been outlined in the calculation sheets to be found in Table VIII.[1] The utility of the results of such a study does not end with the computation of the economic lot size, because the value thus obtained for the production time of the average piece can be employed to even greater advantage in pointing out ways for improvement and still further reductions in cost when occasion warrants.

Interpretation of the Average Piece.—In establishing a general formula for evaluating the total production time T_p, the actual unit time of production consumed by any piece in the lot, as it passes through each operation, cannot be employed, because it is quite evident from an inspection of Figs. 38 to 41 that the first and last pieces cannot be processed at the same rate that any piece in the middle of the lot can be, even though the actual unit time for each operation may be identical for any piece in the lot. If the anticipated length of any manufacturing period, with regard to the size of the lot being processed is to be computed from the data supplied from current time-study or production-control records, the unit time for the average piece in the lot must be used instead. This means that the unit-production time for any piece in a complicated process can be reduced to an average figure which will represent the equivalent unit-

[1] See p. 95.

TABLE XVIII.—EQUATIONS EMPLOYED IN THE DETERMINATION OF THE UNIT-PROCESS TIME FOR THE AVERAGE PIECE

In general let

$$T_p = Q \cdot t_p. \tag{112}$$

Then for non-continuous production

$$t_p = t_N, \tag{113}$$

where

$$t_N = (t_1 + t_2 + t_3 \cdots + t_n),$$

and for semicontinuous production

$$t_p = \frac{t_N}{Q} + \frac{t_C}{2}, \tag{114}$$

where t_N has the value given above and

$$t_C = |t_1 - t_2| + |t_2 - t_3| \cdots + |t_{(n-1)} - t_n|$$

which must be positive. But in this case

$$T_p = t_N + Q \cdot \frac{t_C}{2},$$

and

$$\frac{dT_p}{dQ} = 0 + \frac{t_C}{2},$$

so that in general if

$$T_p = Q \cdot t_p,$$

and

$$\frac{dT_p}{dQ} = t_p.$$

use

$$t_p = \frac{t_C}{2} \tag{115}$$

for substitution only into any expression for Q_e or Q_m.

Then for batch production

$$t_p = t_B, \tag{116}$$

where

$$t_B = \left(\frac{t_N}{n} + \left(1 - \cdot \frac{1}{n}\right) \cdot \frac{t_C}{2}\right).$$

or in the special case of progressive sequence

$$t_p = t'_B, \tag{117}$$

where

$$t'_B = \frac{t_N}{n} + \left(1 - \frac{1}{n}\right) \cdot t_L.$$

For transitions between different types of process, add

$$t_t = \frac{1}{2} \cdot \left[\left(1 - \frac{1}{n_F}\right) \cdot t_F - \left(1 - \frac{1}{n_S}\right) \cdot t_S\right] + \frac{T_m}{Q}, \tag{118}$$

but use

$$= \frac{1}{2} \cdot \left[\left(1 - \frac{1}{n_F}\right) \cdot t_F - \left(1 - \frac{1}{n_S}\right) \cdot t_S\right]$$

for substitution only into any expressions for Q_e or Q_m (for simplified forms of t_t see Table XIV).

For correction in method of delivery or removal of articles add

$$t_d = \frac{1}{2} \cdot \left[\left(1 - \frac{1}{n_i}\right) \cdot t_i + \left(1 - \frac{1}{n_e}\right) \cdot t_e\right], \tag{119}$$

and then

$$t_p = \Sigma\, t_N + \Sigma|t_C| + \Sigma|t_B| + \Sigma|t_t| + t_d. \tag{120}$$

See calculation sheet for practical method. Table VIII, p. 95.

For interpretation of symbols, see text, Chap. XV, or Appendix XIII, page 349.

production time for a non-continuous process that could accomplish the same result in a length of time equal to that consumed by the more complicated process. Furthermore, one's interest in this situation should not be confined merely to the possibilities of a reduction in the actual process time but should be extended so as to include also a consideration of the actual time over which investment charges must be paid upon the capital employed. This of course is the real purpose of this analysis, and the former consideration is only incidental, even though it may lead eventually to a greater productive efficiency. The fact which should be borne in mind is that the true value for the production time T_p should be equivalent to that for the processing of a single piece in the most effective manner, the final value of which is equal to the total of all such individual time actually consumed in the manufacture of the entire lot. In other words, this statement should be interpreted to mean that, whatever the value for t_p and hence T_p, may turn out to be, it represents the time that any one piece taken as typical of the lot remains in process from the instant it enters the manufacturing area or is placed upon the machine to the time it is removed from the process for delivery to stores or to the next stage in its manufacture in some other portion of the plant.

It is fortunate that such a value can be obtained for any process, because it is necessary to know the total time for the whole lot which can be used as a basis for computing the cost of the capital thus employed. On this basis, then, the value of the unit-production time t_p for the average piece for a non-continuous process will be the same as that t_N given in Eq. (113), because there are no corrections to be made, and the symbol t_N will be used hereafter to designate the sum of all such unit-operation times.

The Unit-process Time: Semicontinuous Production.—In the case of semicontinuous production, it will be seen from Fig. 39 that the total production time may be expressed as the sum of the unit time to process one piece plus half the sum of the difference between the total time for each two succeeding operations, provided that the numerical figure of this difference is always used with a positive sign and not the algebraic sum. If this be reduced to the average unit-operating time, t_p can be determined from Eq. (114), where t_C is always positive,

t_1 = the unit-production time for the first operation,

and

t_n = the unit-production time for any other operation being the *nth* in sequence.

When the eventual expression for I_w is differentiated, of which this may be a part, the value for the term containing t_N will become zero, as then it is a constant in this case. Accordingly, for practical purposes, when the value of t_p which corresponds to that for semicontinuous production is substituted in the final expression for either the minimum-cost quantity or the economic-production quantity, it should be that given in Eq. (115).

The Unit-process Time: Batch Production.—Again the situation where batch production occurs may be considered as typical of all cases, because by the use of the appropriate value for the number of batches n either of these two previous expressions may be derived from it. The value of the average unit-operating time t_B for batch production may be found from an inspection of Fig. 40. There it will be seen that the time to produce the quantity in any batch $\frac{Q}{n} \cdot t_N$ is extended by one half the difference in times between the starting and stopping of the remaining number of batches $\left(1 - \frac{1}{n}\right)$ on each operation, with due allowance for the varying lengths of time required for each operation. The principle behind this reasoning, as well as that for semicontinuous production, assumes that it is better to keep material waiting the necessary time before starting any succeeding operation, in order to insure continuous operation of the machine, than to keep the machine idle owing to the lack of material, which occurs when a given operation is more rapid than that preceding it. Accordingly, the process time t_B for the average piece in a lot composed of n batches can be obtained from Eq. (116) where t_C is always positive and is composed of the same group of terms as employed in the expression for semicontinuous production.

The Unit-process Time: Progressive Sequence.—The reverse situation may arise at times where it will be more advantageous to incur some idle machine time than to delay the work in process,

owing to a closely coordinated scheme of production or to the fact that the cost of capital could be lessened by such procedure. This may be quite feasible when the manufacturing operations are more nearly automatic in character, because in that case the direct labor applied to each unit of production may be but a small part of the total labor for each operation, as the greater part of it may take the form of labor incidental to machine changeover and general attendance during the operation, and at no time would any operator be idle because any one of his machines happens momentarily to be out of work. Under such circumstances the expression for the unit-manufacturing cost will be that given in Eq. (77).[1] In order to get the most out of such a scheme all machines in the sequence should be set up for their various operations sufficiently in advance so that the first batch, whatever its size, may proceed continuously from one operation to the next without any delay, and then the succeeding batches should be so scheduled that each one of them will flow likewise through the process without interruption. This method of arranging production is much the simpler to account for, as the total process time for each batch is the same, and the total time for production equals the average time to process one batch plus the sum of the total production time of the longest operation for the remaining number of batches $(n - 1)$. Accordingly, if this be reduced to terms of the average piece, the unit time t'_B can be determined from Eq. (117), where t_L equals the unit-production time for the longest operation and t_N is the same as before.

A General Expression for Any Type of Process.—In order to complete the analogy referred to in the paragraph just preceding the last, it can be shown that the expression for t_p in the case of non-continuous production can be obtained from t_B if $n = 1$, for in that case the whole lot is a batch. Similarly, in the case of semicontinuous production, if each piece is considered a batch because each flows through the process independently, the value of n will become so large that for all practical purposes it may be considered as infinite for the extreme case. So, if the value $n = \infty$ (or $\frac{1}{0}$) is used in the expression for t_B, $1/n = 0$ and the expression for t_C is obtained.

[1] See p. 190.

TABLE XIX.—SIMPLIFIED FORMS OF EQUATION FOR TRANSITIONS BETWEEN SPECIFIC TYPES OF PROCESS

Expressions for t_t for specific transitions (t_t must in all cases be positive)

From non-continuous to batch production

$$t_{NB} = \left(1 - \frac{1}{n_S}\right) \cdot \frac{t_S}{2}. \tag{121}$$

From non-continuous to semicontinuous production

$$t_{NC} = \frac{1}{2} \cdot t_S. \tag{122}$$

From semicontinuous to non-continuous production

$$t_{CN} = \frac{1}{2}(t_F - t_S) - \frac{t_S}{2Q}. \quad \text{(See note.)} \tag{123}$$

From semicontinuous to batch production

$$t_{CB} = \frac{1}{2}\left[t_F - \left(1 - \frac{1}{n_S}\right) \cdot t_S\right]. \tag{124}$$

From batch to non-continuous production

$$t_{BN} = \frac{1}{2}\left[\left(1 - \frac{1}{n_F}\right) \cdot t_F - t_S\right] - \frac{t_S}{2Q}. \quad \text{(See note.)} \tag{125}$$

From batch to semicontinuous production

$$t_{BC} = \frac{1}{2}\left[\left(1 - \frac{1}{n_F}\right) \cdot t_F - t_S\right]. \tag{126}$$

From batch of one type to batch production of another

$$t_{BB} = \frac{1}{2}\left[\left(1 - \frac{1}{n_F}\right) \cdot t_F - \left(1 - \frac{1}{n_S}\right) \cdot t_S\right]. \tag{127}$$

NOTE.—The term $\frac{t_S}{2Q}$ should be omitted if t_t is to be inserted directly into an expression for Q_e or Q_m.

For interpretation of symbols, see text, Chap. XV, or Appendix XIII, p. 349.

An Allowance for the Transition from Processes of Differing Characteristics.—If any process is consistently of the same type throughout and the number of batches remains constant, the average unit operating time t_N, t_C, or t_B can be directly used as the basic time element t_p required in determining the investment charge. If the process is composed of several types of different characteristics, however, the final value for t_p will, of necessity, be the sum of the average unit time for each subdivision of the process, plus the sum of the additional average unit time t_t consumed in the transition from one process to another, as illustrated by Eq. (120). In general the unit time of transition t_t is equal to the positive numerical value of the difference between the production time for a batch on the last operation of a process of one type, and the production time for the batch on the first operation of the process of another type immediately following. In no case can the value of t_t be other than positive; accordingly, by inspection of the diagrams in Fig. 41, it will be seen that in general its value can be obtained from Eq. (118)

where

t_F = the unit-production time for the last or final operation of a process of one type,

t_S = the unit-production time for the first operation of a process of another type starting immediately afterward,

T_m = the total time required to move all the pieces in any batch coming from one process to the next process,

n_F = the number of batches employed in the operation preceding the transition,

and

n_S = the number of batches employed in the operation following the transition.

An Allowance for the Time Consumed in the Transportation of Materials.—Owing to the fact that the move time T_m is always a total figure, because all the pieces in a batch, whatever its size, can probably be moved as a unit, this item will become zero when differentiated as a part of the ultimate unit cost in the solution for the minimum-cost quantity. Only when this method of analysis is used for studying the manufacturing operations need this item be evaluated. Moreover, if conveyors are used to handle material from one process to another, semi-continuous production is implied in both cases, and there is no

transition. The general expression will be found useful in its complete form only when the transition occurs between subdivisions of the process where the number of batches change. The transition between subdivisions where other types of production occur can be expressed in a much simpler form, as illustrated by Eqs. (121) to (127) in Table XIX. Each one of these may be derived from the general expression for t_t [Eq. (118)] by introducing the appropriate value for the number of batches n_S and n_F, in the same manner that the various expressions t_N and t_C were obtained from the general expression for $t_p = t_B$.

Special Allowances for Transitions.—Two special cases of the transition time occur when it is found more practical to deliver all the material for the lot to the production area in bulk and for similar reasons not to remove any finished article until all have been completed, whereupon the entire lot will be delivered to stores as a unit, even though in all other respects the process in no way resembles that for non-continuous production. This additional unit time for the average piece may be represented by t_d, and in general expressed as one half the sum of the time t_i elapsed after the starting of the process up to the instant the last batch goes into production at the first operation, and the time t_c which has elapsed between the completion of the first batch upon the last operation and the finishing of the processes as shown in Eq. (119), where n_i and n_c equal the number of batches employed in the initial and concluding operations of the process, respectively. The various expressions for each type of process can be obtained as before by the use of the correct value of n in each instance. If there is no deviation from the true nature of either of the initial or concluding operations, the time t_i and t_c, as the case may be, will equal zero. For a non-continuous process no correction is possible whether it stands at the beginning or the end of the sequence of operations, because of its inherent characteristics.

General Expression for the Total Process Time T_p.—As a summary of the foregoing paragraphs, a universal expression [Eq. (128)] for the process time of the entire lot based upon the unit-operating time for the average piece can be written when the value of t_p is derived from Eq. (120) and substituted into Eq. (112), if k_d be introduced to account for any arbitrary allow-

ance for delays arising from any cause that is attributable to the lot on a unit-time basis. All other delays not of this order, such as those which affect the lot as a whole, must be treated only as a total charge and included as one of the preparation costs. Often it is found advisable to establish an allowance of this nature as a percentage figure of the unit-production time to provide the necessary leeway for supposedly unavoidable interruptions, instead of arbitrarily extending the observed normal production time derived from time studies of the various operations. Ideally no such factor should be required as delays are actually a form of waste and as such should be eliminated. If they exist, this factor k_d can serve as a measure to show what degree of success has been attained in avoiding this sort of expense. If only the rate of production for any operation is known and the unit time is not, the correct time value can be supplied by using the reciprocal of the rate of production instead. Moreover, if two or more machines are used in any operation, the unit time for the average piece must be divided by the number of machines n_M in whatever operations this situation occurs.

Practical Analytical Technique.—Even though this analysis of the unit-production time is apparently quite complicated, little difficulty will be encountered in obtaining an accurate and representative value should the method of production involve several types of process, if the calculation sheet[1] provided in Table VIII is employed. The arrangement of this sheet has been devised so that the procedure will be self-evident. The use of this method, however, is only recommended in instances where the simple sum of the unit-production times for each operation, as given by t_N in Eq. (113), does not properly represent the true nature of the processes or does not yield a sufficiently reliable time factor for the determination of the investment charge in work in process. In most cases the value obtained from the relation in Eq. (113), when corrected for the number of machines in parallel operation if necessary, will be sufficient, and for this reason the definition of this item on the data collection sheets given in Table V has been based on this simpler relation.

Adjustment of the Average Unit Capital Value for Characteristics of the Process.—Returning to the fundamental relation

[1] See p. 95.

for the investment charge I_w given in Eq. (111), a final expression can be established which will include all of the various dependent items as shown in Eq. (129), and this expression may be used for the investment charge in work in process for either a fabricating process or an assembly process, but in no case for both combined. If the simple method has been employed to obtain the value of t_p from t_N, where no allowance has been made for the nature of the process and k_a has been assumed to be ½, the parenthesis where $c'' = \frac{m + c}{2}$ can be altered arbitrarily to achieve the same purpose. First, if material is introduced to the process continuously and no articles can be withdrawn until all are completed, the parenthesis can be reduced to $(c/2)$. Second, if the reverse situation exists where all the material is introduced at one time and the finished articles can be withdrawn directly upon the completion of each at the last operation, the parenthesis will have the form $(m/2)$. Third, if both material and finished articles flow continuously into and out of the process respectively, a continuous process will be implied and the parenthesis will be equal to zero, demonstrating in a different manner that the investment charges on work in process have in this case no influence upon the economic lot size. If, on the other hand a value for I_w is required in order to calculate the ultimate unit cost it may be obtained from Eq. (129) when q_p, which may be allowed to represent the average number of pieces or the bank of material continually maintained in the process, though no one of them remains there for the full period, is inserted in place of the lot size Q.

Procedure for Processes of Complex Characteristics.—Care must be taken, however, not to assume unjustly from this evidence alone that the whole process is continuous. If, for example, the method of manufacture be similar to that illustrated in Fig. 41, except that in this case the first and last operations be of the semicontinuous type, it will be seen that it would be quite incorrect to have completely neglected the investment charges on work in process, as a consideration of only the flow of material in and out of the process would fail to indicate what transpires with regard to each operation and its effect upon the whole. Whenever this situation exists, t_p must be computed

by means of the method outlined in Table VIII, and the unit value

$$c'' = \frac{m + c}{2}$$

must then be employed in Eq. (129) without alteration.

Evaluation of the Investment Charges on Work in Process.—When it comes time later on to evaluate the ultimate unit cost and the relation for the economic-production quantity Q_e, there is little need to employ the complete expression [Eq. (129)] if it can be divided into those terms which are either constants k_{v_w} or unit values v_w, unit times t_w and total values associated in some manner with the lot size Q. Accordingly, the unit allotment of the investment charge on work in process can be written as

$$\frac{I_w}{Q} = Q \cdot v_w \cdot t_w \cdot i + \frac{P}{Q} \cdot \frac{i \cdot T_M \cdot k_M}{8} + k_{v_w}, \tag{130}$$

where

$$v_w = \left[\left(\frac{1}{k_a} - 1\right) \cdot m + c\right] \cdot k_a, \tag{131}$$

or

$$= c'',$$
$$t_w = t_p \cdot k_d,$$

and

$$k_{v_w} = P \cdot k_d \cdot t_p \cdot i \cdot \left(1 - \frac{k_M}{2}\right).$$

The use of these time-value factors in the evaluation of Q_e [Eq. (52)] is quite parallel to that for the similar factors derived in the previous chapter in connection with the investment charge on articles in stores. Naturally, the capital expenditures and the time during which they have been employed, as applicable to the investment charge on work in process, should be likewise the basis for that portion of the gross return earned upon the investment in the productive phases of manufacture. Accordingly, it will be quite justifiable to introduce the relations represented by v_w, t_w, and k_{v_w} in Eq. (130) wherever these symbols appear in Eq. (52), when the general expression for the economic-production quantity is derived in Chap. XVIII. It will probably be noted that the portion of capital values represented by

$\frac{P}{Q} \cdot \frac{T_M \cdot k_M}{8}$ has not been included with the other capital values upon which a return is earned. The reason for this is based upon the statement in Chap. XX, with reference to this group of terms, and the resulting fact that its eventual influence upon the return earned upon capital employed in production is found to be insignificant, and so this item may be neglected accordingly.

Scope of the Investment Charge on Work in Process.—As the preceding discussion has seemingly dealt only with the investment charges accruing during production, the natural question may arise concerning similar charges[1] upon material or semifinished parts that may be stored in the manufacturing areas under certain conditions. This situation is covered by the fact that the period of production, for which T_p represents the total elapsed time, has been considered to commence with the instant that the material enters the manufacturing area or is delivered to the machines from stores, and to terminate at the instant the last piece has been finished and inspected or withdrawn from work in process and placed in finished parts stores. This method should parallel normal accounting practice, and it may be said generally that the manufacturing period practically extends from the execution of the material requisition to the execution of the final move-order to stores or other manufacturing areas of the plant. Then, for whatever happens to the articles during this time, whether they are in process or in temporary storage, the investment charge I_w will apply to all circumstances. Normally, time which is not consumed by the process is a loss, even if semifinished parts are temporarily withdrawn from production to save further expenditures for fabrication or assembly, as capital is withheld for the time being from performing any useful purpose. As any such situation is for the most part an emergency, and, as economic quantities contemplate standard conditions, special situations like these can be disregarded.

[1] See also Sec. B, Chap. XVI, p. 249.

CHAPTER XVI

SPACE CHARGES

A. IN STORES

The storage of parts and finished products in some industries may present a most difficult problem, owing to their peculiar shape or bulk. Articles of a nature similar to paper boxes, furniture, delicate crockery, and machine tools will require a larger space for their storage in proportion to their value than more compact ones, due to the fact that their form for the most part is similar to a hollow shell the interior of which contains nothing but air, or the fact that their fragile nature prevents stacking one upon another. The exception in this case exists for those articles which can be nested by inserting one within another. This empty space which lies within their overall dimensions cannot be utilized for any purpose, as it would necessitate storing dissimilar articles in the same area, a practice which is vigilantly discouraged on general principles in any well-maintained stores division. Accordingly, such industries have to undergo a charge in excess of that which otherwise might be considered reasonable if the expense of storing articles could be justifiably prorated on value alone.

Value as a Basis for Prorating the Space Charges Unreliable.— In some of the earlier derivations[1] of a formula for the economic lot size a factor was included with the interest rate which accounted for the storage costs on a unit-value basis. If it were conceivable that all articles manufactured in any plant had the same value for the ratio of their overall unit volume to their unit-manufacturing cost, such an accounting procedure might be legitimate. This situation actually occurs in so few instances that it is not fair to each product to attempt to prorate storage costs on any such basis. Should this principle be employed to any extent in the determination of the best lot size, therefore, and that in turn should be utilized as a measure of management

[1] See Table IX, Chap. IX.

for the control of production and inventories, the policies and conclusions adopted under these circumstances would fail to comprehend the true situation. Of course, an excellent argument can be found in current accounting practice against the allotment of storage charges on the basis of the space occupied, because almost universally such charges are a part of overhead and are distributed through the burden rate upon a man-hour or machine-hour basis, either of which is a major factor in the cost or value of a unit of production. This fact alone should not prohibit the adoption of a new method of handling storage costs, if a better cost picture of the manufacturing operations can be obtained in that way no matter how radical or divergent from current practice it may be.

The Bulk of an Article a Preferable Basis.—Another point in favor of the allotment of these costs on the basis of space occupied is that the value of articles in stock should not be enhanced by charges which accrue during the storage function. Unfortunately, this situation cannot be achieved when such charges are included in overhead. To be correct, they should be added to the unit-manufacturing cost at the time a unit of production is removed from stores for assembly or for sale. Only in this way can the storage charge be properly carried over into the value which should be rightly used for the material going to the assembly process or the factory cost for an article to be sold. Herein lies the chief reason for segregating these charges and introducing them as a third element of the economic balance in the determination of the lot size. When this is done, care must be taken to remove all such items from the overhead in order to avoid any errors of duplication.

The Space Charge for Articles in Stores Defined.—The space charges V_s may be defined as the total of all charges accruing to the space set aside for the storage of those articles produced in a lot which must be held in reserve for future orders over the time that any one remains there. In order to evaluate these charges one must obtain all items of cost which can be attributed to the particular storage area during the year, and then reduce them to a factor s in terms of square feet per day by dividing them by the total area utilized for storage purposes, aisles excluded, and the number of working days per year. To conform with this the unit bulk b or the overall volume of the article must be deter-

mined in cubic feet, and then corrected to square feet by dividing it by the total height h to which storage is permissible. If these items be multiplied by the maximum number of articles Q'_z of the same kind that will be in stores at any instant and the time T_v that the space is occupied by any one of them, Eq. (132), in Table XX, can be written so as to express mathematically their relation.

The Unit Cost of Space s.—The derivation of each of the first three items in this expression is quite simple. First the unit-space charge s should contain all cost items which are attributable to a unit of production from any source incidental to its storage and pertain to the particular stores area where it is kept. These items will naturally include rental charges on land, buildings, and equipment such as interest, taxes, depreciation, and maintenance, and departmental charges such as heat, light, storekeepers' salaries, and supplies consumed in the process of supervising, safeguarding, and handling the articles placed in stores. Insurance charges will also have to be taken into account, but owing to the fact that those which apply to the various articles stored have no relation to the space occupied by them or their bulk, they must be segregated from the others which pertain to the building and its equipment, and then these remaining items can be justifiably combined with the rental charges and allocated on a square foot per day basis. Obviously this must be so, because articles in stores can be insured only against loss, in potential money value, by fire and other hazards, and the company is merely interested in maintaining its capital intact, whereby it can in due time replace the physical being of the original articles by others of a like nature. Accordingly, it may be preferable to let this particular item remain in overhead with that portion of the insurance cost applicable to work in process, where both can be prorated on a time basis which to all intents is equivalent to the growth of value through the application of labor. If all charges which accrue during the storage period are not to be allowed to enhance inventory evaluation, however, the insurance charges on articles in stores should be kept as a separate item and reduced to a unit charge per dollar of inventory value similar to the interest rate for the cost of capital. Both can then be combined as they refer to the same item of investment. Similarly, it is an open question whether the department charges

TABLE XX.—EQUATIONS EMPLOYED IN THE DERIVATION OF THE SPACE CHARGES V_s

In general

$$V_s = \frac{s \cdot b}{h} \cdot Q'_x \cdot T_v, \tag{132}$$

where

$$Q'_x = Q - q;$$

or if

$$q = T'_d \cdot S_a,$$

and

$$T'_d = \frac{Q}{D} \cdot k_p,$$

$$Q'_x = Q \cdot \left(1 - \frac{S_a}{D} \cdot k_p\right), \tag{133}$$

where

$$k_p = 1 - \frac{1}{n},$$

and where, from Table XXI.

$$T_v = T_s \cdot k_b = \frac{Q}{S_a} \cdot k_b \tag{140}$$

when T_d is a large portion of T_s

$$k_b = \frac{1}{2} \cdot \left[1 + \frac{1}{n_b} \cdot \left(1 - \frac{S_a}{D} \cdot k_p\right)\right],$$

or when T_d is only a small part of T_s,

$$k_b = \left[1 + \frac{1}{2} \cdot \left(\frac{1}{n_b} - 1\right) \cdot \left(1 - \frac{S_a}{D} \cdot k_p\right)\right], \tag{141}$$

or when the storage space is permanently reserved for a specific type of article

$$k_b = 1$$

and then

$$V_s = \frac{s \cdot b}{h} \cdot \frac{Q^2}{S_a} \cdot \left(1 - \frac{S_a}{D} \cdot k_p\right) \cdot k_b. \tag{142}$$

For interpretation of symbols, see text, Chap. XVI, Sec. A, or Appendix XIII, p. 349.

should be removed from overhead and prorated on a volume basis, but if one is to be consistent these items should be removed as well. The manner of allocating them is then the only indeterminate portion of the problem, and this can best be settled on the ground that bulk is probably more difficult to handle and store than value, so that the distribution of the departmental costs of storage can be justifiably included with that for the rental charges.

The Bulk or Volume of a Unit of Production b.—Second, the overall volume or space b occupied by an article must be determined by the room required to place one article next to or on top of another, either in a bin or container or on the open floor. In either case the outside dimensions must be used so that any projections will be provided for as well as an allowance for any additional space required for picking up and handling each article with the proper amount of facility and safety. The actual physical volume of an article is meaningless in this case, as the theory of voids states that no matter how carefully articles of irregular contour are arranged in any unit of volume at least 28 per cent of the total space will be empty. When articles can be stored in bins or containers, the volume of the total number required expressed in cubic feet will be the measure of the total space that will be occupied by the lot when the maximum number of pieces Q'_x is in stores. On the other hand, when articles are stored on the open floor and stacking is prohibited, because of excessive floor loads, the fragile nature of the product, the danger of warping accurately machined surfaces, or the total height of the product, the actual floor area in square feet occupied by the article may be used, and then there will be no need for any corrective factor for the height ($h = 1$).

The Average Permissible Height of Storage h.—Third, as bins or even containers can be superimposed, the height h to which these can be conveniently reached or the total height from the floor to the top of the uppermost bin or container must be used as a divisor for the total volume occupied or reserved in order to find the exact floor area which lies under either of these and upon which the total space charges are to be determined. As the bins for a given article may not all be on top of each other for reasons of convenience in arrangement of stores, the total height in this case can be that common to all the bins in the storage space,

because this dimension is normally uniform in a given stores area. Sometimes it may be preferable to express the volume of bins or containers in terms of equivalent floor area or space, and then a constant for all storage facilities f_b can be determined which in either case will be equal to the ratio b'/h when h is the uniform height referred to above, and b' is equal to the volume of any unit of available storage space. This may be applied indiscriminately to all articles according to the number n_b of such facilities that are required, but if this be done, the expression b/h in Eq. (132) must be replaced by

$$f_b = n_b \cdot \frac{b'}{h},$$

where

$$n_b = \frac{1}{q_b},$$

and

q_b = the number of articles of a given nature that can be placed in a standard bin or container.

The Length of the Storage Period.—The time factor expressed in days for the period during which any article of the original production lot occupies storage space must be the length of the sales-turnover period T_s. Thus the same value may be used in the determination of the space charges as was employed in the derivation[1] of the investment charge I_s where for uniform demand,

$$T_s = \frac{Q}{S_a},$$

and, when variable demand exists, the more complicated expression[2] must be employed where

$$T_s = \frac{Q}{S_{a_s}} = \frac{Q \cdot T_s}{\int_0^{T_s} S\,dT}.$$

Evidently this will be true because the investment charges cannot accrue over a different period of time from that for the space charges, as the same articles are involved in either case.

[1] See pp. 197–201.
[2] See Appendix II.

The Maximum Storage Space Requirements.—Of greatest consequence is the number of articles which will at any one time be held in stores. Unlike the investment charges, the storage charges cannot be based upon the average quantity in stores, because it is customary to reserve the required space permanently for a given article regardless of the fact that it may be empty for a considerable length of time. The maximum quantity Q'_x, therefore, must be employed in determining the total space charges V_s. This need not be of necessity the total production quantity Q, because in the case of a batch or semicontinuous process a certain part q of the total quantity can be diverted directly to current orders and will never be placed in stores at all. Accordingly, a correction can be made to account for this situation similar to that used in connection with the investment charge I_s,[1] but as the maximum quantity is used and not the average the stock factor k_s should not be applied. On this basis the value for Q'_x can be obtained from Eq. (133) if the demand be uniform, and if the demand be variable the term S_a should be replaced by the appropriate value for S_{as} as derived from the graphical method outlined in Appendix IV.

Permanent Reservation of Storage Space Uneconomical.—The common practice of permanently reserving storage space for a given article has a certain uneconomical aspect. If it were practical to utilize all space to the fullest extent and not let it remain idle merely as a matter of convenience in order to be always able to locate the same article in the same place year in and year out, a saving could be made not only in the cost of the space charged to each article but also in the total capital invested in land, buildings, and fixtures utilized for storage purposes. Should there be any doubt on this point it can best be settled by determining whether the additional cost of procuring articles from stores, if there be any, will exceed the contemplated savings in each individual case. Of course there remains the problem of what should be done with superfluous storage facilities, but this should present no real obstacle if there is something to be gained, because any progressive executive can always find ways and means to capitalize these advantages. In a growing industry such space can probably be used for expanding the manufacturing operations and providing the additional storage

[1] See pp. 206–210.

space thus required by the larger number of articles, without entailing the erection of new buildings and enlarging the storage equipment.

Advantages of Utilizing Entire Storage Facilities All of the Time.—If the assignment of storage space is to conform to the actual stock in hand, savings can be realized only when the maximum number of articles of a kind that will be placed in stores in any period occupy more than a single unit of the usual facilities provided. If this be the case, the diagram shown in Fig. 42 will

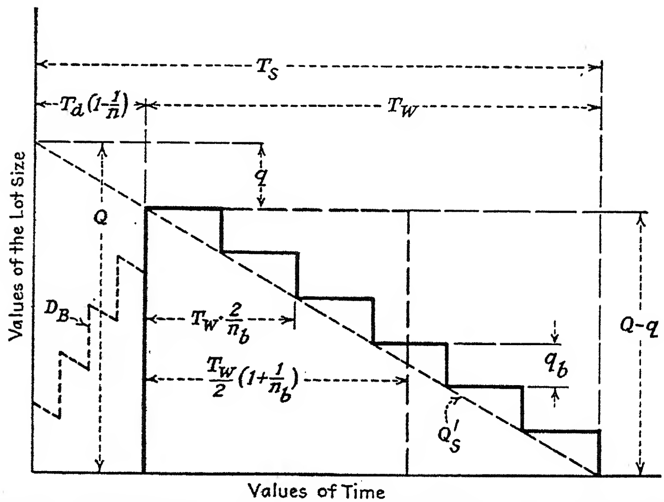

FIG. 42.—The average time of withdrawal of articles from stores when bins are not permanently reserved.

illustrate what occurs. Evidently articles will be delivered to stores over the relatively short period $T_d \cdot \left(1 - \frac{1}{n}\right)$ and will be withdrawn from stores over the longer period T_w. The maximum space requirements will occur at the end of the period T_d, and from then on as each bin or container is emptied, it can be turned over to the storage of articles of another kind, and the charges accruing from its use may be diverted from the article of the first kind to that of the second.

The Average Time of Utilizing Storage Space.—This may be expressed mathematically if n_b is allowed to represent the number of bins required to store the maximum number of articles Q'_x

TABLE XXI.—EQUATIONS EMPLOYED IN THE DETERMINATION OF THE AVERAGE TIME FOR ALLOCATING THE SPACE CHARGE V_s

When space is permanently reserved

$$T_v = T_s, \tag{134}$$

and

$$k_b = 1.$$

When space can be utilized first for one article and then another

$$T_v = T_a, \tag{135}$$

where, according to Fig. 42 and Appendix VIII, if

$$T_{W_a} = \frac{T_W}{2} \cdot \left(1 + \frac{1}{n_b}\right),$$

$$T_a = \frac{T_d}{2} \cdot \left(1 - \frac{1}{n}\right) + \frac{T_W}{2} \cdot \left(1 + \frac{1}{n_b}\right) \tag{136}$$

but as

$$T_W = T_s - T_d \cdot \left(1 - \frac{1}{n}\right), \tag{137}$$

$$T_a = \frac{T_d}{2} \cdot \left(1 + \frac{1}{n}\right) + \left[T_s - T_d \cdot \left(1 - \frac{1}{n}\right)\right] \cdot \frac{\left(1 + \frac{1}{n_b}\right)}{2}, \tag{138}$$

or

$$= \frac{T_s}{2} + \frac{1}{n_b} \cdot \left[\frac{T_s}{2} - \frac{T_d \cdot \left(1 - \frac{1}{n}\right)}{2}\right];$$

and then if

$$T_d = \frac{Q}{D},$$

$$T_s = \frac{Q}{S_a},$$

$$T_a = \frac{Q}{S_a} \cdot \left[\frac{1}{2} + \frac{1}{n_b} \cdot \left(\frac{1}{2} - \frac{S_a}{D \cdot 2} \cdot k_p\right)\right], \tag{139}$$

where

$$k_p = \left(1 - \frac{1}{n}\right);$$

or in general

$$T_a = T_s \cdot k_b, \tag{140}$$

where

$$k_b = \left[\frac{1}{2} + \frac{1}{n_b} \cdot \left(\frac{1}{2} - \frac{S_a}{D \cdot 2} \cdot k_p\right)\right],$$

For interpretation of symbols, see text, Chap. XVI, Sec. A, or Appendix XIII, p. 349.

over the varying periods of time T. Accordingly the average time T_a for determining the space charges may be expressed by Eq. (136) in Table XXI

where

T_d = the duration of the delivery to stores period,

and

T_W = the duration of the withdrawal period.

It can be seen by referring to Fig. 42 that the average withdrawal time $\frac{T_W}{2}$ must be extended by an amount equal to $1 + \frac{1}{n_b}$,[1] because no bin or container can be diverted to other uses until it is empty. Likewise, the delivery to stores time T_d[2] can be influenced by the nature of the process, and so the correction $\left(1 - \frac{1}{n}\right)$ must be applied, as for the investment charge I_s, where n equals the number of batches.[3]

Evaluation of the Bin Factor k_b.—Moreover, if T_d consumes a large part of the whole sales period T_s, the average delivery time $T_d/2$ should be used instead, because under such conditions a saving can be derived from an economical use of the storage facilities over this part of the sales period in a manner similar to that obtained for the remaining period T_W. It will be evident from inspection of Fig. 42 that T_W can be expressed in terms of T_d and T_s as given in Eq. (137), and T_a will take the form shown in Eq. (138). If their respective values from Eqs. (92)[4] and (83)[5] be inserted, the average time T_a can be represented by Eq. (139) or Eq. (140) if the factor k_b is introduced to account for the manner in which the space has been reserved. If the special case is considered where the period T_d is so small a proportion of T_s that little saving can be realized by not reserving all the required space from the time that the first batch is received in stores, the factor k_b will take the form given in Eq. (141), Table XX.

The Normal Value for the Bin Factor.—Ordinarily there will be little need to alter the existing method of permanently reserv-

[1] See Appendix VIII p. 337.

[2] See p. 206.

[3] See p. 210.

[4] See Table XVI, p. 204.

[5] See Table XV, p. 195.

ing storage space, because it has been found that in the majority of cases the bin factor k_b will have a value closely approximating unity. This can be demonstrated for a limiting case if n_b is assumed to be a direct function of the lot size Q, even though it could never actually occur in practice, as it would require placing dissimilar articles in a fraction of a bin where it is impossible to allocate space evenly. The evidence provided by this fact, however, is sufficiently reliable to justify the omission of any consideration of the manner in which space may be reserved unless a study of a particular manufacturing problem should indicate that a worth-while saving can be achieved, and then it may be accounted for by the appropriate value for k_b.

Evaluation of the Space Charges on Articles in Stores.—Now that all the elements which enter into the determination of the space charge V_s have been fully investigated, they may be combined by substituting them in the basic relation [Eq. (132)] as shown in Eq. (142). In order to obtain an expression for use in evaluating the ultimate unit cost which will contain only the basic unit time-value factor $v_v t_v$ and the lot size Q, Eq. (142) can be rewritten so that

$$\frac{V_s}{Q} = Q \cdot v_v \cdot t_v \tag{143}$$

where

$$v_v = \frac{s \cdot b}{h}\left(1 - \frac{S_a}{D} \cdot k_p\right)$$

and

$$t_v = \frac{k_b}{S_a}.$$

B. FOR WORK IN PROCESS

The storage of raw materials and finished or semifinished articles within the manufacturing area should present a similar problem. A certain amount of space must always be set aside for such purposes if order is to be maintained and lots of different units of production are to be kept separate. Naturally, incoming and outgoing materials should be placed in some location conveniently near the transportation facilities so that the move-men will not have to be continuously passing up and down the aisle between the operating equipment or get confused by orders which should remain. Wherever such space has been reserved, some

provision must be made for properly allocating the charges, which normally would apply to each square foot of floor area if it were productively employed. Obviously these charges will be similar in nature to those which are incurred in the storage areas and will include both rental costs and departmental expenses.

Storage Space Requirements in Manufacturing Areas.—This phase of the general subject has been given little or no attention, heretofore, but if it is going to be legitimate to disregard these charges entirely, it will be advisable at least to carry this investigation far enough to ascertain to what extent they may influence the selection of the best lot size. Space of this sort will be required for (*a*) raw materials awaiting the start of a non-continuous process or the time when it is their turn to enter a process which is already underway; (*b*) finished parts or assemblies as they are removed from the process or a completed lot of articles awaiting transfer to stores or another department; and (*c*) semifinished parts or subassemblies which cannot be removed as they must be held back until some other article with which they are later to be assembled catches up with them.

Storage Space for Work in Process.—Articles which are actually undergoing production will naturally be placed alongside of the manufacturing equipment for some period, whether for the full process time or not, and will occupy an area which can be considered a part of the space allotted to the machine. While in this stage of the process the charges for the space are measured by the process time[1] for each piece and can logically be distributed through the overhead as a portion of the machine or the departmental burden rates. Similar pieces which have not as yet been delivered to this area or have been recently removed from it, as well as material which has just been received on orders not yet in production, will naturally occupy some space in the department which has been specifically set aside for the purpose. Here, too, the space charges will accrue to each unit of production at the same rate which would be charged to a productive machine if it were placed there instead.

Basis for Allocating Space Charges on Work in Process.—It will be remembered that in determining the total production time T_p for a lot in Chap. XV,[2] it was assumed that this time

[1] See p. 238.

[2] See p. 229.

should extend from the instant that the first unit of raw material entered the manufacturing department to the instant that the last unit was removed, after all operations upon it had been completed. If this be the case, the time basis for the space charges, wherever any subdivision of the lot may be located in the department, will be the time T_p. Therefore, as this can be broken down to the average unit time t_p, the space charges can be most conveniently allocated to each piece if all such total charges be included in the overhead. Of course, if it were preferable to make a separate item out of them, it would be quite easy to construct the factor V_w for the space charge on work in process, in a manner similar to that used in developing the factor V_s for stores, except that it would refer to the total lot and not to the maximum number of articles Q'_x, and the time per unit would be t_p instead of k_b/S_a.

Storage Space for Semifinished Parts and Subassemblies.—So far in this discussion it has been considered that the space reserved for storing work in process will be occupied first by one kind of article and then by another, without particular reference to the allotment of a specific space to receive each whenever a similar article appears in the department. This naturally is quite permissible as space on the manufacturing floor is valuable and other spaces are provided for their actual storage. If semifinished parts or subassemblies must be definitely stored there as a regular practice, however, individual spaces must be set aside for articles of the same kind, in order that strict account can be kept of them as would be done if they were really in the stores department. In some instances such space may be considered as a storage area and treated as such, even though it may be located in a manufacturing division, as far as the accounting method is concerned. In all events both situations should be treated alike, and then the space charges V_w should be computed in a manner similar to that for V_s. Whether such charges be kept distinct for convenience by placing them in the factor V_w, for the reason that the unit space charge per square foot per day s_w will differ in value from that for the stores division s_s, or be computed separately and then combined, the effect on the lot size will be the same.

Characteristics of Space Charges for Storage in Process.—Accordingly, it would seem to be the best policy not to attempt

to segregate the charges which arise from the storage of articles in work in process, but to let them be taken care of in the usual method of accounting. There is no vital reason for not including them in the manufacturing cost as there is in connection with the space charges on articles in stores. Whatever costs accrue to the article incidental to its manufacture should be legitimately a part of the value assigned to it for inventory purposes. In fact this is the chief reason for differentiating between the storage space charges which are incurred as a part of the process and those which are derived from the necessity of anticipating future orders. In further support of this conclusion, moreover, space charges incurred by articles stored in manufacturing areas will rarely, if ever, be a direct function of the relation of the lot size to the rate of consumption which, in the case of articles in stores inventories, determines the duration of the storage period. For the most part, the time during which articles in the first instance are being stored will depend upon extraneous circumstances. If this be the case, the resulting space charges will have no influence upon the lot size and may be allowed to remain in the general distribution of overhead. Hence, the factor V_w can be disregarded when the time comes to evaluate the ultimate unit cost.

CHAPTER XVII

MINOR COST FACTORS

A number of other factors which are definitely a part of the ultimate cost of any unit of production must also be considered, either because they have certain characteristics similar to those of the major cost factors u_m, I_s, I_w, and V_s, or because they may alter the relation between the quantity produced and the quantity consumed and in so doing affect the time factor upon which the investment charges are based, whether it be for the production period or the sales period. Moreover, there are some factors beside these which should not be overlooked because they may have at some time been employed in a determination of the best lot size for some reason which, after a more thorough understanding of the problem, has little justification, or because there may be some remote possibility that a more intensive analysis may prove it advisable to include them in the fundamental relation. For the most part, it will be found that these minor factors can be totally disregarded, as, in the usual cases where economic lots have a value, their ultimate effect will be of very slight importance. Only in exceptional cases will one of them have to be included in the appropriate formulae for the limits of the economic range of production.

A. DETERIORATION

In industries where the product is of a perishable nature, deterioration of a certain percentage of all articles placed in stores will occur, if the sales or storage period is prolonged or the surroundings are unsuitable and do not provide the necessary protection to safeguard the product. Owing to the fact that the capital invested in the manufacture of these articles cannot be recovered through their sale, the monetary loss thus incurred must be charged to the lot and borne by those articles which are in a saleable condition. There is no way of evaluating deterioration except through experience, and, even then, it can be expressed only by a factor which represents the proportional amount of

the quantity in any lot which may be expected to deteriorate. If sufficient precautions are taken to safeguard the product, the loss can be minimized, and then a fairly constant and reliable value for the deterioration coefficient Δ may be obtained. Owing to the fact that a larger quantity must be produced in any lot than can be eventually sold, the deterioration coefficient also must be incorporated in the expression for the duration of the sales-turnover period T_s, in order to obtain the correct relation between the production quantity and the average daily sales.

Characteristics of Deterioration.—If deterioration is to be considered as a separate item of the ultimate unit cost, the amount of the loss must not be included in overhead. Likewise, deterioration cannot be properly accounted for if it be represented by a factor expressed as the proportion of value lost to the value of articles produced which is to be added to the item i for the interest rate and in this manner combined with capital. This was done in several of the earlier formulae[1] but did not yield satisfactory results. There seems to be ample justification for the treatment of deterioration of articles in stores as a distinct factor, not only because of its influence upon the lot size in specific cases but also because of the fact that the loss thus incurred is in reality a charge which accrues to the remaining saleable articles during the storage period, and therefore should not be included in the evaluation of inventories for the same reasons that the investment charges cannot be. In evaluating the loss, either the unit-manufacturing cost u_m or the ultimate unit cost U may be used for reasons[2] similar to those underlying the derivation of the investment charge I_s on articles in stock. Preferably the former should be used, as it conforms to current accounting practice; however, if U be employed in evaluating I_s, it must likewise be used in this case. In no instance should deterioration be confused with spoilage or depreciation; the first[3] is the result of imperfections or accidental damage in the process of manufacturing, and the second applies to the natural wear and tear from the continued use of machines or equipment and bears no relation to the product.

[1] See p. 125.
[2] See p. 194.
[3] See p. 182.

TABLE XXII.—EQUATIONS EMPLOYED IN THE EVALUATION OF DETERIORATION

Let

$$\Delta = \frac{Q''_s}{Q} \tag{144}$$

and if

$$\begin{aligned} q_d &= Q - Q''_s, & (145)\\ &= Q \cdot (1 - \Delta), \\ L_d &= u_m \cdot q_d, \\ &= Q \cdot \left(c + \frac{P}{Q}\right) \cdot (1 - \Delta), & (146) \end{aligned}$$

or

$$\begin{aligned} L'_d &= U \cdot q_d, & (147)\\ &= U \cdot Q \cdot (1 - \Delta); \end{aligned}$$

but if

$$\begin{aligned} q_d &= \delta \cdot T_s, & (148)\\ \delta \cdot T_s &= Q \cdot (1 - \Delta), \end{aligned}$$

and if

$$T_s = \frac{Q''_s}{S_a},$$

$$\Delta = Q - \frac{\frac{\delta \cdot Q''_s}{S_a}}{Q}$$

or employing Eq. (144) and simplifying

$$\Delta = \frac{1}{1 + \frac{\delta}{S_a}}. \tag{149}$$

If

$$Q''_s = T_s \cdot S_a, \tag{150}$$

so that

$$T_s = \frac{Q''_s}{S_a},$$

and if

$$Q''_s = \Delta \cdot Q,$$

then

$$T_s = \frac{\Delta \cdot Q}{S_a},$$

or if

$$\Delta = \frac{1}{1 + \frac{\delta}{S_a}},$$

$$T_s = \frac{Q}{S_a + \delta}. \tag{151}$$

For an interpretation of symbols, see text, Chap. XVII, Sec. A, or Appendix XIII, p. 349.

Evaluation of the Loss from Deterioration.—The deterioration factor L_d can then be defined as the loss in dollars which has resulted from a certain portion of the articles placed in stores becoming unsaleable from causes usually dependent upon exposure over too long a period of time to conditions unsuited to the continued preservation of the article in its original form. Accordingly, if the coefficient Δ be allowed to represent the ratio of that portion Q''_s of the quantity produced which will remain in saleable condition throughout the period T_s to the total quantity produced Q, as shown in Eq. (144), Table XXII, Eq. (145) can be written to express that portion q_d which has deteriorated in the same period in terms of the quantity Q. Now the total value of all the articles which have deteriorated, or the actual loss L_d, can be obtained from Eq. (146), if the articles were originally evaluated for inventory purposes on the basis of u_m, and, on the other hand, if U was used instead the total value, L'_d will be found by solving Eq. (147). In all probability considerable difficulty will be encountered in determining a suitable value for Δ unless it be expressed as a function of the sales period. This can be done if δ be allowed to represent the total number of articles which will become unsaleable for every day that the storage period lasts, as shown in Eq. (148), and then Δ can be computed from Eq. (149) if Eqs. (145) and (148) for q_d be combined.

The Influence of Deterioration upon the Duration of the Sales Period.—In the case of deterioration, however, the length of the sales period will be determined not by the actual quantity Q produced but by the relation of the number of saleable articles Q''_s in stores to the total sales demand $T_s \cdot S_a$ for the period, so that the correct value of T_s will be that obtained from Eq. (150) and not that previously employed. If the relation given in Eq. (144) for the coefficient Δ be used, Q''_s can be removed from Eq. (150) and then T_s can be expressed directly in terms of Q as in Eq. (151). Accordingly, whenever the loss factor L_d or L'_d is required in the determination of the best lot size, the coefficient Δ must accompany the item Q wherever it appears as the result of its introduction in the evaluation of the time factor T_s. There seems to be little opportunity to question this situation, because it is obvious that the sales period will not extend over as long a time if some of the articles produced will deteriorate

as it would if all articles could be sold. For purposes of evaluating the ultimate unit cost the unit allotment of the loss incurred by depreciation will be expressed as

$$\frac{L_d}{Q} = \left(c + \frac{P}{Q}\right)(1 - \Delta) \tag{152}$$

for the practical solution and

$$\frac{L'_d}{Q} = U \cdot (1 - \Delta). \tag{153}$$

for the exact solution.

B. OBSOLESCENCE

Obsolescence, on the other hand, is a most indeterminate factor and may appear suddenly without warning and make unsaleable a large stock of articles if provision has not been made to forestall its effects. It generally arises from a shift in consumers' desires, from changes in style, or from improvements in the technical features of design. Where progressive obsolescence is employed as a sales policy, the control of this factor is in the hands of the manufacturer. When obsolescence is a result of change in style or design, it may reoccur in some regular cycle which can be forecast with a fair degree of certainty. If it depends upon the whim of the customer, however, little can be done to properly provide for it except to produce a limited quantity, thereby risking the least capital. If this be the case, obsolescence should be handled in the same manner[1] as the conservation of capital, by the production of the smallest quantity within the economic range. Should there be a chance of forecasting its occurrence, production schedules can be proportioned so as to taper off and leave the smallest number of unsaleable articles at the end. As a result it may be apparent that a number of sales-turnover periods can pass before obsolescence takes place, and then a factor θ can be introduced into the economic quantity or minimum-cost quantity formulae to account for it. If this cannot be arranged, judgment only will be the best guide to the most suitable quantity for the lot size.

The Obsolescence Factor Defined.—The obsolescence factor θ can be defined as the proportionate amount that the normal rate of consumption is decreased as the time for complete obsolescence

[1] See pp. 17, 27.

approaches, and indicates the extent to which the usual lot size should be altered in order that the last lot produced will be as nearly exhausted as possible within all practical limits, when this time comes. Numerically it will be the ratio of the total sales that would be normally anticipated over the whole period up to the time when obsolescence occurs, if the rate of consumption remains substantially uniform, to the actual total sales S_{ob} which can be expected under such conditions where the rate of consumption is steadily diminishing. The value for θ can be calculated from Eq. (155), Table XXIII, if the total number N_{ob} of prospective sales-turnover periods is represented by Eq. (154) and S_s stands for the total sales for each turnover period under standard conditions, and Q equals the quantity regularly produced in each lot.

Absorption of Losses from Obsolescence.—If this scheme could be perfected there would be no loss from obsolescence. Under the best conditions, however, there will always remain some articles in stores for which there will be no available market. Naturally, as these will be unsaleable, a loss L_{ob} will be incurred to an amount equal to the capital investment represented by their inventory value, which must be absorbed in some appropriate manner. If need be, this can be evaluated from either Eq. (156) or Eq. (157), depending upon the use of u_m or U in determining their value for inventory purposes, and where q_{ob} represents the actual number of articles for which there can be no sale. Contrary to that which at first glance might seem to be a satisfactory way of handling this loss after having just discussed the loss from deterioration, it is impossible to assess this charge to other articles of the same nature and let them carry the burden, because in this case there will be no articles of this kind produced in the future. As the success of any such scheme can be measured only by the ability of the management to forestall any loss of this nature at all, it is quite logical to consider these charges as a penalty for failing to achieve this goal and post them directly over to the administrative account as an item of the cost of doing business. Unlike the investment charges, which are handled in a similar manner although they are not a penalty, the loss from obsolescence cannot be included in the ultimate unit cost, because as stated above there will be no more articles produced by which these charges can be properly absorbed.

TABLE XXIII.—EQUATIONS EMPLOYED IN EVALUATING THE INFLUENCE OF OBSOLESCENCE

If

$$N_{ob} \cong \frac{S_{ob}}{Q} \tag{154}$$

and by definition

$$\theta = \frac{S_s \cdot N_{ob}}{S_{ob}},$$

then actually since $S_s = Q$

$$\theta = \frac{Q}{S_{ob}/N_{ob}} \tag{155}$$

Now the loss becomes when the practical solution is employed

$$L_{ob} = q_{ob} \cdot u_m, \tag{156}$$

and when the exact solution is employed

$$L'_{ob} = q_{ob} \cdot U. \tag{157}$$

However

$$\frac{S_{ob}}{N_{ob}} = T_{ob} \cdot S_a,$$

but

$$\frac{S_{ob}}{N_{ob}} = \frac{Q}{\theta}, \text{ From Eq. (155)}$$

hence,

$$\frac{Q}{S_a} = T_{ob} \cdot \theta,$$

but

$$T_s = \frac{Q}{S_a}, \text{ From Eq. (83), Table XV.}$$

$$T_s = T_{ob} \cdot \theta. \tag{158}$$

Thus if T_{ob} is to replace T_s,

$$T_{ob} = \frac{Q}{\theta \cdot S_a} \tag{159}$$

should be employed.

For an interpretation of symbols, see text, Chap. XVII, Sec. B, or Appendix XIII, p. 349.

Influence of Obsolescence upon the Lot Size.—Accordingly, if obsolescence is to be accounted for through an adjustment of the lot size to suit the diminishing demand, and should it be found that the limits of the economic range do not provide sufficient latitude, the factor θ may be introduced in conjunction with the time factor T_s in the formulae for either the economic quantity or the minimum-cost quantity. This may be done in a manner similar to that by which θ was first evaluated, because if T_{ob} represents the corrected length of the sales period to conform to the sales S_{ob}/N_{ob} for each remaining one that should be employed eventually in any formula for the lot size, it can be expressed in terms of T_s, as shown in the method of reasoning leading up to Eq. (158). Therefore, when the ultimate unit cost is evaluated in the next chapter, the expression for T_{ob} [Eq. (159)] will be used in place of T_s, and then, when obsolescence is of no importance, θ will have a value of unity and T_{ob} will equal T_s.

C. INTERNAL TRANSPORTATION

The transportation of articles from one manufacturing area to another, from machine to machine, or to and from stores is normally an item of indirect expense, but in some instances it may have characteristics which would seemingly demand special treatment as a separate factor in the composition of the ultimate unit cost. In the majority of cases, however, it is so distinctly a service charge, which rightly belongs in overhead, that little space will be devoted to a discussion of it here. If there is to be any justification for an investigation of this phase of the problem, it can be determined by a consideration of the manner in which such charges accrue to a unit of production and the logical method of distributing them. Ordinarily the costs for operating such a division would be accumulated into one account and prorated to each manufacturing division in accordance with the amount of service rendered. This would imply that a fixed rate would be charged for the time that the service was being performed, regardless of the fact that the rate might conceivably vary with the type of equipment employed or the grade of the operator.

Time the Controlling Element.—Accordingly, the utility of segregating the cost of transportation can be finally determined on the basis of the time required to complete the movement of a

lot from one designated place to another. If this time factor is dependent upon the lot and not the quantity within the lot, the charges will be similar in nature to the preparation costs and should be included with them in that factor. If the time factor is a function of the quantity produced in the lot, the charge can then be reduced to a unit item and prorated to each piece through the overhead. If the time factor is found to be in no way related to either the lot as a whole or the individual units of production, the charge should be distributed as fairly as possible to each operating division as a part of the general overhead.

Legitimately a Factor of Overhead.—It would seem on first thought that the transportation cost would depend upon the lot itself. If the size of the lot be sufficiently large to require more than one trip, however, owing to the fact that only a given number of containers can be moved at one time, it is evident that it really can be prorated to each piece, because the number of containers depends upon the bulk of the article and the total number to be moved. Obviously the time consumed in making each trip is totally independent of the lot or its size, as it is only dependent upon the distance to be traveled. If these two considerations be combined, therefore, it will be seen that the cost can vary only in direct relation to the quantity produced and not to a higher degree, as would be the case when both the time and the rate charged are functions of the quantity. It may be concluded, then, that the transportation costs are not a separate factor and can remain in overhead according to current accounting practice.

The Situation When Conveyor Systems Are Employed.—In the case where a conveyor system is permanently installed for general purposes, the situation is somewhat different. Here the size of the lot has no effect upon the cost of such modes of transportation, because, while the lot is in process, the conveyor is always available for moving each piece as completed. The time when the conveyor is not in use, during the interval while some operation is being performed, must not be overlooked, so that the actual time that the conveyor is being utilized with regard to the lot must be the total process time T_p. Moreover, the conveyor may be employed to transport various types of articles from different lots all going in the same general direction at the same time, or lots of different nature that may be processed

in sequence on the same group of machines, and then the cost of the conveyor cannot be allocated to any specific lot and must be carried over directly into the general overhead.

Special Treatment of Internal Transportation Costs.—In no case will the cost of transportation be a part of the preparation charges, unless a line be drawn so fine between the operating cost and the supervisory cost, for this type of service, that it is worth while to charge the former portion to each piece through the overhead and the latter to the cost of preparation as an item of control.

D. EMERGENCY STOCK

In one of the early formulae for the economic lot size[1] the entire stock of articles of a given kind held in stores inventories over the sales period was employed in the determination of the investment charge I_s, and this has raised the question whether the cost of capital derived from reserve or emergency stocks has any decided influence upon the true lot size. Stocks of any article can be subdivided into three parts: (1) current stock held in anticipation of orders in the immediate future; (2) minimum stock, or, more correctly, that part of the first which has been reserved to fill orders during the manufacturing period of the lot that will eventually be used to replenish the current stock when it becomes exhausted; and (3) reserve or emergency stock, which is held in excess of ordinary demands in order to offset any unexpected increase in the rate of consumption or an exceptionally large order from a desirable or friendly customer.

Circumstances Requiring Emergency Stocks.—The first two classifications of stock will unquestionably be the basis for the investment charge I_s, as this portion of the entire stock is that which is determined by the lot size. The third has no relation to the quantity produced, as the amount to be thus stored is determined in the best way possible through executive judgment. To all intents and purposes this kind of stock is only an expedient employed by the management to remedy defects in their method of anticipating the wants of their customers, or to supply a higher grade of service to enhance good will under conditions when sales and production have failed to be properly coordinated. For the most part the cost of capital invested in emergency stocks

[1] See p. 131.

stands as a penalty against the management for not having utilized the best of their abilities and the most up-to-date tools of management as a means to avoid such expenditures. Otherwise such charges must be considered as unavoidable risks of the business, due to the uncertainties of the trade, and then they can be rightly classed only as an item in the cost of doing business.

The Nature of the Investment Charge on Emergency Stock.—In any event the investment charge for emergency stocks cannot be a part of the ultimate unit cost of any product, and so they must be carried into the general corporate expense account and prorated through the general overhead. The only way in which the lot size will ever be affected by this type of stock is when it must be replenished after some emergency withdrawals have been made. The additional quantity which must be produced together with the usual quantity in order to bring back the emergency stock to its required total will in no way adversely alter the ultimate unit cost of the pieces manufactured in this lot of a temporarily larger size. In fact the greater quantity will actually reduce the ultimate cost, because the preparation charges can be prorated over a larger number of articles, and there will be no investment or other charge to offset this gain for reasons similar to those in the previous paragraph.

The Effect of Abnormal Withdrawals from Emergency Stocks. In other respects no serious situation will arise, because even when the emergency is great enough to exceed the stock in reserve and consumes some of the current stock, the effect will be to advance the order point, and this in turn will indicate that the investment charge I_s for current stock will be less than normally anticipated. Ordinarily the economic range of production quantities will provide sufficient latitude in the selection of the best lot size for any approaching sales period in which undue fluctuations, if at all foreseen, can be adequately anticipated without incurring any extraordinary expenses. In fact, if the method of forecasting future sales is sufficiently perfected, little or no emergency stock need be carried, thereby relieving management of its penalties.

Utility of a Reserve Stock Factor.—In the case referred to at the beginning of this topic, a reserve stock factor was introduced which was supposed automatically to correct for the

effect of this classification of stock in the inventory. By its presence in the formula, withdrawals of an abnormal nature could be accounted for, but no way was provided to adjust for an increase in the lot size in the next period which would be a logical consideration if the accompanying circumstances, as in the former case, be adjudged of sufficient importance to be so emphasized. Moreover, this device can be extended beyond its practical limits of usefulness if the amount of the withdrawals at any one time be so great as to cause a change of sign in the relation of this factor to the average stock factor, developing a fictitious condition which cannot be reproduced in actual practice. Accordingly, it is believed that no consideration should be given to reserve or emergency stocks except as the investment charge thus incurred is a measure of a situation that should be reduced to a minimum.

CHAPTER XVIII

THE FUNDAMENTAL FORMULAE

In the preceding chapters of this section a thorough analysis has been made of the fundamental relationships of all factors which may conceivably influence the best lot size. It has been shown that cost alone is not the sole controlling element, because, if the manufacturing operations are to be performed successfully, the ultimate gross profit must not be impaired by the adoption of a policy which otherwise would operate to the advantage of the production division alone. Moreover, any scheme of manufacture must conserve capital as well as be economical in its use. Little can be gained if a policy of cost reduction be inaugurated and carried to such an extent that a greater investment of capital is required, because in all probability it will result in a decrease in the ultimate earning power of the corporation. Accordingly, all the factors which relate to both cost and earning power must be combined into a single relation if a truly economical situation is to be achieved.

Basic Formula for the Ultimate Unit Cost: Practical Solution.—The measure of this situation for a specific process is contained in the limits of the economic range, and if there be also the question of the best method of production, that can be selected by determining which one has the lowest ultimate unit cost. For this latter reason it will be necessary to evaluate the general expression for the ultimate unit cost first, but as there are two types of solution, one practical and one theoretical, each will have to be considered in turn. This may be accomplished in the case of the practical solution (see Table XXIV), which conforms to current accounting practice, by substituting in Eq. (160) for U_c, as derived in Chap. XII,[1] the various expressions for each factor in its composition which have been already developed in detail in the succeeding chapters. The resulting relation [Eq. (161)] will contain a number of terms, some of which

[1] See Eq. (55), Table XIII.

TABLE XXIV.—EQUATIONS EMPLOYED IN THE EVALUATION OF THE ULTIMATE UNIT COST

In general

$$U_c = m + A \quad (160)$$

where for the practical solution

$$A = (u_m - m) + \frac{I_s + I_w + V_s + V_w + L_d}{Q},$$

and then

$$U_c = u_m + \frac{I_s + I_w + V_s + V_w + L_d}{Q}.$$

Now if

$$u_m = m + l + o + a + \frac{P}{Q}$$

or

$$= c + \frac{P}{Q},$$

$$\frac{I_s}{Q} = Q \cdot v_s \cdot t_s \cdot i + k_{v_s},$$

$$\frac{I_w}{Q} = Q \cdot v_w \cdot t_w \cdot i + \frac{P}{Q} \cdot i \cdot \frac{T_M \cdot k_M}{8} + k_{v_w},$$

$$\frac{V_s}{Q} = Q \cdot v_v \cdot t_v,$$

$$V_w = 0,$$

$$\frac{L_d}{Q} = u_m \cdot (1 - \Delta),$$

$$= \left(c + \frac{P}{Q}\right)(1 - \Delta),$$

$$U_c = c + \frac{P}{Q} + Q \cdot v_s \cdot t_s \cdot i + k_{v_s} + Q \cdot v_w \cdot t_w \cdot i + \frac{P}{Q} \cdot i \cdot \frac{T_M \cdot k_M}{8} + k_{v_w} + Q \cdot v_v \cdot t_v + 0 + \left(c + \frac{P}{Q}\right) \cdot (1 - \Delta),$$

or

$$= [c \cdot (2 - \Delta) + k_{v_s} + k_{v_w}] + \frac{P}{Q} \cdot \left[(2 - \Delta) + \frac{i \cdot T_M \cdot k_M}{8}\right] + Q \cdot [v_s \cdot t_s \cdot i + v_w \cdot t_w \cdot i + v_v \cdot t_v]. \quad (161)$$

For an interpretation of symbols see Appendix XIII, p. 349.

will be constant unit cost items, others total costs allocated to each unit of production, and still others charges computed upon a unit time-value factor which varies with the quantity produced. If these be grouped according to their specific characteristics,[1] it will be found that Eq. (161) takes the form of the basic expression [Eq. (63)] for U_c as given in Chap. XII.[2]

Fundamental Formula for the Minimum-cost Quantity.—Now, if the terms u', F and f, in this last equation be assigned to their respective groups of terms according to their original definitions, it will be possible to obtain the expression [(Eq. (163)] for the minimum-cost quantity Q_m from Eq. (162) by reintroducing the groups in accordance with the arrangement of the terms F and f under the radical, as shown in Table XXV. The mathematical steps which are required in the determination of the minimum point of any curve through differentiation have been reproduced in this table as well in order to show the transition. Equation (163) can be considered as the fundamental expression for Q_m for the practical method of solution, because, when the actual expressions for each time factor and each value factor, as well as the numerical value for the rates of interest and depreciation, have been determined for a specific case,[3] they can be introduced without further hesitation concerning their true relationship.

Fundamental Formula for the Economic-production Quantity. Since the minimum-cost quantity Q_m thus derived is the upper limit of the economic range and is also the basis for determining the lower limit in order that it may refer to the same type of process, the economic quantity Q_e may be evaluated by introducing Eq. (163) for Q_m into Eq. (164) for Q_e as derived in Chap. XI.[4] The various steps involved in the determination of the ultimate expression for Q_e, which will contain the least repetition of terms, have been outlined in Table XXVI or may be found in full upon reference to Appendix IX, page 339. Equation (165) expresses the fundamental relation of all factors which enter into the determination of an economic-production quantity Q_e when the practical solution is employed, and may be evaluated

[1] See p. 163.

[2] See p. 170.

[3] See Chap. XIX, or Chap. V.

[4] See Eq. (52), p. 158.

TABLE XXV.—EQUATIONS EMPLOYED IN THE DERIVATION OF THE FUNDAMENTAL FORMULA FOR THE MINIMUM-COST QUANTITY

PRACTICAL SOLUTION

If

$$U_c = u' + \frac{P}{Q} \cdot F + Q \cdot f, \tag{162}$$

where

$$u' = c \cdot (2 - \Delta) + k_{v_s} + k_{v_w},$$

$$F = 2 - \Delta + \frac{i \cdot T_M \cdot k_M}{8},$$

$$f = (v_s \cdot t_s + v_w \cdot t_w) \cdot i + v_v \cdot t_v;$$

and if

$$\frac{dU_c}{dQ} = 0,$$

for the minimum point of the curve U_c, or

$$= \frac{du'}{dQ} + \frac{d\left(\frac{P \cdot F}{Q}\right)}{dQ} + \frac{d(Q \cdot f)}{dQ},$$

then

$$0 = 0 - \frac{P \cdot F}{Q^2} + f$$

or

$$\frac{P \cdot F}{Q^2} = f.$$

Now, Q in this case will be the minimum-cost quantity, so

$$Q_m = \sqrt{\frac{P \cdot F}{f}}.$$

If the expressions for F and f be reintroduced

$$Q_m = \sqrt{\frac{P \cdot \left(2 - \Delta + \frac{i \cdot T_M \cdot k_M}{8}\right)}{(v_s \cdot t_s + v_w \cdot t_w) \cdot i + v_v \cdot t_v}}. \tag{163}$$

For an interpretation of symbols see Appendix XIII, p. 349.

for a specific case when the appropriate expression for each factor has been ascertained by an analysis of the characteristics of the problem[1] as described for the minimum-cost quantity.

Fundamental Formula for the Point of Maximum Return.—In some instances it may be of importance to know the point within the economic range at which the maximum return can be earned upon the invested capital. By this is meant the point U_R where the greatest spread exists between the actual ultimate unit cost of any product and the unit margin of profit which must be obtained to satisfy the expectations of a gross return at the rate r normally anticipated by the owners of the business. One can readily see upon reference to Fig. 19[2] that this point will occur when the curve U for the actual ultimate unit cost is at its greatest divergence from the curve U' for the maximum allowable ultimate unit cost of production within the economic range. Obviously this situation will occur when the sum $\frac{R_o}{S_y} + U$ of the ultimate unit cost and unit margin of profit is a minimum. Accordingly, if the unit margin of profit on capital invested in work in process and articles in stores alone be expressed[3] by

$$\frac{R'_o}{S_y} = r \cdot [Q \cdot (v_s \cdot t_s + v_w \cdot t_w) + k_{v_s} + k_{v_w}],$$

an expression for the point of maximum return Q_R can be obtained when the sum $(i + r)$ of the interest rate and the expected rate of return be inserted in the expression for the minimum-cost quantity Q_m in place of the term i alone. This procedure is quite justifiable, because if one digressed to evaluate separately the relation $\left(\frac{R'_o}{S_y} + U\right)$ for the value of Q at the point where the sum of these two factors is a minimum, it would be found that the resulting expression would be identical with that for U_m in all respects except for the fact that i would be replaced by $(i + r)$. Therefore, in order to avoid a tedious duplication of the mathematical analysis, it can be stated directly with unquestionable authority that

$$Q_R = \sqrt{\frac{P \cdot F}{(v_s \cdot t_s + v_w \cdot t_w) \cdot (i + r) + v_v \cdot t_v}}. \tag{166}$$

[1] See p. 267.
[2] See p. 152.
[3] See Table XII, Chap. XI.

TABLE XXVI.—EQUATIONS EMPLOYED IN THE EVALUATION OF THE FUNDAMENTAL FORMULA FOR THE ECONOMIC-PRODUCTION QUANTITY

PRACTICAL SOLUTION

If

$$Q_c = \frac{Q_m \cdot P \cdot F}{P \cdot F + Q_m{}^2 \cdot r \cdot (v_s \cdot t_s + v_w \cdot t_w)} \tag{164}$$

and

$$Q_m = \sqrt{\frac{P \cdot F}{(v_s \cdot t_s + v_w \cdot t_w)i + v_v \cdot t_v}}, \text{ From Eq. } (163)$$

then

$$Q_c = \frac{\sqrt{\dfrac{P \cdot F}{(v_s \cdot t_s + v_w \cdot t_w) \cdot i + v_v \cdot t_v}}}{1 + \dfrac{P \cdot F}{P \cdot F} \cdot \left[\dfrac{(v_s \cdot t_s + v_w \cdot t_w) \cdot r}{(v_s \cdot t_s + v_w \cdot t_w) \cdot i + v_v \cdot t_v}\right]} \text{ For transition see Appendix IX.}$$

or

$$= \sqrt{\frac{P \cdot F \cdot [(v_s \cdot t_s + v_w \cdot t_w) \cdot i + v_v \cdot t_v]^2}{[(v_s \cdot t_s + v_w \cdot t_w) \cdot i + v_v \cdot t_v] \cdot [(v_s \cdot t_s + v_w \cdot t_w) \cdot (i + r) + v_v \cdot t_v]^2}},$$

and then by expanding and clearing off fractions

$$= \sqrt{\frac{P \cdot F}{(v_s \cdot t_s + v_w \cdot t_w) \cdot \left\{ i + 2 \cdot r + \dfrac{r^2}{\left[i + \dfrac{v_v \cdot t_v}{v_s \cdot t_s + v_w \cdot t_w}\right]} \right\} + v_v \cdot t_v}}. \tag{165}$$

For an interpretation of symbols see Appendix XIII, p. 349.

General Formulae for the Critical Points of Economical Production.—Now that the fundamental formulae have been established for the limits of the economic range Q_m[1] and Q_e[1] as well as for the point of maximum return Q_R,[1] they will be applicable to any situation that may at any time be encountered in industry no matter how complex or how simple it may be. The form of any one of these expressions, which will be most appropriate for use in determining the manufacturing policy when a specific case is being considered, can be evolved with little difficulty if a study of the characteristics of the process is carried out along the lines proposed in the next chapter. Should it be evident that a particular situation demands a consideration of all factors, the general formulae for the economic-production quantity, the minimum-cost quantity, or the point of maximum return, whichever it may be desirable to employ, will be found in Table XXVII, where the fundamental expressions have been fully expanded in acordance with the procedure there outlined. When it comes time to use one of the Eqs. (168), (169), or (170) in actual practice, it will be a simple matter to substitute in the expression selected the data obtained from current cost and production records in the places indicated by the respective symbols.

The Introduction of a General Expression for the Duration of the Sales Period.—In the evaluation of any one of these general expressions, a basic expression as shown in Eq. (167) for the length of the sales period T_s has been employed which includes the factors for deterioration Δ and obsolescence θ. It has been found advisable to do this because it is the author's intention to develop a single formula for Q_e, Q_m, and Q_R individually which will be universal in their application. For similar reasons the symbol S_{a_s} has been employed throughout to indicate the position of the average daily rate of consumption irrespective of the nature of the demand. In the actual application of any one of the general formulae to a specific problem, the factors of obsolescence and deterioration can be assigned a value of unity when it is recognized that either one or both have little or no influence upon the results. Likewise if the demand be uniform the constant value for S_a can be inserted in place of S_{a_s}, which by its presence ordinarily would suggest the condition of variable demand, as the former is naturally a special case of the latter.

[1] See Table XXVII.

TABLE XXVII.—EQUATIONS EMPLOYED IN THE FINAL EVALUATION OF THE GENERAL FORMULA FOR THE MINIMUM-COST QUANTITY AND THE ECONOMIC-PRODUCTION QUANTITY

PRACTICAL SOLUTION

If

$$v_s = c$$

$$t_s = \frac{1}{\theta \cdot S_{a_s}} \cdot \left(\frac{k_s}{\dfrac{\delta}{\theta \cdot S_{a_s}} + 1} - k_p \cdot \frac{\theta \cdot S_{a_s}}{2 \cdot D} \right)$$

$$v_w = \left[\left(\frac{1}{k_a} - 1 \right) \cdot m + c \right] \cdot k_a,$$

or

$$= c''$$

where for specific cases

$$c'' = \frac{m + c}{2},$$

$$= \frac{c}{2}, \quad \text{See p. 236.}$$

$$= \frac{m}{2}.$$

$$t_w = k_d \cdot t_p,$$

$$v_v = \frac{s \cdot b}{h},$$

$$t_v = \frac{k_b}{\theta \cdot S_{a_s}} \cdot \left(\frac{1}{\dfrac{\delta}{\theta \cdot S_{a_s}} + 1} - k_p \cdot \frac{\theta \cdot S_{a_s}}{D} \right),$$

$$F = 2 - \Delta + \frac{T_M \cdot k_M \cdot i}{8},$$

$$(1 - \Delta) = \left(\frac{1}{\dfrac{\theta \cdot S_{a_s}}{\delta} + 1} \right),$$

and

$$T_s = \frac{\Delta \cdot Q}{\theta \cdot S_{a_s}}, \tag{167}$$

or as a special case

$$\frac{\Delta}{\theta \cdot S_a} = \frac{\Delta}{\theta \cdot S_{a_s}}.$$

$$Q_m = \sqrt{\frac{P \cdot S_{a_s} \cdot \theta \cdot \left[1 + i \cdot \dfrac{T_M \cdot k_M}{8} + \dfrac{1}{\left(\dfrac{\theta \cdot S_{a_s}}{\delta} + 1 \right)} \right]}{c \cdot i \cdot \left(\dfrac{k_s}{\dfrac{\delta}{\theta \cdot S_{a_s}} + 1} - k_p \cdot \dfrac{\theta \cdot S_{a_s}}{2 \cdot D} \right) + \left[\left(\dfrac{1}{k_a} - 1 \right) m + c \right] \cdot k_a \cdot k_d \cdot \theta \cdot S_{a_s} \cdot t_p \cdot i + \dfrac{s \cdot b}{h} \cdot k_b \cdot \left(\dfrac{1}{\dfrac{\delta}{\theta \cdot S_{a_s}} + 1} - k_p \cdot \dfrac{\theta \cdot S_{a_s}}{D} \right)}} \tag{168}$$

$$Q_e = \sqrt{\frac{P \cdot \theta \cdot S_{a_s} \cdot \left[1 + i \cdot \frac{T_M \cdot k_M}{8} + \frac{1}{\left(\frac{\theta \cdot S_{a_s}}{\delta} + 1\right)}\right]}{\left\{c \cdot \left(\frac{k_s}{\frac{\delta}{\theta \cdot S_{a_s}} + 1} - k_p \cdot \frac{\theta \cdot S_{a_s}}{2 \cdot D}\right) + \left[\left(\frac{1}{k_a} - 1\right) \cdot m + c\right] \cdot k_a \cdot k_d \cdot \theta \cdot S_{a_s} \cdot t_p\right\} \cdot \ldots}} \ldots$$

$$\ldots \cdot \left\{ i + 2r + \left[\frac{r^2}{1 + \left[\frac{\frac{s \cdot b}{h} \cdot k_b \cdot \left(\frac{1}{\frac{\delta}{\theta \cdot S_{a_s}} + 1} - k_p \cdot \frac{\theta \cdot S_{a_s}}{D}\right)}{c\left(\frac{k_s}{\frac{\delta}{\theta \cdot S_{a_s}} + 1} - k_p \cdot \frac{\theta \cdot S_{a_s}}{2 \cdot D}\right) + \left[\left(\frac{1}{k_a} - 1\right) \cdot m + c\right] \cdot k_a \cdot k_d \cdot \theta \cdot S_{a_s} \cdot t_p}\right]}\right]\right\} + \ldots$$

$$\ldots + \frac{s \cdot b}{h} \cdot k_b \cdot \left(\frac{1}{\frac{\delta}{\theta \cdot S_{a_s}} + 1} - k_p \cdot \frac{\theta \cdot S_{a_s}}{D}\right) \tag{169}$$

$$Q_R = \sqrt{\frac{P \cdot \theta \cdot S_{a_s} \cdot \left[1 + i \cdot \frac{T_M \cdot k_M}{8} + \frac{1}{\frac{\theta \cdot S_{a_s}}{\delta} + 1}\right]}{\left\{c\left(\frac{k_s}{\frac{\delta}{\theta \cdot S_{a_s}} + 1} - k_p \cdot \frac{\theta \cdot S_{a_s}}{2 \cdot D}\right) + \left[\left(\frac{1}{k_a} - 1\right) \cdot m + c\right] \cdot k_a \cdot k_d \cdot \theta \cdot S_{a_s} \cdot t_p\right\} \cdot (i + r) + \ldots}} \ldots$$

$$\ldots + \frac{s \cdot b}{h} \cdot k_b \cdot \left(\frac{1}{\frac{\delta}{\theta \cdot S_{a_s}} - 1} - k_p \cdot \frac{\theta \cdot S_{a_s}}{D}\right) \tag{170}$$

For an interpretation of symbols see Appendix XIII, p. 349.

TABLE XXVIII.—EQUATIONS EMPLOYED IN THE EVALUATION OF THE ULTIMATE UNIT COST

EXACT SOLUTION

Again if (171)

$$U_i = m + A,$$

$$A = (u_m - m) + \frac{I'_s + I_w + V_s + V_w + L'_d}{Q};$$

then

$$U_i = u_m + \frac{I'_s + I_w + V_s + V_w + L'_d}{Q}.$$

Now if

$$u_m = m + l + o + a + \frac{P}{Q}$$

or

$$= c + \frac{P}{Q},$$

$$\frac{I'_s}{Q} = Q \cdot U_i \cdot t_s \cdot i,$$

$$\frac{I_w}{Q} = Q \cdot v_w \cdot t_w \cdot i + \frac{P}{Q} \cdot i \cdot \frac{T_M \cdot k_M}{8} + k_{v_w},$$

$$\frac{V_s}{Q} = Q \cdot v_v \cdot t_v,$$

$$V_w = 0,$$

$$\frac{L'_d}{Q} = U_i \cdot (1 - \Delta),$$

$$U_i = c + \frac{P}{Q} + U_i \cdot Q \cdot t_s \cdot i + Q \cdot v_w \cdot t_w \cdot i + \frac{P}{Q} \cdot i \cdot \frac{T_M \cdot k_M}{8} + k_{v_w} + \cdots$$
$$Q \cdot v_v \cdot t_v + 0 + Ui \cdot (1 - \Delta),$$

$$= [c + k_{v_w}] + \frac{P}{Q} \cdot \left[1 + i \cdot \frac{T_M \cdot k_M}{8}\right] + Q[v_w \cdot t_w \cdot i + v_v \cdot t_v + U_i \cdot t_s \cdot i] + \cdots$$
$$U_i \cdot (1 - \Delta). \quad (172)$$

For an interpretation of symbols see Appendix XIII, p. 349.

Basic Formula for the Ultimate Unit Cost: Exact Solution.—Even though the exact solution will rarely be employed in the determination of the best lot size, owing to the fact that it does not conform to the accepted methods of accounting, no approach to this subject would be complete if a similar treatment for the relation of the factors in the expression U_i for the ultimate unit cost were omitted. Therefore the basic expression [Eq. (172)] for U_i may likewise be developed by introducing into Eq. (171)[1] the various expressions for each factor contained therein, as shown in Table XXVIII. Then if the terms in this expression be grouped according to their various characteristics in relation to Q or U_i, as shown in Eq. (173), the fundamental relation for the minimum-cost quantity, Q_m in Eq. (174), can be evolved through a process of differentiation, as illustrated in Table XXIX. This relation can then be evaluated by reintroducing the terms represented by the symbols u'', F', f', f'', f''', so that the fundamental expression for Q_m in this case can be written as shown in Eq. (175).

Fundamental and General Formulae Based on the Exact Method of Solution.—Once the relation of the factors for the minimum-cost quantity has been established, it is a simple matter to repeat the procedure (see Table XXX) of combining Eqs. (175) and (176) in order to obtain the fundamental formula [Eq. (177)] for the economic-production quantity which will correspond to the requirement of the exact method of solution. Likewise, the fundamental expression for the point of maximum return can be evaluated by introducing the parenthesis $(i + r)$ in place of i in Eq. (175) for reasons identical to those given in the preceding case.[2] Furthermore, if there be cause to employ one of these formulae, as derived for the exact solution in actual practice, any one of them can be fully expanded so as to include all items which enter into the composition of each factor. As it is doubtful whether any such need will arise, owing to the greater complexity of these equations and the questionable advantage to be gained from their use, the various general expressions for Q_m, Q_e, and Q_R under these conditions will not be evaluated here. It will be comparatively easy, when the occasion demands, to introduce the appropriate terms for

[1] See Eq. (64), p. 170.

[2] See p. 269.

TABLE XXIX.—EQUATIONS EMPLOYED IN THE DERIVATION OF THE FUNDAMENTAL FORMULA FOR THE MINIMUM-COST QUANTITY

EXACT SOLUTION

$$U_i = u'' + \frac{P}{Q} \cdot F' + Q \cdot (U_i \cdot f' + f'') + U_i \cdot f''', \tag{173}$$

where

$$u'' = c + k_{v_w},$$
$$F' = 1 + i \cdot \frac{T_M \cdot k_M}{8},$$
$$f' = t_s \cdot i,$$
$$f'' = v_w \cdot t_w \cdot i + v_v \cdot t_v,$$
$$f''' = (1 - \Delta),$$

and then

$$U_i = \frac{u'' + \frac{P}{Q} \cdot F' + Q \cdot f''}{1 - f' \cdot Q - f'''}.$$

Hence, if for the minimum point of the curve U_i where Q becomes Q_m

$$\frac{dU_i}{dQ} = 0,$$

or

$$= + u'' \cdot f' - \frac{P \cdot F'}{Q_m{}^2} \cdot (1 - f''') + 2 \cdot \frac{P \cdot F'}{Q_m} \cdot f' + f'' \cdot (1 - f'''),$$
$$= Q_m{}^2 \cdot \left[\frac{u'' \cdot f'}{1 - f'''} + f''\right] + 2 \cdot Q_m \cdot \frac{P \cdot F' \cdot f'}{1 - f'''} - P \cdot F';$$

then by use of the formula (Appendix XII)

$$Q_m = \frac{-P \cdot F' \cdot f'}{u'' \cdot f' + f'' \cdot (1 - f''')} \pm \sqrt{\left[\frac{-P \cdot F' \cdot f'}{u'' \cdot f' + f'' \cdot (1 - f''')}\right]^2 + \frac{P \cdot F' \cdot (1 - f''')}{u'' \cdot f' + f'' \cdot (1 - f''')}} \tag{174}$$

or when the expression for each term is reintroduced

$$Q_m = \frac{-P \cdot F'}{c + k_{v_w} + v_w \cdot \Delta \cdot \frac{t_w}{t_s} + \Delta \cdot \frac{v_v}{i} \cdot \frac{t_v}{t_s}} \pm$$
$$\sqrt{\left[\frac{-P \cdot F'}{c + k_{v_w} + v_w \cdot \Delta \cdot \frac{t_w}{t_s} + \Delta \cdot \frac{v_v}{i} \cdot \frac{t_v}{t_s}}\right]^2 + \frac{\Delta \cdot P \cdot F'}{(c + k_{v_w}) \cdot t_s \cdot i + \Delta \cdot v_w \cdot t_w \cdot i + \Delta \cdot v_v \cdot t_v}}. \tag{175}$$

This equation can be further evaluated by the insertion of the appropriate expressions in accordance with the conditions for t_s, v_w, t_w, v_v, t_v, and Δ.

For an interpretation of symbols see Appendix XIII, p. 349.

TABLE XXX.—EQUATIONS EMPLOYED IN THE EVALUATION OF THE FUNDAMENTAL FORMULA FOR THE ECONOMIC-PRODUCTION QUANTITY

EXACT SOLUTION

In order to simplify the complex relations in Eqs. (175) and (52), let $\Delta = 1$ and

$$K_{sw} = v_s \cdot t_s \cdot i + v_w \cdot t_w \cdot i,$$

$$v_s = c,$$

where

$$K_{vk} = v_v \cdot t_v + k_{v_w} \cdot t_s \cdot i,$$

$$K_z = t_s \cdot i\left[-1 \pm \sqrt{1 + \frac{(K_{sw} + K_{vk})}{P \cdot F' \cdot t_s^2 \cdot i^2}}\right].$$

Then

$$Q_m = \frac{P \cdot F' \cdot K_z}{K_{sw} + K_{vk}}$$

and

$$Q_l = \frac{Q_m}{1 + \dfrac{Q^2{}_m \cdot r \cdot \dfrac{K_{sw}}{i}}{P \cdot F'}};$$

whereupon

$$Q_{l_i} = \frac{\dfrac{P \cdot F' \cdot K_z}{K_{sw} + K_{vk}}}{1 + \dfrac{P^2 \cdot F'^2 \cdot K_z^2}{(K_{sw} + K_{vk})^2} \cdot \dfrac{r}{i} \cdot \dfrac{K_{sw}}{P \cdot F'}}. \qquad (176)$$

By combining terms, this equation can be reduced to the form where

$$Q_e = Q_{l_i} = \frac{k_e}{\dfrac{K_{sw}}{i \cdot P \cdot F'} + r \cdot k_e^2},$$

if

$$k_e = \frac{t_s}{1 + \dfrac{K_{vk}}{K_{sw}}} \cdot \left[-1 \pm \sqrt{1 + \frac{(K_{sw} + K_{vk})}{P \cdot F' \cdot t_s^2 \cdot i^2}}\right]. \qquad (177)$$

For an interpretation of symbols see Appendix XIII, p. 349.

each unit value and time factor when the requirements for a specific case have been determined.[1]

Adjustments for Special Accounting Procedures.—If it be the accepted practice in any concern to distribute overhead upon the basis of the manufacturing cost u_m in the manner illustrated by Eq. (78)[2], the terms c, P, and m in any of these general formulae should be replaced by $c_o \cdot (1 + d_u)$, $P_o \cdot (1 + d_u)$ and $m \cdot (1 + d_u)$. Upon reference to Chap. XIII it will be seen that

$$c_o = (m + l) \cdot (1 + d_u)$$

or

$$= m \cdot (1 + d_u) + l \cdot (1 + d_u),$$

and so wherever the material value m appears, it must be increased by the values of $(1 + d_u)$ in order that the portion of the overhead, which in this case is also carried by the material cost, as it is a part of the manufacturing cost, will not be overlooked. Since the unit value factor v_w in I_w contains the term m as well as c, this factor must be rewritten so that

$$v_w = \left[\left(\frac{1}{k_a} - 1\right) \cdot m + c_o\right] \cdot (1 + d_u) \cdot k_a.$$

These alterations in any general formula can be performed quite easily whenever the need arises, and on that account there will be little advantage if Eqs. (168), (169), and (170) be rewritten here merely to conform with the requirements of this special case.

[1] See Chap. XIX or Chap. V.

[2] See p. 190.

CHAPTER XIX

SIMPLIFICATION IN ACTUAL PRACTICE

In the preceding chapter the fundamental[1] formulae for both the economic-production quantity and the minimum-cost quantity were established in a form which can be employed for the determination of the best lot size or the limits of the economic range of production under any conditions that may be encountered in industry, no matter how complex or how simple. In addition, general formulae[2] were derived from these which supposedly contain all the necessary items for computing the lot size in any case where every contributory factor is by force of circumstances required. Such expressions, for the most part, are of major importance only in a theoretical consideration of this subject when taken as a whole, and will be of little practical value to the ordinary type of executive who abhors, on general principles, any procedure which from appearance alone would seem to involve mathematical mysteries and would tend to increase the indirect expense of production control, even though much benefit could be derived from its use.

Simplified Formulae Justifiable.—Fortunately it has been found, as a result of a survey of a large number of cases where economic lot sizes can be employed to advantage, that the limits of the economic range depend for the most part upon but one of the three major elements, which, in conjunction with the preparation costs, determine the economic balance. In a few instances two elements will be required, but the probability of having to employ all three is so remote that it is well worth while to investigate the possibilities of simplifying the general formulae so as to make them conform to the specific conditions which actually govern a given problem. Naturally, if one or more of the major elements are omitted, the value obtained for the lot size from a simplified form will only approximate the true value which would otherwise have been obtained from a

[1] See Tables XXV and XXVI.

[2] See Table XXVII.

general formula. It will be shown further on in this chapter,[1] however, that the deviation between these results will be so small that for all practical purposes it can be entirely neglected, within certain prescribed limits.

Simplification Must Be Carefully Controlled.—If simplification of any general formula is to be at all feasible, it cannot be achieved in a haphazard manner. To assume without proper justification that any one or two elements can be disregarded would unquestionably lead to serious errors, and there would be no possible way of ascertaining whether the lot size thus determined actually met the conditions or was commensurate with that which would have been obtained from a solution of the general formula.[2] Some sound mathematical basis must be established for the guidance of those who in actual practice desire to avail themselves of this opportunity. Then it will be possible to determine reasonable limits of accuracy for the selection of an appropriate simplified form. In this manner one can foretell to what extent the details of computation need be carried, and as a result a simple procedure[3] can be established, which will minimize the labor of calculation even to the point where mechanical means[4] of solution can be adopted.

The Method of Simplification.—In order to obtain a logical basis for the method of simplification it has been found advisable to subdivide the analytical procedure into a number of steps.

1. It will be necessary to determine the type of solution, whether it be the exact or the practical one, which will conform to the accounting methods most commonly employed in industry, so that the special forms may be derived from the appropriate general expressions for the minimum-cost quantity, the economic-production quantity and the point of maximum return.

2. All factors which are found to apply only in extraordinary cases should be eliminated.

3. The most suitable general expressions being thus reduced to their simplest proportions, so that they contain only the major cost factors, means should be provided for selecting, if possible, the single element in the denominator which is the controlling factor.

[1] See p. 292.

[2] See Table XXVII.

[3] See Table VI, Chap. V.

[4] See p. 41.

4. In order to ascertain the permissible degree of approximation which will be required by the use of a simplified form, limits should be determined for the allowable variation in the data consistent with the permissible variation in the results that will be obtained from a solution of any one of these special forms.

5. A maximum limit of permissible variation in the results should be established which will depend upon the characteristics of the problem, so that it can be used as a measure to aid in the selection of a form having the desired degree of approximation.

6. Once the appropriate form has been selected which bears most distinctly upon a specific problem, the terms in this expression should be rearranged and all known constant items introduced, so that the final form of these expressions, as released for actual use, will contain the fewest repetitions of similar items.

Selection of the Most Practical Type of Solution.—In the first place it should be remembered that the exact solution was based upon the hypothesis[1] upheld by C. H. Scovell that interest charges should be included in the manufacturing cost of any unit of production and that it was justifiable to employ this unit value for inventory purposes. As this theory has met with almost universal opposition by other cost accountants, it would be impractical to insist upon its use solely for the determination of the best lot size. Moreover, as the practical solution has been constructed in conformance with current accounting practice and differs only from the preceding method in that the cost of capital is not compounded, there is every reason to support the choice of this type of solution. Even appearances alone would indicate to the practical-minded executive that whatever general expressions might be developed from the exact solution,[2] they would be altogether too complex for daily use when compared with those derived from the other type. Accordingly, the method of simplification as developed in this chapter will be confined to a consideration of the general expressions represented by Eqs. (178) for Q_m, (179) for Q_R, and (180) for Q_e.[3]

[1] See reference 3, p. 160.

[2] Compare Eqs. (163) and (165) with (175) and (177) in Chap. XVIII, Tables XXV, XXVI, XXIX, and XXX.

[3] See Table XXXI, p. 282.

TABLE XXXI.—SIMPLIFIED FUNDAMENTAL FORMULAE FOR Q_m, Q_R AND Q_e

$$Q_m = \sqrt{\frac{P}{(v_s \cdot t_s + v_w \cdot t_w) \cdot i + v_v \cdot t_v}}, \tag{178}$$

$$Q_R = \sqrt{\frac{P}{(v_s \cdot t_s + v_w \cdot t_w) \cdot i \cdot \left(1 + \frac{r}{i}\right) + v_v \cdot t_v}}, \tag{179}$$

$$Q_e = \sqrt{\frac{P}{(v_s \cdot t_s + v_w \cdot t_w) \cdot i \cdot \left\{1 + 2 \cdot \frac{r}{i} + \frac{r^2}{i^2 \cdot \left[1 + \frac{v_v \cdot t_v}{(v_s \cdot t_s + v_w \cdot t_w) \cdot i}\right]}\right\} + v_v \cdot t_v}}, \tag{180}$$

or if the elements only be indicated

$$Q_m = \sqrt{\frac{K_p}{K_s + K_w + K_v}}, \tag{181}$$

$$Q_R = \sqrt{\frac{K_p}{(K_s + K_w) \cdot f_R + K_v}}, \tag{182}$$

$$Q_e = \sqrt{\frac{K_p}{(K_s + K_w) \cdot f_r + K_v}}. \tag{183}$$

Then in general the ideal quantity will be

$$Q_i = \sqrt{\frac{K_p}{(K_s + K_w) \cdot f_r + K_v}}, \tag{184}$$

where for the minimum-cost quantity

$$f_r = 1,$$

for the point of maximum return

$$f_r = f_R = \left(1 + \frac{r}{i}\right),$$

for the economic-production quantity,

$$f_r = 1 + 2\frac{r}{i} + \frac{r^2}{i^2 \cdot \left[1 + \frac{K_v}{K_s + K_w}\right]},$$

and

$$\begin{aligned} K_p &= P, \\ K_s &= v_s \cdot t_s \cdot i, \\ K_w &= v_w \cdot t_w \cdot i, \\ K_v &= v_v \cdot t_v. \end{aligned}$$

For interpretation of symbols, see text, Chap. XIX, or Appendix XIII, p. 349.

Elimination of Extraordinary Factors.—In the second place, if the factors of deterioration Δ[1] and obsolescence θ[2] are of sufficient importance to demand their retention in any formula, the problem will in all probability be so complex that it would be inadvisable to employ any simplified form. Moreover, a situation will be encountered so seldom where the sales demand is found to vary from one sales period to another to such an extent as to warrant special treatment, that for all practical purposes, where simplified forms will be advantageous, the demand may be considered as uniform and a constant numerical value can be assigned to the average stock factor k_s.[3] Therefore, in the majority of cases it will be assumed that obsolescence, deterioration, and variable demand can be disregarded. Accordingly, if the coefficients θ and Δ be assigned a value of unity and the factor k_s be allowed to retain its normal value of one half, a new group of formulae can be written to take the place of the general ones referred to above.

Elimination of Minor Factors.—If the simplification of the general expressions is to be complete in order to dispense with all unnecessary computations, three other items can be omitted as well. In the numerator of the fundamental formulae a number of minor items, which may at times have some influence upon the preparation cost P, have been grouped under the symbol F. These represent an adjustment for the interest charge derived from the value added to a unit of production through the expense of machine set-ups, and that portion of the allotment of the preparation charges which becomes a loss due to deterioration and which must be borne by the remaining saleable articles. In the latter case the factor $\frac{P}{Q} \cdot (1 - \Delta)$ can be disregarded if Δ is assumed to be unity when special forms are employed. Likewise, in the former case $\frac{P \cdot i}{Q} \cdot \frac{T_M \cdot k_M}{8}$ can be neglected, because the value of $i/8$, when i is assigned a normal value, such as 6 per cent, and then expressed in terms of the same unit employed in designating the time consumed in machine changeover T_M, will be so small that the terms $\frac{i}{8} \cdot T_M \cdot k_M$ will have little influence

[1] See Sec. A, Chap. XVII.
[2] See Sec. B, Chap. XVII.
[3] See p. 211.

in changing the value of the factor F should it have been originally assumed to be unity.

Moreover, the bin factor k_b, appearing in the space charges V_s can be omitted, as it is almost universal practice to assign permanently a specific number of bins for the storage of a particular article, regardless of the fact that some of the bins will remain empty over some portion of the storage period. As the delay factor k_d associated with the actual unit process time t_p was introduced merely to show where to make a correction for unavoidable delays as a matter of convenience, should it be necessary, this factor can be left out of the expression for the special forms in order that only the fewest terms will appear. Accordingly, as a result of this stage of the simplification process, the general expressions for Q_m, Q_R, and Q_e will take the forms illustrated in Table XXXI by Eqs. (181), (182), and (183).

Basic Elements of the General Formulae.—Before proceeding with the third stage of simplification it will be advisable to stop for a moment and study the composition of these equations. It will be immediately noticeable that Eqs. (181) and (182) for Q_m and Q_R are almost identical in form, except that in the latter case the first two groups of terms in the denominator are increased by the value of the parenthesis $\left(1+\frac{r}{i}\right)$. Likewise, it is evident that the various groups of terms in Eq. (183) are comparable with those in either of the other two expressions. Now it so happens that each of these groups represents one of the major cost factors which control the best lot size, and as the next step will be devoted to a consideration of their relative importance, these groups will henceforth be designated as elements of the general expression, and specific symbols will be employed to denote each particular group, as indicated in Table XXXI.

Devices Employed in Simplification.—Thus the necessity for writing out such expressions in full each time they are referred to can be avoided, and then it will be a simple matter to substitute the actual expressions for the particular elements required when the need arises. In a similar manner the composition of any simplified form which may eventually be developed can be conveniently indicated. Another device will be employed hereafter to designate these special forms for the determination

of the best lot size, and contemplates the use of an additional subscript appended to the general symbols Q_m, Q_R, Q_e, etc., to indicate the elements which in each case may have the greatest influence. For example, $Q_{e_{sv}}$ would indicate that the economic-production quantity Q_e for a given set of conditions could best be determined from the special form which contains the storage and space charge elements K_s and K_v in the denominator.

A Composite Expression for the Ideal Lot Size.—The selection of the appropriate elements for use in these special forms will be the third step in the process of simplification. Owing to the fact that the general expressions for Q_m, Q_e, and Q_R, as given in Table XXXI, contain the same typical groups of terms and in their final form differ only in the terms which can be represented by the factor f_r, where, for the minimum-cost quantity Q_m,

$$f_r = 1,$$

for the point of maximum return Q_R

$$f_r = f_R = \left(1 + \frac{r}{i}\right),$$

and for the economic-quantity Q_e,

$$f_r = 1 + \frac{2r}{i} + \frac{r^2}{i^2\left[1 + \dfrac{K_c}{K_s + K_w}\right]},$$

the method to be employed in the selection of special forms can be constructed upon the basis of a composite general expression where f_r is introduced, as indicated Table XXXI, in Eqs. (181), (182), and (183), so that Eq. (184) for Q_i will designate the lot size when determined on the basis of any one of these general expressions. However, as the minimum-cost quantity Q_m is employed in the derivation of the quantities Q_R and Q_e so as to relate them in the proper manner to the conditions imposed by the characteristics of a specific problem, the limits of accuracy which will govern the choice of a special form, for Q_m, Q_R, and Q_e as the circumstances may demand, can be determined on the basis of Q_m alone. As a result the factor f_r need not be considered in the selection of a special form.

The Influence of Simplification upon the Ultimate Unit Cost.—If the process of simplification at this stage proposes to eliminate

one or more of the elements in the denominator of these general expressions which will have the least influence upon the problem, the quantity Q_u, as determined from a simplified form of any one of the general expressions, will be greater than that obtained from the corresponding expression which contains all elements. This is obviously true because the numerator K_p in any form will always be the same, and the denominator K of a special form Q_u will be composed of fewer elements than appear in the denominator K_c for a complete form Q_i. This situation will be all the more evident if Eq. (184) for Q_i be compared with the equations listed in Table IV,[1] or should the diagram in Fig. 7[1] be referred to. In this diagram it will be noticed, if the line $K_c \cdot Q$ represents the sum of the unit investment and storage space charges for each value of Q, and, similarly, if the line $K \cdot Q$ represents the sum of only those unit charges which are found to have the greatest influence, that the minimum-cost point where in general

$$\frac{P}{Q} = Q \cdot K$$

will be moved to the right and at the same time lowered from the position it ordinarily would occupy if no elements were neglected. The fact that the curve U is also shifted in position has little meaning, because the intent of this device is to obtain a reasonable approximation through Q_u of the true value of Q_i, it being understood that the actual minimum ultimate unit cost or any other unit cost on the curve U is in no way affected by this assumption.

Permissible Variation from the Ideal Quantity.—Consequently, this divergence between the values of Q_i and Q_u due to the elimination of certain insignificant cost elements in the determination of any quantity lying within the economic range, will serve as an index or measure for controlling the selection of the most suitable form for a specific problem, provided that a definite maximum permissible limit of variation E_M[2] can be established to prevent the occurrence of an error which would be undesirable when the characteristics of the problem are considered. Accordingly, the coefficient E will be introduced to show the amount of divergence between Q_u and Q_i for any special form Q_u and can be employed to signify the permissible variation

[1] See Chap. V.

[2] See p. 305 or Table VI, Chap. V.

TABLE XXXII.—EQUATIONS EMPLOYED IN DEMONSTRATING THE RELATION OF THE FORM INDEX TO THE PERMISSIBLE VARIATION IN THE RESULTS

If the approximate value for the lot size be

$$Q_u = \sqrt{\frac{K_p}{K}}, \tag{185}$$

and the ideal value be

$$Q_i = \sqrt{\frac{K_p}{K_c}}, \tag{186}$$

where

K = the elements employed in the denominator of a special form,

and

$K_c = (K_s + K_w)f_r + K_v$ or the elements in the denominator of any one of the fundamental formulae,

$$E = \frac{Q_u}{Q_i}, \tag{187}$$

or

$$= \sqrt{\frac{K_p/K}{K_p/K_c}},$$

$$E = \sqrt{\frac{K_c}{K}}. \tag{188}$$

Now if by definition

$$e = \frac{K}{K_c}, \tag{189}$$

$$E = \frac{1}{\sqrt{e}}, \tag{190}$$

or

$$e = \frac{1}{E^2}, \tag{191}$$

or

$$= \frac{1}{(1 + E')^2}.$$

For an interpretation of symbols, see text, Chap. XIX, or Appendix XIII, p. 349.

in the answer, if the value of E be nearer to unity than that of E_M, when E is expressed by Eq. (187).

The Form Index.—If the selection of the appropriate elements for a simplified form is to be governed by the relation of the coefficient E to the permissible limit E_M, it will be necessary to express E in terms of the elements to be employed. This can be accomplished if K is allowed to represent any one or a group of two major elements used in the denominator, and if K_c represents the denominator in any one of the general expressions [Eq. (188)]. Then Q_u and Q_i will take the form shown in Eqs. (185) and (186). If these be introduced into Eq. (187), it will be found that the degree of variation in the answer E depends upon the reciprocal of the square root of the variation in the data e employed [Eq. (190)], as designated by the ratio of the elements K in the denominator of the special form to those K_c in the denominator of the complete form. This ratio e as expressed in Eq. (189) will be designated as the form index f_c when compared with the maximum value of e, and its relation to the permissible variation in the answer may be shown by Eq. (190). At this point it is quite interesting to note when E' and e represent decimally the variation respectively above and below the ideal value of Q_m expressed as unity, in accordance with Eq. (191), that for a given variation in the answer a much larger variation in the data will be allowable. As a result, much more latitude is permitted in the choice of a simplified form than could be if e were only inversely proportional to E.

Selection of Special Forms by Index Ratios.—Now if a value for the maximum allowable variation in the data e_M is obtained from E_M through the use of Eq. (191), and if a value for the form index f_c for each possible combination of elements in the denominator, as shown by Eq. (202) in Table XXXIII, can be obtained, the simplest form can be readily selected by a comparison of e_M and f_c, such that f_c is always greater than e_M. The appropriate form will be that one for which the form index f_c most nearly approximates the value of e_M, provided that the previous condition always holds. If it were actually necessary to calculate the values of the form index f_c in each case from the complete expression for each element employed, little advantage could be realized from the use of simplified forms. This can be avoided, however,

TABLE XXXIII.—EQUATIONS EMPLOYED IN DERIVING A METHOD OF SELECTING SPECIAL FORMS

If by definition

$$R_s = \frac{K_s \cdot f_r}{K_s \cdot f_r} = \frac{v_s \cdot t_s \cdot i \cdot f_r}{v_s \cdot t_s \cdot i \cdot f_r}, \tag{192}$$

$$R_w = \frac{K_w \cdot f_r}{K_s \cdot f_r} = \frac{v_w \cdot t_w \cdot i \cdot f_r}{v_s \cdot t_s \cdot i \cdot f_r}, \tag{193}$$

$$R_v = \frac{K_v}{K_s \cdot f_r} = \frac{v_v \cdot t_v}{v_s \cdot t_s \cdot i \cdot f_r}, \tag{194}$$

$$R_c = \frac{K_c}{K_s \cdot f_r} = \frac{K_s \cdot f_r + K_w \cdot f_r + K_v}{K_s \cdot f_r}, \tag{195}$$

$$R = \frac{K}{K_s \cdot f_r}, \tag{196}$$

and if when simplified in accordance with text (See p. 283).

$$v_v = \frac{s \cdot b}{h},$$

$$t_v = \frac{f_p}{S_a},$$

when

$$S_{a_s} = S_a,$$

and where

$$f_p = \left(1 - k_p \cdot \frac{S_a}{D}\right), \quad \text{(See p. 290)}$$

or

$$= \left[1 - \frac{S_a}{D} \cdot \left(1 - \frac{1}{n}\right)\right],$$

$$v_w = c'',$$

where c'' may be expressed as

$$= \frac{m + c}{2},$$

$$= \frac{c}{2},$$

$$= \frac{m}{2},$$

as the nature of the process may demand. (See p. 236.)

$$t_w = t_p,$$

$$v_s = c,$$

$$t_s = \frac{f_p}{2S_a},$$

when

$$k_s = \tfrac{1}{2}.$$

Then

$$R_s = 1, \tag{197}$$

$$R_w = f_m \cdot t_p \cdot \frac{S_a}{f_p}, \tag{198}$$

where

$$f_m = \frac{v_w}{v_s/2} = \frac{c''}{c/2}, \quad \text{(See p. 290).}$$

$$R_v = \frac{\phi}{f_r} \cdot \frac{b}{c}, \tag{199}$$

where

$$\phi = \frac{2 \cdot s}{i \cdot h},$$

a constant in a specific plant, and

$$f_r = 1 + 2\frac{r}{i} + \frac{r_2}{i_2} \cdot \left[\frac{1 + R_w}{R_c}\right].$$

$$R_c = 1 + R_w + R_v. \tag{200}$$

Then if

$$f_c = \frac{K}{K_c} = \frac{\dfrac{K}{K_s} \cdot f_r}{\dfrac{K_c}{K_s} \cdot f_r}, \tag{201}$$

$$f_c = \frac{R}{R_c}, \tag{202}$$

$$R = f_c \cdot R_c, \tag{203}$$

or for the limiting case where

$$f_c = e_a$$

or

$$= e_M,$$

$$R = e_a \cdot R_c \tag{204}$$

or

$$= e_M \cdot R_c. \tag{205}$$

For an interpretation of symbols, see text, Chap. XIX, or Appendix XIII, p. 349.

by employing a series of index ratios R[1] expressed in terms of a common denominator, so that each will contain only the controlling items and will represent the relative importance of a particular element. Thus all items which otherwise might be duplicated can be eliminated, and, as the value of each index depends upon a ratio, sufficient accuracy can be obtained by the use of two significant figures, so that the least amount of computation will be required.

The Plant Constant and the Process Factor.—It has been found upon further study that in each of these ratios there are certain combinations of items which can be grouped together and assigned a characteristic symbol to indicate their identity, because these various items influence the selection of a special form as a group and not individually, and consequently in most instances a constant value can be assigned to each, depending upon certain general characteristics of the problem. Typical of this is the plant constant ϕ which represents the ratio of the space charges per cubic foot per year s/h to one half the annual rate of interest $i/2$ on borrowed capital, and, as it is a constant for all problems in any plant, its value need be calculated but once. Similarly, the factor f_r for the limits of the economic range will be unity in all cases, except where an intricate problem requires special analysis in the selection of a form for the economic-production quantity Q_e or the point of maximum return Q_R. Again, the flow of material factor f_m has been introduced in order to simplify the computation of the ratio $\dfrac{2 \cdot c''}{c}$ when c'' can be expressed in various ways[2] to account for the manner in which the flow of material is affected by the type of process.[3] Finally, the process factor f_p has been employed to combine all items which depend either upon the characteristics of the process or the rate of consumption as they may be used to account for the overlapping of the production and sales or storage periods. It is interesting to note that this factor can be disregarded when the value of f_p for any problems exceeds 0.9. This situation can be justified if f_e represents the correction which must be made in E in order to permit the omission of the terms in the factor f_p from Q_u, as illustrated in Table XXXIV. Since $f_e = \sqrt{f_p}$

[1] See Eqs. (192) to (196) or (197) to (200) in Table XXXIII.

[2] See p. 236.

[3] See p. 29 or 201.

TABLE XXXIV.—EQUATIONS EMPLOYED IN DEMONSTRATING THE EFFECT OF THE NATURE OF THE PROCESS UPON THE PERMISSIBLE VARIATION IN THE RESULTS

If by definition see Table XXXII.

$$Q_{u_{ms}} = \sqrt{\frac{K_p}{K_s}} \text{ (or) } = \sqrt{\frac{K_p}{K'_s \cdot f_p}}, \tag{206}$$

where

$$f_r = 1,$$

$$K'_s = \frac{c \cdot i}{2S_a},$$

and

$$f_p = \left(1 - k_p \cdot \frac{S_a}{D}\right),$$

and

$$Q'_{u_{ms}} = \sqrt{\frac{K_p}{K'_s}}, \tag{207}$$

$$Q_{u_{ms}} = \frac{1}{\sqrt{f_p}} \cdot Q'_{u_{ms}}$$

However, if

$$E \cdot Q_i = Q_{u_{ms}},$$

$$E'' \cdot Q_i = E \cdot f_e \cdot Q_i = Q'_{u_{ms}},$$

where

$$E'' = \frac{Q'_{u_{ms}}}{Q_i}$$

and

$$f_e = \frac{E''}{E},$$

$$E \cdot Q_i = \frac{1}{\sqrt{f_p}} \cdot Q'_{u_{ms}},$$

or

$$= \frac{E \cdot f_e \cdot Q_i}{\sqrt{f_p}},$$

Hence

$$f_e = \sqrt{f_p}. \tag{208}$$

But if E″ cannot exceed E, E must be corrected to allow for the additional error incurred by the omission of f_p in $Q'_{u_{ms}}$, so that

$$\frac{E}{\sqrt{f_p}} = E'' \tag{209}$$

must be employed as the limit of permissible variation for the form $Q'_{u_{ms}}$.

For an interpretation of symbols, see text, Chap. XIX, or Appendix XIII, p. 349.

and if $f_p = 0.9$ as a limiting value, the correction in E will only amount to 0.055, as f_e indicates that E need only be reduced to 0.945 of its original value to compensate for this omission. Accordingly, it would seem reasonable in actual practice to disregard the effect of any slight overlap between the production and sales periods, provided that f_p, as a result, always exceeds the limiting value of 0.9.

Choice of the Basic Elements for a Special Form.—Now that the identity of the index ratios R_v, R_w, and R_s have been established, the best form may be selected by comparing their individual values with that of R_c multiplied by the value for e_M in order that the desired advantage can be gained through the existence of a permissible variation in the results. The eventual form will be that indicated by the characteristic subscript for the single index ratio or a group of two, the value of which exceeds this value of $e_M \cdot R_c$, because this is the same as saying that the form index f_c bears the proper relation to e_M or E_M. The mathematical justification of this procedure can be explained by reference to Eqs. (201) to (205) in Table XXXIII, if the general index R be allowed to represent the value of the index ratio which will most nearly equal but always exceed the value of $e_M \cdot R_c$. If no single ratio meets this requirement, some combination of two can be found that does, and then the two elements represented by these ratios will have to be employed in place of a single one. In this manner the appropriate special forms for Q_m, Q_R, and Q_e can be selected, depending upon the respective values for f_r as required by the general forms for each of these expressions. Since it has been found that the storage space element K_v is of importance in only rare instances, reliable results can be obtained when the method of selection is confined to the elements in the minimum-cost quantity Q_m alone, and then, when special forms for Q_R and Q_e are required, the same combination of elements can be used with equal satisfaction. Thus unnecessary preliminary computations can once again be avoided.

The Arbitrary Value for the Form Index.—This is the method which underlies the procedure outlined in Part I of the calculation sheets or the graphical method of analysis which were recommended for common use in Chap. V,[1] and in both of which f_r was assumed to be unity in accordance with the discussion just

[1] See Table VI or Fig. 9.

preceding. It has been found, however, that under ordinary conditions a wide range of latitude in the selection of special forms can be obtained when the form index f_c is assumed to be greater than $e = 0.666$ without impairing the reliability of the results. Therefore in drawing up these calculation sheets the arbitrary value $e_a = 0.666$ was inserted in order to dispense with the necessity of determining E_M or e_M for every problem. Of course, if any peculiar conditions exist which may give sufficient cause to doubt the reliability of this assumption, these limiting values may be determined in advance. For this reason Part II[1] of the calculation sheets has been prepared, but its use has been allowed to remain optional.

A Short-cut Method for Computing the Ideal Quantity.—If for any reason the absolute value for Q_m, Q_R, or Q_e is desired after an approximate value has been computed from a special form, a short-cut method may be applied by employing the form index as a corrective factor. This will avoid any recourse to the more complicated general expressions for its solution and may be achieved in accordance with the Eqs. (210) to (214) given in Table XXXV. Numerous uses for this short cut suggest themselves, especially when a simple slide rule can be constructed for determining the lot size when the element K_s is employed alone. If the degree of variation can thus be so readily obtained, the true value of the lot size can be easily determined by the application of the form index f_c as a corrective factor. In this manner apparently complex calculations can be reduced to a routine matter.

The Final Simplification of a Special Form.—Owing to the fact that the determination of a controlling value for the form index from the maximum permissible limits of variation is a problem in itself, and that it is to be discussed at length in the next chapter, there remains for further consideration only the last stage of simplification. This has mainly to do with the arrangement of terms in the particular form selected for a given purpose and may be accomplished in two ways. First, as soon as the actual expressions for K_s, K_w, and K_v have been introduced according to the elements required, the various items in the special form should be so grouped that there will be the least repetition of terms. Second, all items should be inspected to see whether,

[1] See Table VII or Chap. XX.

TABLE XXXV.—EQUATIONS EMPLOYED IN SUPPORT OF THE SHORT-CUT METHOD OF DETERMINING Q_i

If by definition

$$E = \frac{1}{\sqrt{f_c}} = \frac{Q_u}{Q_i}, \tag{210}$$

then in general

$$Q_i = \sqrt{f_c} \cdot Q_u, \tag{211}$$

or for Q_m

$$Q_m = \sqrt{f_{c_m}} \cdot Q_{u_m}, \tag{212}$$

or for Q_R

$$Q_R = \sqrt{f_{c_R}} \cdot Q_{u_R}, \tag{213}$$

or for Q_e

$$Q_e = \sqrt{f_{c_e}} \cdot Q_{u_e}. \tag{214}$$

where in general f_{c_m}, f_{c_R}, and f_{c_e} can be derived from

$$f_c = \frac{R}{1 + R_w + R_v}.$$

If

$$R_v = \frac{\phi \cdot \dfrac{b}{c}}{f_r},$$

where for Q_m and f_{c_m}

$$f_r = 1,$$

for Q_R and f_{c_R}

$$f_r = f_R,$$

for Q_e and f_{c_e}

$$f_r = f_r \quad \text{See Table XXXI.}$$

Hence

$$Q_i = \sqrt{\frac{R}{R_c} \cdot \frac{K_p}{K}}. \tag{215}$$

For an interpretation of symbols, see text, Chap. XIX, or Appendix XIII, p. 349.

for a given manufacturing plant, certain ones will remain constant regardless of the particular problem involved, and then these definite constant values should be introduced and combined wherever it is found to be practicable. The first of these operations is entirely optional with the person making the calculations, but it will be found that a regrouping of the various items will reduce the final labor of calculation to a minimum, when it comes time to introduce the appropriate numerical values for a specific case, because undoubtedly certain items which appear both in the numerator and denominator will cancel out or become combined. The second may be achieved by the proper selection of values for such terms as r, i, k_s, k_p, n, h, and s. Any one at all familiar with the handling of mathematical expressions would naturally perform these steps before attempting to compute the answer; however, this suggestion at least will not be amiss, as it will serve to remind those interested that such treatment will always be of assistance.

CHAPTER XX

THE PROBLEM INDEX AND MAXIMUM LIMITS OF VARIATION

The foregoing discussion has established the mathematical basis for the selection of special forms; however, if the method of simplification as a whole is to be governed by the characteristics of the problem, definite limits for the permissible variation in the results must be likewise determined, as there will be some instances where the arbitrary limit of variation in the data e_a will not be applicable. If such limits are to be developed, they must be derived from some condition which represents the fundamental situation. The one such condition which will satisfy this situation may be found in the relative increase in cost that would be incurred by the production of a quantity varying to some degree from the quantity which would have been produced should the lot size have been determined from one of the general expressions for Q_m, Q_e or Q_R. As this increase in cost is in reality a loss to be designated hereafter by L_s, for the same reasons that were advanced when determining the economic quantity from the minimum-cost quantity, the same procedure can be employed here in evaluating it, except for the fact that in this case any large divergence from the ideal conditions will actually alter the gross return normally expected from the sale of the articles.

The Influence of Simplification upon the Ideal Lot Size.—In the case of the economic-production quantity Q_{i_e}, (see Fig. 7)[1] the use of a special form will in no way adversely affect the profits, because the resulting approximate value Q_{u_e} will be larger than the ideal Q_{i_e}, and then the corresponding actual ultimate unit cost U_{e_u} for Q_{u_e} on the curve U will be less than U_e, and the difference $U_e - U_{e_u}$ will be a gain which will enhance the unit margin of profit over that required by the expected rate of return r, but it will not permit the full realization of the benefits of conservation in capital. In the case of the minimum-cost quan-

[1] See Chap. V, p. 54.

tity and the point of maximum return, however, the expected unit margin of profit will be slightly diminished. In the first instance little concern need be had, provided that the value of Q_{u_R} from a special form does not exceed Q_m as the upper limit of economic range, because, as long as the lot size remains within this range, the normal return can be earned. In the latter case the value of Q_{u_m} from any special form for Q_m will always exceed the limits of the economic range, as shown in Fig. 7, and then the resultant increase in cost $U_{m_u} - U_m$ will incur a distinct reduction in profits. The extent to which such a loss may be permissible can be determined only through its effect upon the expected gross return and its relation to the anticipated savings from the simplification of departmental routine resulting from the use of special forms.

The Permissible Variation in the Minimum-cost Quantity Is the Final Measure.—Owing to the fact that an approximate value for Q_e will not impair the expected return and that the ultimate expression for Q_e, whatever its form may be, always depends upon the form of the expression for Q_m, as shown by Eq. (52)[1], there will be no need for establishing the limits of permissible variation in this case, as those which apply to the minimum-cost quantity Q_m will be equally appropriate in the selection of a special form for Q_e. Since the forms of the expressions for Q_R and Q_m are identical except for the value of the factor f_r, any method which may be devised for determining the limits for a special form for Q_m can be employed satisfactorily in the determination of the limits of variation for Q_R, provided that i in the latter case be replaced by $(i + r)$, because

$$i \cdot f_r = i$$

where

$$f_r = 1 \text{ for } Q_m,$$

and

$$i \cdot f_r = (i + r)$$

where

$$f_r = \left(1 + \frac{r}{i}\right) \text{ for } Q_R.$$

Accordingly, the method that will be recommended for the determination of the limits of permissible variation in the results

[1] See p. 158.

TABLE XXXVI.—EQUATIONS EMPLOYED IN THE DERIVATION OF THE LIMITS OF PERMISSIBLE VARIATION

By definition

$$L_s = U_{m_u} - U_m, \tag{216}$$

where

$$U_{m_u} = u' + \frac{P \cdot F}{Q_{u_m}} + Q_{u_m} \cdot f,$$

and

$$U_m = u' + \frac{P \cdot F}{Q_m} + Q_m \cdot f;$$

or if

$$Q_m = \sqrt{\frac{P \cdot F}{f}},$$

and

$$Q_m \cdot f = \frac{P \cdot F}{Q_m},$$

$$U_m = u' + 2 \cdot \frac{P \cdot F}{Q_m}, \tag{217}$$

and

$$U_{m_u} = u' + \frac{P \cdot F}{Q_{u_m}} + Q_{u_m} \cdot \frac{P \cdot F}{Q_m^2}.$$

Hence

$$L_s = u' + \frac{P \cdot F}{Q_{u_m}} + \frac{Q_{u_m}}{Q_m^2} \cdot P \cdot F - u' - \frac{2 \cdot P \cdot F}{Q_m},$$

or when expressed in terms of Q_{u_m}

$$\frac{Q^2{}_{u_m}}{Q_m^2} - 2 \cdot \frac{Q_{u_m}}{Q_m} \cdot \left(\frac{L_s \cdot Q_m}{2 \cdot P \cdot F} + 1\right) + 1 = O.$$

Then solving for Q_{u_m} by the use of the formula (Appendix XII).

$$Q_{u_m} = Q_m\left[\left(\frac{L_s \cdot Q_m}{2 \cdot P \cdot F} + 1\right) \pm \sqrt{\left(\frac{L_s \cdot Q_m}{2 \cdot P \cdot F} + 1\right)^2 - 1}\right],$$

or

$$Q_{u_m} = Q_m[K_e \pm \sqrt{K_e^2 - 1}], \tag{218}$$

where

$$K_e = 1 + \frac{L_s \cdot Q_m}{2 \cdot P \cdot F};$$

then if by definition

$$E_l = \frac{Q_{u_m}}{Q_m}, \tag{219}$$

$$E_l = K_e \pm \sqrt{K_e^2 - 1}, \tag{220}$$

when

$$K_e = 1 + \lambda \cdot K_o, \text{ From Table XXXVII}$$

where

$$\lambda = \frac{L_s}{U_m}$$

and

$$K_o = \frac{Q_m \cdot U_m}{2 \cdot P \cdot F}.$$

Then

$$E_{max} = K_e + \sqrt{K_e^2 - 1} \tag{221}$$

and

$$E_{min} = K_e - \sqrt{K_e^2 - 1}; \tag{222}$$

whereupon

$$Q_{max} = E_{max} \cdot Q_m \tag{223}$$

and

$$Q_{min} = E_{min} \cdot Q_m. \tag{224}$$

For an interpretation of these symbols, see text, Chap. XX, or Appendix XIII, p. 349.

will be derived on the basis of a minimum-cost quantity, and the loss L_s will be evaluated by finding the difference between the ultimate unit cost U_{m_u} and U_m for an approximate minimum-cost quantity Q_{u_m} and the ideal Q_m, respectively, as shown in Eq. (216) Table XXXVI, the latter being the lowest ultimate unit cost attainable for the type of process employed.

Evaluation of the Limits of Permissible Variation.—Now if the expression for this loss L_s be solved for Q_u in terms of L_s and

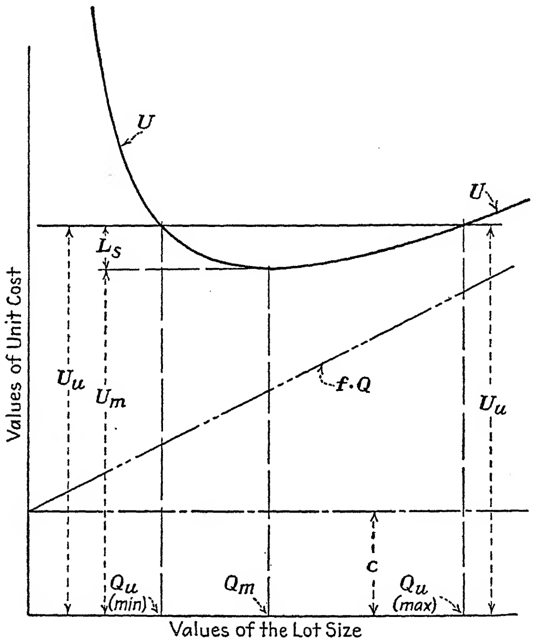

FIG. 43.—The loss due to the production of an uneconomic quantity Q_u.

the ideal quantity Q_m, as obtained from the general expression, Eq. (220) will be obtained, which is the expression for the limits E_l of permissible variation. Up to this point the term E has been allowed to represent the permissible variation of the quantity Q_u from Q_i or in this case Q_m. As it is evident from Fig. 43, however, that the quantity Q_u can have two values for a stipulated loss L_s, because for any ultimate unit cost greater than the minimum ultimate unit cost there will be two points on the curve U which have the same value, the coefficient E can also have two

values. Since E must always designate the variation for a specific Q_u, the coefficient E_l will be introduced to designate the limits of variation which are controlled by the characteristics of the problem. Then if the value of E_l be that obtained by the use of the positive sign, it will correspond to $E_{\max}$ for the largest quantity $Q_{u_{\max}}$ that can be produced within these limits, and, similarly, if the value of E_l be that obtained by the use of the minus sign it will correspond to the value of $E_{\min}$ for the smallest quantity $Q_{u_{\min}}$. Moreover, for the present, since the value of Q_u from a special form cannot be less than the ideal value of Q_m, one need only consider the maximum limit of variation $E_{\max}$ as a guide in applying the method of selection. Nevertheless, there will be occasion under other circumstances to employ the minimum limit $E_{\min}$, as well.

The Loss Factor λ Defined.—Until the terms represented by K_e in Eq. (218) can be replaced by others, the values for which can be easily obtained from the data employed in the determination of the lot size itself, little practical value can be realized from the expression for E_l. This can be accomplished, if for the moment another expression for the loss L_s be developed by introducing the loss factor λ to show its relation to the ultimate unit cost at the minimum point as given in Eqs. (225) and (226), Table XXXVII. This procedure is quite logical because one naturally tends to think of the loss in terms of the best possible performance that can be obtained for a given method of production, and this of course is measured by the lowest cost, namely, U_m, that can be realized under the conditions imposed by the process. The loss factor λ can then be defined as the ratio of the difference between the ultimate unit costs $U_{m_u} - U_m$ for a lot of uneconomic proportions Q_{u_m} and one of ideal proportions $Q_i = Q_m$ to the ultimate unit cost U_m, which is the measure of the best practice. Then a definite value can be assigned to λ which will represent, according to the best executive judgment, the greatest proportions that the loss L_s can assume in relation to the ideal conditions. Further on, means will be devised for the guidance of executives in their choice of a suitable value for λ when its utility has become more evident.

The Problem Index.—Now if this second expression for the loss L_s be substituted in Eq. (227) for K_e, and then those terms

TABLE XXXVII.—EQUATIONS EMPLOYED IN THE DERIVATION OF THE PROBLEM INDEX

If by definition

$$\frac{L_s}{U_m} = \frac{U_{m_u} - U_m}{U_m}, \tag{225}$$

then

$$L_s = \lambda \cdot U_m; \tag{226}$$

and if

$$K_e = 1 + \frac{L_s \cdot Q_m}{2 \cdot P \cdot F}, \quad \text{From Eq. (218), Table XXXVI.} \tag{227}$$

then

$$K_e = 1 + \lambda \cdot \left[\frac{U_m \cdot Q_m}{2 \cdot P \cdot F}\right],$$

or

$$K_e = 1 + \lambda \cdot K_o, \tag{228}$$

when

$$K_o = \frac{1}{2} \cdot \left[\frac{U_m}{\frac{P \cdot F}{Q_m}}\right]. \quad \text{The "problem index."} \tag{229}$$

Then for practical purposes if

$$U_m = u' + \frac{2 \cdot P \cdot F}{Q_m},$$

$$K_o = \left[u' + \frac{2 \cdot P \cdot F}{Q_m}\right] \cdot \frac{Q_m}{2 \cdot P \cdot F},$$

or

$$K_o = \frac{u' \cdot Q_m}{2 \cdot P \cdot F} + 1;$$

but as

$$u' = c + k_{v_s} + k_{v_w}, \quad \text{From Eq. (162) (see Table XXV, where } \Delta = 1.)$$

and as $\frac{k_{v_s}}{2 \cdot P \cdot F}$ and $\frac{k_{v_w}}{2 \cdot P \cdot F}$ are relatively small compared with $\frac{c}{2 \cdot P \cdot F}$ (see page 302), and as F can be neglected (see page 283), for use in selecting special forms

$$K_o = 1 + \frac{c \cdot Q_m}{2 \cdot P}. \tag{230}$$

However, if

$$Q_m = \sqrt{e_s \cdot Q_{u_{ms}}}, \quad \text{From Eq. (215), Table XXXV.} \tag{231}$$

or

$$Q_m = \sqrt{\frac{R_s}{R_c} \cdot \frac{K_p}{K_s}},$$

where

$$K = K_s$$

and

$$R = R_s = 1 \qquad \text{See Table XXXIII.}$$

$$Q_m = \sqrt{\frac{1}{R_c} \cdot \frac{2 \cdot P \cdot S_a}{c \cdot i \cdot f_p}}.$$

$$K_o = 1 + \sqrt{\frac{1}{R_c} \cdot \frac{c^2}{4 \cdot P^2} \cdot \frac{P}{c} \cdot \frac{2 \cdot S_a}{f_p \cdot i}}, \tag{232}$$

$$= 1 + \sqrt{\frac{1}{R_c} \cdot \frac{c}{P} \cdot \frac{S_a}{2 \cdot i \cdot f_p}}. \qquad \text{See Table VII, Part II of Calculation Sheets, Chap. V, p. 62.}$$

For an interpretation of symbols, see text, Chap. XX, or Appendix XIII, p. 349.

which are multiplied by λ be placed in a group K_o by themselves, it can be said that K_e will depend upon the product of the two factors λ and K_o [Eq. (228)]. This situation is of more than ordinary importance because, if one inspects the relation designated by K_o, it will be noticed that it contains the ratio of the ultimate unit cost U_m under ideal conditions to the unit allotment PF/Q of the total preparation charges. Since this ratio embodies the fundamentals which enter into any problem of determining the best lot size as well as the measure of the most desirable method of production, because economical production must depend upon the relative proportion of these two major cost factors, K_o will henceforth be designated as the "problem index." Obviously then, if the extent of the loss can be regulated as a matter of general policy by the factor λ and the characteristics of any problem can be introduced by the factor K_o, the resulting value for K_e under a given set of conditions will definitely control the limits of permissible variation employed in the selection of special forms.

Evaluation of the Problem Index for Practical Purposes.—Accordingly if K_o is one of the basic factors, its value may be computed for practical purposes from Eq. (230), which was obtained from Eq. (229) by inserting the expression for U_m from Eq. (217)[1], provided that the terms k_{v_s} and k_{v_w} in u' are of little relative significance. This latter feature can be demonstrated by the fact that if

$$\frac{k_{v_s}}{2P} = \frac{i}{2 \cdot S_a}\left(k_s - k_p \cdot \frac{S_a}{2D}\right)$$

and

$$\frac{k_{v_w}}{2P} = \frac{i \cdot t_p}{2}\left(1 - \frac{k_M}{2}\right)$$

where k_d and F have been omitted for the reasons[2] given in Chap. XIX, the first can be disregarded, because the parenthesis $\left(k_s - k_p \cdot \dfrac{S_a}{2D}\right)$ under the most undesirable circumstances cannot exceed the value of one half, and the ratio i/S_a will be very small as the normal value for i will in all probability not be greater than 0.0003, and $1/S_a$ for reasonable values of S_a will be always less

[1] See Table XXXVI.
[2] See p. 283.

than $1/10$. Likewise, the second can be disregarded, because t_p as normally expressed will be less than unity, i will have the same low value, and $\frac{1}{2}\left(1 - \frac{k_M}{2}\right)$ at most should not exceed a value of one half.

Procedure for Determining the Permissible Variation.—If Eq. (230) for K_o is to be employed in the calculation sheets for the determination of the limits E_l, it will be necessary to eliminate the term Q_m in order that the utility of this procedure may be fully realized. This can be accomplished by introducing the short-cut method, described in the last chapter,[1] whereby Q_m can be replaced by items which have been previously employed in the calculation sheets so that no lengthy computations will be required. Accordingly, if the expression for Q_m be introduced where $e_s = R_s/R_c$ and $Q_{u_{m_s}} = \sqrt{K_p/K_s}$, as shown in Eq. (206), and the respective values for R_s, R_c, K_p, and K_s be inserted, the expression for K_o can be rewritten as shown in Eq. (232). Here it will be found that the values for R_s and R_c, as well as that for the parenthesis, have already been determined in Part I of the calculation sheets, so that it will be necessary at this point only to compute the ratio of c/P and multiply it by $1/2i$ and then combine all items under the radical. Accordingly, it has been considered practical to use Eq. (232) as a basis for the procedure required by Part II of the calculation sheets.

Executive Policy Introduced through the Loss Factor.—Since the characteristics of a given problem can be introduced in this manner into the factor K_e which controls the permissible limits of variation E_l, there remains only the necessity of determining the most suitable value for the loss factor λ that will apply indiscriminately to all problems which may arise in a given plant. It was for this reason that it seemed advisable to express the loss L_s in terms of U_m, the measure of the best known practice for any specific case, because the value for λ could then be determined as a part of a uniform manufacturing policy so that all cases may be treated alike.

The Influence of Simplification upon the Financial Policy.—The situation which controls the magnitude of any loss depends upon the extent to which such a loss will impair the gross return

[1] See p. 293.

normally expected by the owners of the business. This case is not different from that which was encountered when an increase in cost was incurred by the conservation of capital,[1] except for the fact that the increase in cost due to the production of a lot, the size of which only approximates the ideal, cannot be compensated for in the same manner, because one cannot avoid its effect in actually diminishing the gross return. This is not so serious as might be supposed, because if reasonable latitude in the selection of special forms can be obtained for a net reduction in the expected rate of return of less than 0.1 per cent, the financial policy of the company will be in no way seriously impaired, especially if the savings from a simplification of production control and departmental routine are sufficient to offset the loss thus incurred. In other words, if the expected return be 18 per cent on the capital invested and these advantages can be obtained under conditions which will actually yield a return of 17.9 per cent, the 0.1 per cent difference between expectation and reality will be of little moment, if the attendant reduction in administrative costs is equal to 0.1 per cent of the working capital.

Evaluation of the Loss Factor for Practical Purposes.—For all practical purposes in determining any manufacturing policy the loss factor λ can be assumed to vary in direct proportion to the reduction in the expected rate of return from its normal value r, so that if the value for λ increases, the expected rate of return must be reduced by a like amount. This situation can be illustrated if it be assumed that the initial investment of working capital C_i/Q[2] in the manufacture of any lot, upon which both the cost of capital and the unit margin of profit are based, is proportional to the manufacturing cost u_m, as would be in the more common case where the cost of the storage space and the loss from deterioration are carried in the overhead. Now if conditions in the trade will not permit the original minimum-sales price to be altered to adjust this loss, the desired return R_m/S_y cannot be earned, and the owners of the business will have to be content with a smaller rate of return, because r will have been reduced to r_u. The reduction in profit then must be such that it will just offset the loss L_s. If these facts be evaluated and the

[1] See p. 154.
[2] See p. 146.

increase in capital due to the production of a lot of larger size be accounted for in terms of the investment under ideal conditions through a coefficient β, it will be found when

$$\beta \cong 1$$

that

$$\frac{\lambda}{r - r_u} = \frac{\alpha}{1 + \alpha \cdot i},$$

a constant for a given problem.

The mathematical analysis for these deductions will be found in detail in the appendix.[1] Therefore, if the ratio of λ to $r - r_u$ is a constant, a change in the loss factor will require an equivalent correction in the expected rate of return r_u since the term r is fixed in value. Accordingly, if in the judgment of the executives a 0.1 per cent decrease in the gross return is not unreasonable, an increase in cost of 0.1 per cent for the time being will give ample opportunity to achieve savings in the methods of production control through a simplification of routine. In this manner an approximate value of λ can be determined, which, when employed in connection with Part II of the calculation sheets,[2] should yield satisfactory results.

Graphical Determination of the Limits of Permissible Variation.—Even though means have been devised for evaluating the controlling factors which through the term K_e govern the limits of permissible variation E_l, some more rapid method of solving Eq. (218)[3] than one of a purely mathematical nature must be devised in order to avoid the implied calculations. The simplicity of the relation in this equation immediately suggests a graphical method of solution (see Fig. 13),[4] because, if the values of E_l be plotted on the scale of abscissa and the values of K_o be plotted on the scale of ordinates, a series of curves can be drawn showing the relation of these two factors, when each one of the curves represents a definite value of λ. The family of curves thur obtained will center about a straight vertical line for the value of $E_l = 1$, which will be found to indicate the ideal conditions whers the lot size has been determined from the general expression foe the minimum-cost quantity. The curves which lie to the right

[1] See Appendix XI, p. 344.
[2] See Table VII, Chap. V.
[3] See Table XXXVI, p. 298.
[4] See p. 76.

of this line have been derived from Eq. (221) for the maximum limits of variation E_{max} where the positive sign is employed in Eq. (220) for E_l, and those which lie to the left have been derived from Eq. (222) for the minimum limits of variation E_{min} by using the minus sign instead.

Special Charts for Specific Instances.—Such a chart is universal in its application and can be even employed to determine the limits of the economic range or the point of maximum return for all problems which do not require a separate consideration of the storage space charges V_s, that is, when R_v is quite small in relation to R_s or R_w. If a desirable limiting value λ_d for the loss factor, however, can be determined in advance by executive policy, a similar chart can be prepared for use in a given plant, and then only the curve based on this value of λ need be plotted, thereby relieving the subordinates in the production-control division of any responsibility in the choice of a correct value. Whenever the limits of variation E_l have been determined in this manner, the limiting values for Q_{u_m}, or any uneconomic quantity Q_u, can be obtained by the use of Eqs. (223) and (224). Where the value of E_l in Eq. (219) is greater than unity, Q_{u_m} or Q_u will be Q_{max}, and where it is less than unity, Q_{u_m} or Q_u will be Q_{min}. The former only will be applicable to the method of selecting special forms, however, as the deviation in this case must always depend upon the use of the positive sign in determining values of E_l.

Justification of the Arbitrary Value for the Form Index.—This chart can also be employed to translate any value of permissible variation in results E directly into values of e, the allowable variation in the data, or the form index, f_c, for use in the calculation sheets, if a second scale of abscissa be drawn based upon the relation in Eq. (191).[1] Then e or f_c can be read off directly, without further computations and introduced into the expression, Eq. (203),[1] for R in terms of R_c. Moreover, a check is also provided upon the rationality of the arbitrary value for the form index $e_a = 0.666$. For instance, it may be seen from the chart in Fig. 13[2] that if λ be determined by executive decision to be 0.1 per cent, because it will only change the expected rate

[1] See Table XXXIII, Chap. XIX.

[2] See p. 76.

TABLE XXXVIII.—EQUATIONS EMPLOYED IN THE EVALUATION OF THE LOSS FACTOR

$$p''_u - p''_m = \frac{R_u}{S_y} - \frac{R_m}{S_y} + U_u - U_m, \text{ From Fig. 44.} \tag{233}$$

and

$$p''_u - p''_m = \frac{R_u}{S_y} - \frac{R'_u}{S_y}, \tag{234}$$

but if by definition

$$\lambda \cdot U_m = U_u - U_m = L_s,$$

$$p''_u - p''_m = \frac{R_u}{S_y} - \frac{R_m}{S_y} + \lambda \cdot U_m = \frac{R_u}{S_y} - \frac{R'_u}{S_y}.$$

Hence

$$\lambda \cdot U_m = \frac{R_m}{S_y} - \frac{R'_u}{S_y},$$

where

$$\frac{R_m}{S_y} = r \cdot [Q_m \cdot (v_s \cdot t_s + v_w \cdot t_w) + k_{v_s} + k_{v_w} + C_{f_s} \cdot t_s + C_{f_u} \cdot t_w].$$

From Eq. (44), Table XII, Chap. XI.

Then likewise

$$\frac{R'_u}{S_y} = r_u \cdot [Q_u \cdot (v_s \cdot t_s + v_w \cdot t_w) + k_{v_s} + k_{v_w} + C_{f_s} \cdot t_s + C_{f_w} \cdot t_w],$$

or, in a simpler form, where

$$v \cdot t = (v_s \cdot t_s + v_w \cdot t_w),$$

and

$$C'_F = k_{v_s} + k_{v_w} + C_{f_s} \cdot t_s + C_{f_w} \cdot t_w,$$

$$\frac{R_m}{S_y} = r \cdot (Q_m \cdot v \cdot t + C'_F)$$

and

$$\frac{R'_u}{S_y} = r_u \cdot (Q_u \cdot v \cdot t + C'_F);$$

then

$$\lambda = \frac{(r \cdot Q_m - r_u \cdot Q_u) \cdot v \cdot t + (r - r_u) \cdot C'_F}{U_m}. \tag{235}$$

For interpretation of symbols, see text, Chap. XX, or Appendix XIII p. 349.

of return, 18 per cent to 17.9 per cent, e_a can be reliably employed in the selection of special forms for all problems for which the problem index K_o is greater than 20. As experience has shown that those problems for which simplification will be of most advantage will have an index greater than this amount, the arbitrary value of e_a can be employed with complete reliability, and need be corrected only in the few cases where K_o is less.

Mathematical Determination of the Limits of Permissible Variation.—Should it be inadvisable to assume for practical purposes that the loss factor λ bears a direct relation to the

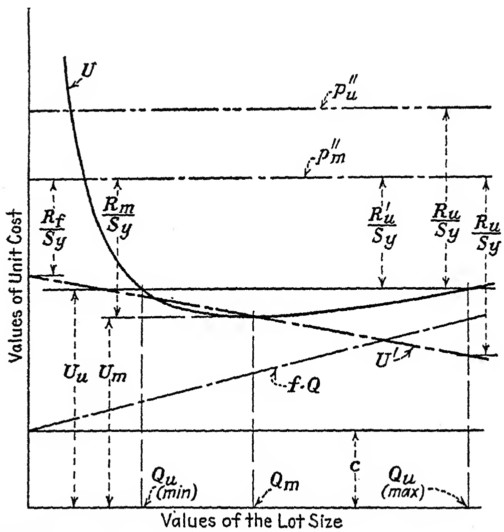

FIG. 44.—The influence of the financial policy upon the maximum- and minimum-production quantity in relation to the selling price.

reduction in the expected rate of return, an exact value for the maximum or minimum production quantities $Q_{\max}$ and $Q_{\min}$, respectively, can be obtained by introducing the true relation of λ in terms of r and r_u, where the actual rate of return r_u is equal to $\rho \cdot r$, and ρ represents the allowable variation in r as determined by executive judgment. The situation which exists under these conditions can best be illustrated by reference to Fig. 44 where p''_m represents the actual minimum-sales price for which the rate of return is r, and p''_u is the price which must be obtained if the same unit margin of profit is earned upon an ultimate unit cost U_u instead of U_m. The normal unit margin of profit for a lot size

Q_m will be, as formerly, R_m/S_y and for the larger lot Q_u; where the rate of return is the same it will be R_u/S_y. Now if the minimum-sales price p''_m is to be maintained, the unit margin of profit that can be obtained must be R'_u/S_y, where the rate of return is r_u instead of r.

The Maximum Limit of Permissible Variation.—Now if the loss L_s due to the use of special forms is to be expressed in terms of r through the medium of these respective margins of profit, it will be seen from Fig. 44 that in order to adhere to the price p''_m the difference between p''_u and p''_m must be offset by a reduction in the value of r to that of r_u. The basic mathematical relation can be illustrated by reference to Eqs. (233) and (234) in Table XXXVIII, and if these be combined, the expression for λ [Eq. (235)] can be expressed in terms of r where r_u has been replaced by $\rho \cdot r$ as shown in Eq. (236). If this expression for λ and the fundamental relation, Eq. (229), for the problem index K_o be substituted in Eq. (228) for K_e, this equation [Eq. (237), Table XXXIX] can be employed in Eq. (220), Table XXXVI, when the positive sign is used [see Eq. (239)] to determine the value of the maximum permissible variation $E_{\max}$. Then if this value is introduced into Eq. (222) the maximum allowable production quantity [Eq. (240)] can be directly determined, provided that the appropriate value for Q_m has been previously calculated. As the mathematical procedure is quite complex it has been omitted from Table XXXIX and placed in Appendix XI.[1] To simplify the eventual calculation in a specific case certain relations previously employed in the method of selection[2] have been introduced.

The Minimum Limit of Permissible Variation.—Even at best it must be recognized that this method of determining the maximum limits of variation or the maximum-production quantity is far too complicated for actual practice. It was for this reason that the simpler but approximate method as developed on page 304 was proposed for use in conjunction with the chart in Fig. 13.[3] As there is nothing to hinder the use of that method in the left-hand portion of the chart in order to determine a minimum-production quantity $Q_{\min}$, a mathematical check upon

[1] See p. 344.
[2] See Table VI, Chap. V.
[3] See p. 76.

TABLE XXXIX.—EQUATIONS EMPLOYED IN THE DERIVATION OF THE MAXIMUM AND MINIMUM LIMITS OF PERMISSIBLE VARIATION

If it be assumed that

$$\rho = \frac{r_u}{r},$$

and if in general

$$E = \frac{Q_u}{Q_m},$$

then from Eq. (235), Table XXXVIII

$$\lambda = \frac{r \cdot Q_m \cdot v \cdot t \cdot (1 - \rho \cdot E) + r \cdot (1 - \rho) \cdot C'_F}{U_m}; \tag{236}$$

so that if

$K_e = 1 + \lambda \cdot K_o$, From Eq. (228), Table XXXVII.

$$K_e = 1 + \frac{r \cdot Q_m}{2 \cdot P \cdot F}[Q_m \cdot v \cdot t \cdot (1 - \rho \cdot E) + (1 - \rho) \cdot C'_F]; \tag{237}$$

and then if Eq. (220), Table XXXVI is expanded

$E_l^2 - 2 \cdot E_l \cdot K_e + 1 = 0$, See Appendix XI for detailed analysis. (238)

$$E_l = \frac{2 \cdot P \cdot F + r \cdot Q_m \cdot [Q_m \cdot v \cdot t + C'_F \cdot (1 - \rho)]}{2 \cdot P \cdot F + 2 \cdot \rho \cdot r \cdot Q_m^2 \cdot v \cdot t} \pm \sqrt{\left\{\frac{2 \cdot P \cdot F + r \cdot Q_m \cdot [Q_m \cdot v \cdot t + C'_F \cdot (1 - \rho)]}{2 \cdot P \cdot F + 2 \cdot r \cdot \rho \cdot Q_m^2 \cdot v \cdot t}\right\}^2 - \cdots \cdots \frac{P \cdot F}{P \cdot F + \rho \cdot r \cdot Q_m^2 \cdot v \cdot t}}, \tag{239}$$

from which can be obtained

$$Q_{\max} = \frac{f_n}{f_d}\left[1 + \sqrt{1 - \frac{2 \cdot f_d}{f_n^2} \cdot \frac{P \cdot F}{v_s \cdot t_s \cdot i}}\right] \tag{240}$$

and

$$Q_{\min} = \frac{f_n}{f_d}\left[1 - \sqrt{1 - \frac{2 \cdot f_d}{f_n^2} \cdot \frac{P \cdot F}{v_s \cdot t_s \cdot i}}\right], \tag{241}$$

where

$$f_n = \sqrt{\frac{P \cdot F}{v_s \cdot t_s \cdot i \cdot R_c}} \cdot \left[2 \cdot R_c + \frac{r}{i} \cdot (1 + R_w)\right] + \frac{r}{i} \cdot (1 - \rho) \cdot \frac{C'_F}{v_s \cdot t_s}$$

and

$$f_d = 2 \cdot R_c + 2 \cdot \rho \cdot \frac{r}{i} \cdot (1 + R_w),$$

R_w and R_c being introduced to simplify eventual computations.

For interpretation of symbols, see text, Chap. XX, or Appendix XIII, p. 349.

the results thus achieved can be obtained from Eq. (241) since through a similar analysis it will be found that $E_{\min}$ and then $Q_{\min}$ can be calculated from Eqs. (239), when the minus sign is employed, and (223), respectively. Ordinarily there will be little use for these minimum limits of variation from the point of minimum cost, as the limits of the economic range perform this function much more satisfactorily. The only instance where it may be valuable to compute a minimum-production quantity may arise when it seems logical, in a case where the economic-production quantity is not applicable, to still reap the benefit of conservation of capital through the production of a smaller quantity than a larger one, as long as the increase in the ultimate unit cost to be incurred can be determined and therefore controlled.

PART III

APPENDIX

APPENDIX I

MATHEMATICAL DERIVATION OF THE LIMITS FOR THE ECONOMIC RANGE OF PRODUCTION

In Table XII[1] where the various fundamental steps in the derivation of a relation for the limits of the economic range have been outlined, the transition from Eq. (50) to Eq. (51) was omitted, because it seemed inadvisable to burden the reader at that point with certain details of a purely mathematical character. For those who are interested in following through the mathematical analysis required to establish the reliability of Eq. (51), the remaining steps will now be presented in detail.

In Eq. (46) it was shown that

$$L_r = \frac{R_m}{S_y} - \frac{R_e}{S_y} = U_e - U_m,$$

and that this relation could be expanded to that in Eq. (50) where

$$L_r = r \cdot (Q_m - Q_l) \cdot (v_s \cdot t_s + v_w \cdot t_w) = P \cdot F \cdot \left(\frac{1}{Q_l} - \frac{1}{Q_m} + f \cdot (Q_l - Q_m)\right).$$

Now if the terms $r \cdot (v_s \cdot t_s + v_w \cdot t_w)$ are represented by the symbol Z, and if Eq. (50) be divided by $P \cdot F$ throughout, so that the relation

$$\frac{f}{P \cdot F} = \frac{1}{Q_m{}^2}$$

can be introduced, a new expression is obtained where the terms depending upon Q_l can be grouped on one side and the remaining ones will appear on the other.

Accordingly,

$$\frac{Z}{P \cdot F} \cdot (Q_m - Q_l) = \left(\frac{1}{Q_l} - \frac{1}{Q_m}\right) + \frac{f}{P \cdot F} \cdot (Q_l - Q_m)$$

[1] See p. 156.

or

$$\frac{1}{Q_l} + \frac{Q_l}{Q_m^2} + \frac{Q_l \cdot Z}{P \cdot F} = \frac{1}{Q_m} + \frac{Q_m}{Q_m^2} + Q_m \cdot \frac{Z}{P \cdot F}.$$

From this latter relation a quadratic equation in terms of Q_l can be obtained which can be solved for Q_l by the use of the formula for quadratic equations.[1]

Hence if

$$Q_l^2 \cdot \left(\frac{1}{Q_m^2} + \frac{Z}{P \cdot F}\right) - Q_l \cdot \left(\frac{2}{Q_m} + Q_m \cdot \frac{Z}{P \cdot F}\right) + 1 = 0,$$

then

$$Q_l = \frac{\left(\frac{2}{Q_m} + Q_m \cdot \frac{Z}{P \cdot F}\right) \pm \sqrt{\left(\frac{2}{Q_m} + Q_m \cdot \frac{Z}{P \cdot F}\right)^2 - 4 \cdot \left(\frac{1}{Q_m^2} + \frac{Z}{P \cdot F}\right)}}{2 \cdot \left(\frac{1}{Q_m^2} + \frac{Z}{P \cdot F}\right)},$$

which can be simplified by the cancellation of like terms of opposite sign under the radical to an expression where

$$Q_l = \frac{\dfrac{2 + Q_m^2 \cdot \dfrac{Z}{P \cdot F}}{Q_m} \pm \sqrt{\dfrac{Q_m^2 \cdot Z^2}{P^2 \cdot F^2}}}{\dfrac{2}{Q_m^2} \cdot \left(1 + Q_m^2 \cdot \dfrac{Z}{P \cdot F}\right)}.$$

It is but a simple matter now to rearrange the various items so that Q_l can be expressed in terms of Q_m and a factor also depending upon Q_m. Thus when the full expression for Z is reintroduced, Eq. (51) is immediately obtained, where

$$Q_l = Q_m \left[\frac{\dfrac{2}{Q_m^2} + \dfrac{r}{P \cdot F} \cdot (v_s \cdot t_s + v_w \cdot t_w) \pm \dfrac{r}{P \cdot F} \cdot (v_s \cdot t_s + v_w \cdot t_w)}{2\left\{\dfrac{1}{Q_m^2} + \dfrac{r}{P \cdot F} \cdot (v_s \cdot t_s + v_w \cdot t_w)\right\}} \right].$$

[1] See Appendix XII, p. 348.

APPENDIX II

EVALUATION OF THE AVERAGE DAILY SALES S_{a_s} FOR VARIABLE DEMAND

When the sales demand over any general period is decidedly variable, a reliable value for the average daily sales cannot be obtained by the simple method employed when the demand is approximately uniform. In this case the total sales for the desired period must be obtained in some manner, but as the length of this period T_s is dependent upon the relation between the quantity produced and the actual daily sales, one must resort to higher mathematics in order to obtain a reliable value. For those who are familiar with calculus it will be evident that the total sales for a period of any length will be the area under the curve (see Fig. 27) plotted for the rate S at which sales occur each day, between the limits where $T_o = 0$ and $T_n = T_s$. For the more practically minded reader this is equivalent to adding together the sales which have occurred prior to any day to those which occur on that day and plotting the total number on the proper ordinate corresponding to that day on the scale at the bottom. Repeating this process for each day in the sales period or the year, a curve can be drawn through the plotted points which will be the cumulative sales curve desired. Even though this latter method may be a more obvious one for evaluating the total sales, one must start with the former method in order to introduce the proper equation for the actual curve for the daily rate of sales.

Accordingly, the mathematical basis for the derivation of the actual sales period under conditions of variable demand may be shown, if the probable rate S at which articles which will be sold on any day T during this period, is expressed as

$$S = f(T). \tag{242}$$

Then the equation for the curve for the daily rate of sales (Fig. 27) will have the general form

$$S = \alpha + \beta T + \gamma T^2 + \epsilon T^3 \ . \ . \ . \text{ etc.} \tag{243}$$

If seasonal fluctuations in business occur with any reasonable or periodic regularity, the curve for the daily rate of sales may be assumed to follow the sine wave and in this case the expression for S may be written as

$$S = \alpha \cdot \sin (\beta \cdot T + \gamma) + \epsilon.$$

The use of this expression, however, results in an extremely complicated equation for the minimum-cost quantity, in terms of the sixth degree of the (cos T) which has no practical value, and as it is of interest only from a mathematical point of view, no further consideration will be given to it here. From the expression in Eq. (242) the value for the total or cumulative sales S_c from the beginning of the period $T_o = 0$ to any day T_n may be determined by finding the area under the curve S, by integration, so that

$$S_c = \int_{T_o}^{T_n} S \cdot dT. \tag{244}$$

From this, in turn, the total expected sales S_s for the whole sales-turnover period T_s may be evaluated if the duration of the period is expressed by the limits $T_o = 0$ and $T_n = T_s$, whereupon

$$S_s = \int_o^{T_s} S \cdot dT. \tag{245}$$

Now the quantity Q'_s remaining in stock on any day, where the total prior sales have been S_c, can be expressed by

$$Q'_s = Q - S_c, \tag{246}$$

and at the end of the period, where $T_n = T_s$, Q'_s will be equal to zero, as all articles Q produced and placed in stock to meet the demand in this period will have been exhausted. At the same time, as shown by Eq. (245),

$$S_c = S_s,$$

so that

$$Q'_s = Q - S_s = 0$$

or

$$Q = S_s = \int_o^{T_s} S \cdot dT. \tag{247}$$

However, the average daily sales S_{a_s} over any period T_s, where the demand is likely to be variable, is equal to the total sales S_s divided by the length of the period in days, so

$$S_{a_s} = \frac{S_s}{T_s} = \frac{1}{T_s} \cdot \int_o^{T_s} S \cdot dT; \tag{248}$$

but if the relations expressed in Eqs. (247) and (248) are combined, it will be seen that

$$S_{a_s} = \frac{Q}{T_s} = \frac{1}{T_s} \cdot \int_o^{T_s} S \cdot dT. \tag{249}$$

If from this analysis it has been shown that both S_{a_s} and S_a equal Q/T_s, either can be employed in the evaluation of the duration of the sales period T_s, the only difference being that the value of S_a can be obtained by the usual method of averaging, whereas S_{a_s} must be obtained by integration. For practical purposes a method of obtaining a value for the average daily sales S_{a_s} under conditions of variable demand has been described in Appendix IV.

APPENDIX III

EVALUATION OF THE AVERAGE STOCK FACTOR FOR VARIABLE DEMAND

To obtain a representative value for the average stock factor k_s under conditions of variable demand, the same general method can be pursued as employed in the evaluation of the average stock Q_v for uniform demand. In that case an expression for Q_v was obtained which bore a definite relation to the quantity produced Q. This relationship was expressed by the factor k_s. The purpose of this device was to obtain an average value for the quantity in stock which, if held in stores over the whole sales period, would incur the same investment charges as were actually incurred by a stock quantity which was uniformly being drawn upon.

If now a similar relationship is to be obtained under conditions of variable demand where the quantity Q'_s in stock at any instant cannot be represented by a straight line, as shown in Figs. 29, 35*a*, and 35*b*, a value for the average stock factor k_s must be obtained which will represent the deviation of the values for Q'_s in the case of variable demand from those for Q'_s when uniform demand can be assumed. It will be of importance, therefore, to study the shape of the curve Q'_s, from its equation given in Appendix II, as it may apply over the whole sales period between the limits $T_o = 0$ and $T_n = T_s$. When the expression for S_c from Eq. (244) has been introduced, the general expression [Eq. (246)] for this curve will then become

$$Q'_s = Q - \int_{T_o}^{T} S \cdot dT. \tag{250}$$

Since the average stock factor, in reality, is the ratio of the area A' under the curve Q'_s to the area A_o of the circumscribed rectangle *abcd* as shown in Figs. 36*a*, or 36*b*, the effect upon the investment charges of the shape of the curve Q'_s, as it may vary under differing conditions, can be measured by the relation of the specific value for k_s in such cases to its value of one half when the demand is uniform. Accordingly, k_s for variable

demand can be evaluated when the area A_o of the circumscribed rectangle is expressed as

$$A_o = Q \cdot T_s,$$

and the area *acd* under the curve Q'_s between the limits $T_o = 0$ and $T_n = T_s$ is written as

$$A_1 = \int_o^{T_s} Q'_s \cdot dT.$$

Now if $k_s = A_1/A_o$, according to the above definition

$$k_s = \frac{\int_o^{T_s} Q'_s \cdot dT}{Q \cdot T_s},$$

or

$$= \left[\frac{\int_o^{T_s} Q \cdot dT - \int_o^{T_s} S_c \cdot dT}{Q \cdot T_s} \right]; \qquad (251)$$

or if the relation for S_c in terms of S be introduced

$$= 1 - \left[\frac{\int_o^{T_s} dT \int_o^{T_s} S \cdot dT}{T_s} \right] \cdot \frac{1}{Q}.$$

Since this last expression cannot be employed as the basis of any practical method of evaluating k_s, until the function of S_c or S with regard to T is known or has been obtained by a study of the respective curves, this analysis must stand merely as evidence of the principles involved in interpreting as well as evaluating k_s.

If it be found, however, that the daily rate of sales varies according to the relation

$$S = a \cdot T, \text{ See Eq. (243)}$$

then the value of k_s can be determined, because the cumulative sales for the period $T_o = 0$, $T_n = T_s$ can be expressed by

$$S_c = \int_o^{T_s} S \cdot dT = \frac{a \cdot T_s^2}{2},$$

and then

$$k_s = 1 - \frac{\int_o^{T_s} \frac{a \cdot T^2}{2} \cdot dT}{Q \cdot T_s} = 1 - \frac{a \cdot T_s^3}{3 \cdot 2} \cdot \frac{1}{Q \cdot T_s}.$$

But, if from Eq. (249)

$$S_{a_s} = \frac{\int^{T_s} S \cdot dT}{T_s} = \frac{a \cdot T_s^2}{2 \cdot T_s},$$

then

$$k_s = 1 - \frac{S_{a_s} \cdot T_s}{3 \cdot Q};$$

and if

$$T_s = \frac{Q}{S_{a_s}}$$
$$= 1 - \tfrac{1}{3} = \tfrac{2}{3}$$

In this particular case for the specific period $T_o = 0$, $T_n = T_s$, the value for k_s, as employed in determining the lot size, should be two thirds instead of one half. In any other period preceding or following, the shape of the cumulative sales curve will probably require a different value of k_s. A graphical method of solution has been described in Appendix IV.

To show that this method has a universal application, it can be applied as well when the demand is uniform, although in that case there is little need for it as the rate of sales is constant. If this fact can be expressed by

$$S = a,$$

then

$$S_c = \int_o^{T_s} a \cdot dT = \frac{a \cdot T_s}{1};$$

whereupon

$$k_s = 1 - \frac{\int_o^{T_s} a \cdot T \cdot dT}{Q \quad T_s},$$
$$= 1 - \frac{a \cdot T_s^2}{2 \cdot Q \cdot T_s},$$

which, when

$$S_{a_s} = a \text{ (a constant)}$$

is introduced

$$k_s = 1 - \frac{T_s^2 \cdot S_{a_s}}{2 \cdot T_s \cdot Q};$$

and again, if

$$\frac{1}{T_s} = \frac{S_{a_s}}{Q},$$
$$k_s = 1 - \tfrac{1}{2} = \tfrac{1}{2},$$

which was the value obtained in Chap. XIV by merely a logical process of reasoning.

APPENDIX IV

GRAPHICAL METHODS EMPLOYED IN THE DETERMINATION OF ECONOMIC LOT SIZES FOR VARIABLE DEMAND

When the daily rate of sales fluctuates to a marked degree between consecutive sales periods owing to the seasonal characteristics of the demand, it is unwise to assume a uniform rate of consumption S_a for all periods, which is legitimate when the demand is more nearly constant throughout the year, in determining the best lot size for any one of the periods. This is due to the fact that if uniform demand had been assumed, there would be certain periods, where the demand is slight, in which the investment charges would be greatly increased, and other periods, where the demand is great, in which the investment charges would be less than that estimated. In the latter case an opportunity has been lost for temporarily producing a larger lot, for which the investment charges will be no greater than in the period of slack demand, at a less ultimate unit cost due to a greater quantity in the lot over which the preparation charges can be distributed. In the former case the lot size should also be corrected in order that the investment charges will not too greatly increase the ultimate unit cost. Evidently, if production schedules can be made to conform to the average daily rate of sales in each of the periods, a more economical manufacturing situation can be achieved.

This involves the determination of the average daily rate of consumption S_{a_s} under conditions of variable demand as well as the average stock factor k_s for each period. To attempt to calculate a value for either of these items from Eqs. (249) and (251), respectively, as derived in Appendices II and III, would not only consume much valuable time but would also require a familiarity with the methods employed in higher mathematics. For the average production-control clerk this would be impracticable and in all probability well-nigh impossible. The principles underlying the mathematical determination of both S_{a_s} and k_s,

however, may be employed to develop a graphical means of solution which can be readily applied in practice.

The basis of this method depends upon the ability of the sales department to furnish the production-control division with an estimate of the sales for each month or preferably each day for the ensuing year. Practical means of forecasting have been developed by some of the largest industries in this country which have shown a high degree of accuracy, so that it is possible to gage future sales closer than 5 per cent of the actual. The seasonal characteristics can be determined by a study of sales for the past few years, and a curve can be drawn which will show the approximate daily sales for a given unit of production. If the trend of sales in any year seems to be greater or less than that shown by this curve, the values read off from any of the points can be corrected by means of an appropriate factor. From this curve or from the data supplied by the sales department, a cumulative sales curve can be constructed by adding to the sales on any day all sales which have occurred prior to it since the beginning of the period or year. It is advisable to construct this cumulative curve complete for the coming year on a large scale and then any part of it can be used without difficulty. Such curves for a typical case are shown in Fig. 27.

To obtain the average daily rate of sales S_{a_s} for any period of time T_s, read off from the cumulative sales curve the total sales at the beginning and the end of the period and find the difference between these figures. The value of S_{a_s} will be found by dividing this difference by the number of working days in the period. In order to obtain a reasonable initial approximation to the length of any sales period, which naturally depends on the lot size or quantity produced at any one time, compute the lot size assuming uniform demand where S_a will equal the total sales for the year divided by the total working days in the year, and then divide the resulting quantity Q by the average uniform rate of consumption S_a. Using this number of days, determine S_{a_s} as described above and compute a new lot size, which when divided by S_{a_s} should give a period slightly greater or less than that for uniform demand. If greater accuracy is desired, determine S_{a_s} again using this last value for the length of the period. The lot size thus obtained should be sufficiently accurate for general production purposes. The best lot size for the remaining periods in the year may then be computed

by repeating the process for each in turn, though a single calculation will ordinarily suffice, except where the sales period for the computed lot size does not coincide within reason with that used in the calculations. In each case use the total of all lots previously produced in the year as the point on the cumulative curve for the beginning of the next period and use the length of the prior period as the basis for the time that it will last, reading off at the end of this time the total or cumulative sales to date for the computation of the average rate of consumption to be employed in computing the next lot size.

In most cases it will be found that a straight line can be used almost always to approximate the actual curved line for the cumulative sales curve within any period. Whenever this can be done the average stock factor can be assumed to be one half. If the divergence between the curved and straight lines seems to be unreasonable, the value of k_s must be computed. Again the mathematical method will be too complicated for practical purposes. If as earlier suggested, the cumulative curve has been drawn to a large enough scale, the value of k_s may be determined by drawing a vertical and horizontal line through the points on the cumulative curve at the beginning and end of the period. If these lines be extended till they intersect, a rectangle will be formed with the curved line passing inside it. Now the area of this rectangle and the area above the curve, as shown by the shaded portion of Fig. 45, may be obtained in square inches by the aid of an Amsler polar planimeter, the instrument used in finding the horsepower of a steam engine from the indicator cards. Since the value of k_s is equal to the ratio of the area above this curve to the area of the circumscribed rectangle, the dimensional units employed in measuring these areas is of little moment. Accordingly, if this ratio be computed, the resulting decimal can be inserted in any formula for the lot size in place of the value of k_s where it would otherwise have been assumed to be one half. In no case has it been found necessary to recompute the value of k_s because a longer or shorter sales period than anticipated was the result. This is due to the fact that k_s is a ratio and that the sales period T_s varies directly with the lot size Q. If the change in the value of k_s seriously alters the length of the sales period, it may be advisable to recalculate the lot size on the basis of S_{a_s} for the longer or shorter period, as the case may be.

When, in this discussion, a redetermination of the lot size is recommended, it does not infer that one must repeat all calculations. If the formula, whichever one be employed, be treated according to the elements in its composition, one need only apply a corrective factor[1] f'_s to the constant portion of each element

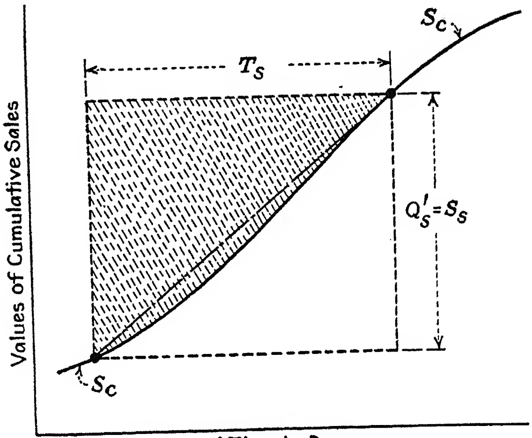

FIG. 45.—The graphical method of determining the average stock factor k_s for variable demand.

to obtain the new value for the lot size. This factor represents the ratio of the rate of consumption for uniform demand to that for variable demand. Accordingly if

$$f'_s = \frac{S_a}{S_{a_s}},$$

then in general

$$Q = \sqrt{\frac{P}{(K'_s \cdot f_r + K'_v) \cdot f'_s + K'_w \cdot f_r}},$$

where the constants

$$K'_s = \frac{c \cdot i}{2 \cdot S_a},$$

$$K'_v = \frac{s \cdot b}{h \cdot S_a},$$

$$K'_w = \left(\frac{m + c}{2}\right) \cdot t_p \cdot i,$$

and the factor[2] f_r designates the point within the economic range to which Q refers.

[1] See also p. 80.

[2] See item 11, Table VI, for evaluation of f_r.

Another method of determining the value of the average stock factor k_s was divised by Fletcher and Cammann which can be described in much fewer words but which does not lend itself too readily to a rapid solution of the problem. This was published[1] as a part of a general discussion of economic-production quantities.

[1] *Trans.* A.S.M.E. vol. 50-MAN, No. 10, January–April, 1928.

APPENDIX V

ANALYSIS OF THE GEOMETRIC RELATIONSHIPS ENTERING INTO THE DERIVATION OF THE AVERAGE STOCK FOR BATCH PRODUCTION

In order to evaluate the average stock Q_{v_B} so that it will be representative of the varying stock conditions imposed by the withdrawal of articles from a process where the lot has been subdivided into batches, a special form of procedure must be adopted. The reason for this is evident upon inspection of Figs. 32 and 34, because it can be seen that as a result of batch production the period where manufacture overlaps sales is composed of an alternating series of deliveries to stores and withdrawals from stores until the last batch has been completed. Accordingly, the saw-tooth line *af* (Fig. 34) must be employed to represent this condition instead of a straight line, which could be used in the case of semicontinuous production when deliveries to stores were uniformly maintained over the manufacturing period. Now the average stock Q_{v_B} is the altitude of a rectangle having a base T_s and an area equal to the unshaded area shown in Fig. 32 the value of which can be expressed by the quantity-time relation $Q_{v_B} \cdot T_s$. If Q_{v_B} is to be expressed in terms of Q, the lot size, however, some means must be devised for evaluating the irregular shaded area in Fig. 32, so that $Q_{v_B} \cdot T_s$ can be expressed as the difference between this area $A_2 = aef$ and the area $A_1 = aed$, where $A_1 = \frac{Q \cdot T_s}{2}$, as shown in Fig. 34.

By a study of the geometric relationships in Fig. 34, it can be seen that the saw-tooth line *af* divides the area of the parallelogram *aefg* in halves and can therefore be replaced by the straight line *af*. This fact can be definitely proved if the intercepted areas on either side of the straight line *af*, lying between it and the saw-tooth line, are summed algebraically, because it will be found that the net result in this case will be zero. Accordingly then, the area $A_2 = aef$ will equal the area *afg* or one half the area *aefg*. Since the area of a parallelogram is equal to the area of a

rectangle having the same base and altitude, it is evident that the area A_2 will equal one half the area *aehi*.

Now the area of this rectangle can be evaluated directly in terms of the lot size Q and T'_d, where T'_d represents the time during which the manufacturing period can overlap the sales period when articles are delivered to stores in batches. This is obvious because the altitude *ae* of this rectangle is equal to Q, and the base *ai* is naturally equal to the time element T'_d. If the total time to process the lot over the last operation is T_d and the time to process any batch is T_d/n, where n equals the number of batches, the period of overlap can be expressed by $T_d - \frac{T_d}{n}$, because the first batch need be the only one processed over the last operation of the production sequence in advance of the new sales period. Accordingly,

$$T'_d = T_d \cdot \left(1 - \frac{1}{n}\right);$$

and then

$$A_2 = \frac{1}{2}Q \cdot T_d \cdot \left(1 - \frac{1}{n}\right)$$

or

$$= \frac{1}{2} \cdot \frac{Q^2}{D} \cdot \left(1 - \frac{1}{n}\right), \text{ See Eq. (95), Table XVI}$$

because T_d is known to equal Q/D as derived for the case of semi-continuous production.[1] From this point on, the evaluation of Q_{v_B} may be accomplished by following through the steps indicated by Eqs. (94) to (98) inclusive in Table XVI.

[1] See p. 205.

APPENDIX VI

EVALUATION OF THE AVERAGE VALUE FACTOR k_a

It should be quite evident to anyone at all familiar with manufacturing operations that the manner in which value accumulates to a unit of production will have considerable influence upon the final value for the investment charges on work in process. Naturally, wage rates will vary with the skill required of the worker who is assigned to each particular operation in the manufacturing sequence, and it is conceivable that the burden rates may be prorated on such a basis that they will vary also. In some instances costly operations may occur early in the sequence and in others late, while in most cases the accumulation of value will fluctuate widely throughout the whole process. Rapid accumulation early in the process will mean high investment charges, whereas, if it occurs near the end, these charges would be considerably less, even though the expenditure of working capital be the same in either case. Obviously the investment charge on work in process depends on a value-time relationship in just the same manner as do similar charges upon articles in stores. Accordingly, a corrective factor for the accumulation of value must be introduced into the former case for the same purpose that the average stock factor was employed in the latter.

Even though in actual practice a constant value can be justifiably assigned to the factor k_a,[1] occasions may arise where it will be desirable to determine a more representative value instead. In order to furnish the necessary procedure for such computations, the factor k_a must be given a mathematical interpretation. Since the investment charges depend upon a value-time relationship, the area $A_c = acd$ under the curved or broken line ac in Fig. 37, which represents the value of a unit of production at any moment in the process, is the measure of their extent when compared with the maximum for such charges, which can be represented by the area A_o of the circumscribed rectangle $abcd$ for which the base is equal to the total unit process time $\Sigma(t_o)$ and

[1] See p. 222 and Table XVII, p. 219.

the altitude is equal to the total unit value $(l + o)$ of labor and overhead accruing to the piece. On this basis, then, the average value factor can be expressed as

$$k_a = \frac{A_c}{A_o}, \text{ See Eq. (110), Table XVII.}$$

where

$$A_o = (l + o) \cdot \sum_o^n (t_o)_n, \tag{252}$$

the terms of which can easily be evaluated by multiplying the combined unit costs of labor and overhead by the summation of the unit-process time for each operation.

On the other hand, the evaluation of the area A_c is not so simple. In this case each increment of cost must be multiplied by one half the process time for the operation in which it was incurred plus the unit times for all remaining operations. Thus the value-time relation for the first increment is

$$(l_{o_1} + o_{o_1}) \cdot \left(\frac{t_{o_1}}{2} + t_{o_2} + t_{o_3} + \cdots t_{o_n}\right),$$

and for the second increment it is

$$(l_{o_2} + o_{o_2}) \cdot \left(\frac{t_{o_2}}{2} + t_{o_3} + \cdots t_{o_n}\right) \text{ etc.,}$$

until for the last increment it becomes

$$(l_{o_n} + o_{o_n}) \cdot \frac{t_{o_n}}{2}$$

Now if these be summed, the value for A_c can be expressed as

$$A_c = \cdot \sum (l_o + o_o)_n \cdot \left(\frac{t_{o_n}}{2} + t_{o_{n+1}} + t_{o_{n+2}} \cdots \text{etc.}\right) \tag{253}$$

where the subscript n denotes any particular operation by its numerical position or order in the sequence of manufacture. Thus when actual values are inserted in the expressions for A_c and A_o, a representative value for k_a can be obtained for any process where the accumulation of value is not fairly uniformly distributed among the various operations.

In most cases, however, it will be found that the value of labor l and its overhead o will accumulate at a fairly uniform rate, because in any related group of operations the basic wage rates d_l for each worker, when they can be reduced to an hourly

basis, will be very nearly alike, and the burden rates d_o will in all probability be the same for all operations. This situation simplifies the determination of k_a, because the broken line ac can be replaced by the straight dotted line ac in Fig. 37, the equation for which is

$$(l + o)_n = (d_l + d_o) \cdot (t_1 + t_2 + t_3 \cdots t_n) = (d_l + d_o) \cdot [f(t)].$$

The area under this curve A'_c can then be represented as

$$A'_c = (d_l + d_o) \cdot \int_o^{t_o'} t \cdot dt$$

or

$$= (d_l + d_o) \cdot \frac{t'^2_o}{2} = \frac{1}{2} \cdot (l + o) \cdot \sum_o^n (t_o)_n,$$

where the limit t'_o is equal to the sum of the unit-process times for each operation $\left[\sum_o^n (t_o)_n\right]$. Hence, if these be combined in the original expression for k_a, it becomes evident that its value for practical purposes may be one half, because

$$k_a = \frac{A'_c}{A_o} = \frac{\frac{1}{2} \cdot (l + o) \cdot \sum_o^n (t_o)_n}{(l + o) \cdot \sum_o^n (t_o)_n} = \frac{1}{2}.$$

APPENDIX VII

THE INFLUENCE OF MACHINE CHANGEOVER COST M UPON THE ACCUMULATION OF VALUE TO WORK IN PROCESS

For ordinary purposes it would seem reasonable not to attempt to segregate the machine changeover costs M from the total preparation charges P when it is necessary to evaluate the investment charges on work in process[1] as a part of the computation for the economic lot size. Anyone acquainted with manufacturing methods however, will quickly appreciate the fact that this situation, if followed blindly, might lead to certain errors unless the true relationship of these factors has been emphasized.

Obviously the machine changeover costs cannot be considered as a whole and as being incurred at the start of any process, because each portion of it is really only fully accumulated at specific moments throughout the process just prior to the start of any individual operation (see Fig. 37). On the other hand, it is legitimate to consider that the other factors which go to make up the total preparation costs can be combined and treated as if the investment charge on this portion of the capital expenditure in manufacturing operations was uniform over the whole manufacturing period T_p. This is due to the fact that engineering or technical expense, tool preparation, production control, and general administrative costs are for the most part incurred in advance of production and represent an initial accumulation of value before any unit of production in the lot has begun to assume a concrete form.

In order then to conform to the actual situation, the unit allotment of the investment charges which are derived from the value added by the preparation cost must be expressed by the equation

[1] See p. 222 and Table XVII, p. 219.

$$\frac{i \cdot P}{Q} \cdot \left(\frac{T_M}{2} + T_p\right) = i \cdot \left[\frac{M_1 \cdot T_{M_1}}{Q \cdot 2} + \frac{M_2 \cdot T_{M_2}}{Q \cdot 2} + \frac{M_3 \cdot T_{M_3}}{Q \cdot 2} \cdots \text{etc.}\right]$$
$$+ i \cdot \left[\frac{M_1 \cdot Q}{Q} \cdot \sum_1^n (t_o)_n + \frac{M_2 \cdot Q}{Q} \cdot \sum_2^n (t_o)_n + \right.$$
$$\left. \frac{M_3 \cdot Q}{Q} \cdot \sum_3^n (t_o)_n \cdots \text{etc.}\right] + i \cdot t_p \cdot Q \cdot$$
$$\cdots \left(\frac{O + E_g + J + G}{Q}\right) \quad (254)$$

where

M_n = the total machine changeover cost for setting up and dismantling any operation,
T_{M_n} = the time required to change over the machine for any operation,
t_o = the time to process one piece through any operation,
O = total production-control cost per lot,
E_g = engineering or technical expense per lot,
J = tool expense per lot,
G = allotment of total administrative costs per lot,

and the subscript n or its numerical value is employed to designate any operation by its number or order in the production sequence. An accurate determination of this portion of the investment charges would require the solution of this expression, but since the real purpose is to employ this general relationship in a formula for the determination of the lot size without too much complication, it must be materially simplified.

This can be accomplished by special treatment of each group of terms and by the use of the machine changeover factor $k_M = M/P$. If this be done the first group can be reduced to the form

$$\frac{i \cdot P \cdot k_M \cdot k_t}{Q \cdot 2},$$

as long as it can be assumed that the accumulation of value proceeds at a uniform rate for all machine changeovers. Hence, if in general

$$M_n = d_M \cdot T_{M_n}$$

the first group omitting $\frac{i}{Q \cdot 2}$ for the moment may be expressed as

$$d_M \cdot (T_{M_1}{}^2 + T_{M_2}{}^2 + T_{M_3}{}^2 + T_{M_4}{}^2) = d_M \cdot T_M{}^2 \cdot k_t = P \cdot k_M \cdot T_M \cdot k_t,$$

where

$$T_M = (T_{M_1} + T_{M_2} + T_{M_3} \cdot \cdot \cdot \text{ etc.}),$$

and

$$k_t = \frac{1}{1 + 2 \cdot k'_t},$$

$$k'_t = \frac{T_{M_1} \cdot T_{M_2} + T_{M_1} \cdot T_{M_3} + T_{M_2} \cdot T_{M_3} + \cdot \cdot \cdot \text{ etc.}}{T_{M_1}{}^2 + T_{M_2}{}^2 + T_{M_3}{}^2 + \cdot \cdot \cdot \text{ etc.}}$$

Now a study of the relation represented by k'_t for values of machine changeover time which would normally be employed in the series designated by T_M shows that the value of k'_t will in all probability remain within the limits $0.95 < k'_t < 1.5$. Under these conditions the value of k_t can be assumed to be no greater than one quarter and then, for the purposes of this analysis, the first group can be evaluated from

$$\frac{i \cdot P \cdot k_M \cdot T_M}{Q \cdot 8}.$$

The second group can be easily reduced to the form

$$i \cdot M \cdot t_p \cdot k'_a$$

by following the method of reasoning in determining the average accumulation of value from the application of labor and its overhead by substituting the unit allotment of the machine changeover costs M/Q for the unit labor and overhead cost $l + o$, as applied to the total process time $T_p = Q \cdot t_p$. The only difference in this case is that the machine changeover costs have already accumulated and this fact has been accounted for in the first group of terms. Since the factor k'_a takes into account the summation of the various series $\sum_1^n (t_o)_n$, $\sum_2^n (t_o)_n$, $\sum_3^n (t_o)_n$, etc.,[1] the investment charge from this phase may be expressed as

$$i \cdot P \cdot k_M \cdot k'_a \cdot t_p$$

for further use in evaluating I_w.

The last group needs little special treatment except to eliminate the numerous general items which it contains and to express it in terms of P and k_M. This can be achieved by means of the relation

$$\begin{aligned} O + E_g + J + G &= P - M \\ &= P \cdot (1 - k_M), \end{aligned}$$

[1] The derivation of k'_a is identical with that for k_a, see p. 331, Appendix VI.

so that for ease in evaluation the final expression containing all three groups can be written as

$$\frac{i \cdot P}{Q} \cdot \left(\frac{T_M}{2} + T_p\right) = i \cdot P \cdot \left\{\frac{k_M \cdot T_M}{8 \cdot Q} + t_p \cdot [1 - k_M \cdot (1 - k'_a)]\right\}$$

or

$$= i \cdot P \cdot \left[\frac{k_M \cdot T_M}{8 \cdot Q} + t_p \cdot \left(1 - \frac{k_M}{2}\right)\right]$$

if $k'_a = \frac{1}{2}$ for the same reason that k_a does.

APPENDIX VIII

EVALUATION OF THE AVERAGE TIME OF WITHDRAWAL OF STOCK WHEN STORAGE BINS ARE NOT PERMANENTLY RESERVED

Owing to the fact, when storage bins are not permanently reserved,[1] that the storage space charges incurred by the use of each bin can only accumulate to a given lot of articles in stores in accordance with the time required to empty each one in turn, it is impossible to assume that the average withdrawal time is equal to one half the total time during which the stock is being depleted. Reference to Fig. 42 will give conclusive evidence of this fact because even though there be but one piece in a bin, the cost of that bin must be charged to the lot for the full time that this last piece remains there. This must be so because it is considered poor practice to place dissimilar articles in the same unit of storage space and only in this manner could the average time be made equal to one half the total time of withdrawal.

Resorting again to the fact that charges such as these eventually depend upon a quantity-time relation in the same way that the investment charges did, the average time of withdrawal T_{W_a} can be evaluated from the relation

$$Q'_x \cdot T_{W_a} = q_{b_1} \cdot T_W \cdot \left(\frac{1}{n_b}\right) + q_{b_2} \cdot T_W \cdot \left(\frac{2}{n_b}\right) + q_{b_3} \cdot T_W \cdot \left(\frac{3}{n_b}\right) + \cdots q_{b_n} \cdot T_W \cdot \frac{n_b}{n_b},$$

where

$$q_{b_1} = q_{b_2} = q_{b_3} = q_{b_n} = \frac{Q_x}{n_b} = q_b,$$

and q_{b_n} is equal to the number of pieces placed in any bin, it being assumed that the maximum number of articles Q'_x placed in stock are evenly divided among the fewest number n_b of bins which could hold them all. Here again the subscript n designates each bin by the number representing the order or sequence in which articles are removed from the bins in turn. Since the

[1] See p. 246, and Table XXI, p. 247.

total number of articles are evenly distributed, the original expression may be rewritten so that

$$Q'_x \cdot T_{W_a} = q_b \cdot T_W \cdot \left(\frac{1 + 2 + 3 \cdots n_b}{n_b}\right),$$

or

$$Q'_x \cdot T_{W_a} = \frac{Q'_x}{n_b} \cdot T_W \cdot \left(\frac{1 + 2 + 3 \cdots n_b}{n_b}\right);$$

whereupon

$$T_{W_a} = \frac{T_W}{n_b} \cdot \left(\frac{1 + 2 + 3 \cdots n_b}{n_b}\right).$$

Now the series in this expression can be evaluated definitely, when the actual number of bins n_b are known, by the use of a much simpler relation where

$$\frac{1 + 2 + 3 \cdots n_b}{n_b} = \frac{1 + n_b}{2}.$$

If this be introduced into the previous expression for T_{W_a}, the form which was assigned to it in Table XXI can be immediately obtained from

$$T_{W_a} = \frac{T_W}{n_b} \cdot \left(\frac{1 + n_b}{2}\right),$$

whence

$$T_{W_a} = \frac{T_W}{2} \cdot \left(1 + \frac{1}{n_b}\right). \quad \text{See Eq. (136).}$$

APPENDIX IX

EVOLUTION OF THE FUNDAMENTAL RELATION FOR THE ECONOMIC-PRODUCTION QUANTITY

In order to avoid in Table XXVI, Chap. XVIII, any confusion in the mind of the reader, certain routine steps in the transition from Eq. (164) to Eq. (165) were omitted. If the evaluation of Eq. (164) for the economic-production quantity by the introduction of Eq. (163) in its more fully expanded form is to be thoroughly understood, however, these remaining steps should be presented. As a result of this substitution the following expression was obtained in Table XXVI where

$$Q_e = \frac{\sqrt{\dfrac{P \cdot F}{(v_s \cdot t_s + v_w \cdot t_w) \cdot i + v_v \cdot t_v}}}{1 + \dfrac{P \cdot F}{P \cdot F}\left[\dfrac{(v_s \cdot t_s + v_w \cdot t_w) \cdot r}{(v_s \cdot t_s + v_w \cdot t_w) \cdot i + v_v \cdot t_v}\right]}.$$

Naturally it would be desirable to arrange the terms in this expression so that they will appear in some form comparable to the expression for Q_m, Eq. (163). To accomplish this the above expression must be cleared of the compound fractions and all terms must be placed under the radical. Accordingly, if

$$Q_e = \frac{\sqrt{\dfrac{P \cdot F}{(v_s \cdot t_s + v_w \cdot t_w) \cdot i + v_v \cdot t_v}}}{\dfrac{(v_s \cdot t_s + v_w \cdot t_w) \cdot i + v_v \cdot t_v + (v_s \cdot t_s + v_w \cdot t_w) \cdot r}{(v_s \cdot t_s + v_w \cdot t_w) \cdot i + v_v \cdot t_v}},$$

then

$$Q_e = \sqrt{\frac{P \cdot F \cdot [(v_s \cdot t_s + v_w \cdot t_w) \cdot i + v_v \cdot t_v]^2}{[(v_s \cdot t_s + v_w \cdot t_w) \cdot i + v_v \cdot t_v][(v_s \cdot t_s + v_w \cdot t_w) \cdot (i + r) + v_v \cdot t_v]^2}},$$

and by cancellation and expanding the denominator

$$Q_e = \sqrt{\frac{P \cdot F}{\dfrac{[(v_s \cdot t_s + v_w \cdot t_w) \cdot i + v_v \cdot t_v]^2 + 2 \cdot [(v_s \cdot t_s + v_w \cdot t_w) i + v_v \cdot t_v](v_s \cdot t_s + v_w \cdot t_w) \cdot r + (v_s \cdot t_s + v_w \cdot t_w)^2 \cdot r^2}{(v_s \cdot t_s + v_w \cdot t_w) \cdot i + v_v \cdot t_v}}}.$$

Now if the fraction in the denominator is simplified by again cancelling out similar groups of terms and rearranging the remaining ones, so that

$$Q_e = \sqrt{\frac{P \cdot F}{(v_s \cdot t_s + v_w \cdot t_w) \cdot (i + 2r) + \dfrac{(v_s \cdot t_s + v_w \cdot t_w) \cdot r^2}{\dfrac{(v_s \cdot t_s + v_w \cdot t_w)i + v_v \cdot t_v}{v_s \cdot t_s + v_w \cdot t_w}} + v_v \cdot t_v}},$$

the final expression as given by Eq. (165) can be obtained by a further simplification and regrouping. Hence

$$Q_e = \sqrt{\frac{P \cdot F}{(v_s \cdot t_s + v_w \cdot t_w) \cdot \left\{ i + 2r + \dfrac{r^2}{\left[i + \dfrac{v_v \cdot t_v}{v_s \cdot t_s + v_w \cdot t_w} \right]} \right\} + v_v \cdot t_v}}.$$

APPENDIX X

SUBSTANTIATION OF THE APPROXIMATE METHOD FOR EVALUATING THE LOSS FACTOR λ

In order to justify the method presented on page 304 in Chap. XX, for a rapid determination of an approximate value for the loss factor which can be employed by any executive in connection with Fig. 13 page 76, and avoid the more lengthy computations[1] required in its evaluation for calculating the maximum-production quantity, certain reasonable assumptions will have to be made. Since the greater portion of the ultimate cost U of any unit of production is represented by the working capital invested in its manufacture or is in some way dependent upon it, it can be stated with reasonable accuracy if

$$U = u'_m + i \cdot \frac{C_i}{Q},$$

where the average unit initial investment of capital

$$\frac{C_i}{Q} = \alpha \cdot u'_m$$

that

$$U = u'_m + i \cdot \alpha \cdot u'_m$$

or

$$= u'_m \cdot (1 + i \cdot \alpha).$$

In this analysis the unit manufacturing cost u'_m has been considered as containing all storage space charges and other less important items in the composition of U, since for this approximation it will be permissible for them to be placed in the overhead as a part of the unit-production cost c in the relation

$$u'_m = \left(c + \frac{P}{Q}\right).$$

[1] See Eq. (235), Table XXXVIII, p. 307.

Now in Chap. X it was stated that the unit margin of profit was a function of the average unit initial investment of capital, so that if

$$\frac{R_u}{S_y} = r \cdot \frac{C_i}{Q},$$

the above relation for C_i/Q can be employed to express R_u/S_y in terms of u'_m, whereupon

$$\frac{R_u}{S_y} = r \cdot \alpha \cdot u'_m.$$

Since the loss L_s due to the production of a quantity different from the minimum-cost quantity can be expressed not only as

$$L_s = \lambda \cdot U = \lambda \cdot u'_m (1 + i \cdot \alpha)$$

but also as

$$= \frac{R_m}{S_y} - \frac{R_u}{S_y} = r \cdot \frac{C_{i_m}}{Q_m} - r_u \cdot \frac{C_{i_u}}{Q_u},$$

where R_m/S_y and R_u/S_y represent the unit margin of profit for the minimum-cost quantity Q_m and any other quantity Q_u, respectively, and the rates r and r_u are the measures of the unit margin of profit R_m/S_y and R_u/S_y above, the loss factor λ can be expressed in terms of these respective rates of return. As a result it will be possible to determine the effect of a given percentage of loss upon the expected rate of return and determine to what extent it may be reduced in order to justify a reasonable variation in production cost that will permit a desirable degree of flexibility in control.

Accordingly, if the capital investment for any unit in a lot differing in size from the lot for minimum cost be assumed to be proportional to that for the minimum-cost quantity

$$\frac{C_{i_u}}{Q_u} = \beta \cdot \frac{C_{i_m}}{Q_m},$$

then

$$\lambda \cdot u'_m \cdot (1 + i \cdot \alpha) = r \cdot \alpha \cdot u'_m - r_u \cdot \beta \cdot \alpha \cdot u'_m,$$

or cancelling out u'_m

$$\lambda = (r - \beta \cdot r_u) \cdot \left(\frac{\alpha}{1 + i \cdot \alpha}\right),$$

whereupon

$$\frac{\lambda}{(r - \beta \cdot r_u)} = \frac{\alpha}{1 + i \cdot \alpha} \quad \text{(a constant).}$$

Thus it can be assumed that an increase in λ will cause a proportionate decrease in the expected rate of return r_u for the lot size Q_u since r is fixed, as the difference in the rates must increase in the same relative manner. The influence of the coefficient β in the parenthesis will have little effect upon general conclusions, because within the range of small values of λ the value β will vary infinitesimally from unity.

APPENDIX XI

EVOLUTION OF THE GENERAL EXPRESSION FOR THE LIMITS OF PERMISSIBLE VARIATION

As the purpose of Table XXXIX in Chap. XV is only to show the basis upon which the final expression for the limits of permissible variation [Eq. (239)] depends and not the method employed in its derivation, the major steps in its development will be outlined in this appendix in order to justify the form of this equation.

Given the appropriate expression for K_e [Eq. (237)] the limits E_l of permissible variation can be determined by solving for E_l when Eq. (237) has been substituted in Eq. (238). The procedure from this point on can be represented as follows. If Eq. (237) be simplified for the moment so that it becomes

$$K_e = 1 + \alpha \cdot (1 - \rho \cdot E) + \beta \cdot (1 - \rho),$$

where

$$\alpha = \frac{r \cdot Q_m{}^2 \cdot v \cdot t}{2 \cdot P \cdot F}$$

and

$$\beta = \frac{r \cdot Q_m \cdot C'_F}{2 \cdot P \cdot F},$$

Eq. (238) may be rewritten, when E is represented by E_l as the limits of variation, so that

$$E_l{}^2 - 2 \cdot E_l \cdot \gamma + 2 \cdot E_l{}^2 \cdot \alpha \cdot \rho + 1 = 0$$

or

$$E_l{}^2 \, (1 + 2 \cdot \alpha \cdot \rho) - 2 \cdot E_l \cdot \gamma + 1 = 0,$$

where

$$\gamma = 1 + \alpha + \beta \cdot (1 - \rho);$$

and then by the use of the formula for solving a quadratic equation (Appendix XII),

$$E_l = \frac{2 \cdot \gamma \pm \sqrt{4 \cdot \gamma^2 - 4 \cdot (1 + 2 \cdot \alpha \cdot \rho) \cdot 1}}{2 \cdot (1 + 2 \cdot \alpha \cdot \rho)},$$

Now if the constant 2 be canceled out and the values of α, β, and γ be reintroduced

$$E_l = \frac{2 + \dfrac{r \cdot Q_m{}^2 \cdot v \cdot t}{P \cdot F} + \dfrac{r \cdot Q_m \cdot C'_F \cdot (1 - \rho)}{P \cdot F}}{2 + \dfrac{2 \cdot \rho \cdot r \cdot Q_m{}^2 \cdot v \cdot t}{P \cdot F}} \pm$$

$$\sqrt{\left[\frac{2 + \dfrac{r \cdot Q_m}{P \cdot F}[Q_m \cdot v \cdot t + C'_F \cdot (1 - \rho)]}{2 + 2 \cdot \rho \cdot r \cdot \dfrac{Q_m{}^2}{P \cdot F} \cdot v \cdot t}\right]^2 - \frac{1}{1 + \rho \cdot r \cdot \dfrac{Q_m{}^2}{P \cdot F} \cdot v \cdot t}},$$

which is an equivalent expression to that given in Eq. (239), Table XXXIX, except for the fact that in this case the terms $P \cdot F$ have not been canceled out.

If the relation in Eq. (163), Table XXV, be employed to eliminate $Q_m{}^2$ when $f = v \cdot t \cdot i + v_v \cdot t_v$ and the terms $P \cdot F$ are now canceled out wherever possible

$$E_l = \frac{2 + \dfrac{r \cdot v \cdot t}{v \cdot t \cdot i + v_v \cdot t_v} + \dfrac{r \cdot Q_m \cdot C'_F \cdot (1 - \rho)}{P \cdot F}}{2 + \dfrac{2 \cdot \rho \cdot r \cdot v \cdot t}{v \cdot t \cdot i + v_v \cdot t_v}} \pm$$

$$\sqrt{\left[\frac{2 + \dfrac{r \cdot v \cdot t}{v \cdot t \cdot i + v_v \cdot t_v} + \dfrac{r \cdot Q_m \cdot C'_F \cdot (1 - \rho)}{P \cdot F}}{2 + \dfrac{2 \cdot \rho \cdot r \cdot v \cdot t}{v \cdot t \cdot i + r_v \cdot t_v}}\right]^2 - \frac{1}{1 + \dfrac{\rho \cdot r \cdot v \cdot t}{v \cdot t \cdot i + v_v \cdot t_v}}}.$$

Without question any of these expressions thus far developed will be of little practical value for industrial use because of the complex relations. It will be noticed, however, that a certain group of terms is often repeated and that it is the same group which is employed in Chap. XIX in evaluating the index ratios R_s and R_w. Since the calculation sheets provide a reliable method of computing the value of these ratios, there seems to be no good reason why they cannot be employed here, as well, to simplify these equations. Accordingly, if

$$\frac{r}{i} \cdot \frac{v \cdot t \cdot i}{v \cdot t \cdot i + v_v \cdot t_v} = \frac{r}{i} \cdot \frac{(v_s \cdot t_s + v_w \cdot t_w) \cdot i}{[(v_s \cdot t_s + v_w \cdot t_w) \cdot i + v_v \cdot t_v]} \cdot \frac{v_s \cdot t_s \cdot i}{v_s \cdot t_s \cdot i} = \frac{r}{i} \cdot \frac{(R_s + R_w)}{R_c}$$

and

$$R_s = 1,$$

an expression can be written where

$$E_l = \frac{2 + \frac{r}{i} \cdot \frac{(1 + R_w)}{R_c} + r \cdot \frac{Q_m}{P \cdot F} \cdot C'_F \cdot (1 - \rho)}{2 + \frac{2 \cdot \rho \cdot r}{i} \cdot \frac{(1 + R_w)}{R_c}} \pm$$

$$\sqrt{\left[\frac{2+\frac{r}{i} \cdot \frac{(1 + R_w)}{R_c} + r \cdot \frac{Q_m}{P \cdot F} \cdot C'_F \cdot (1 - \rho)}{2+\frac{2 \cdot \rho \cdot r}{i} \cdot \frac{(1 + R_w)}{R_c}}\right]^2 - \frac{1}{1+\frac{\rho \cdot r}{i} \cdot \frac{(1+R_w)}{R_c}}}.$$

Since the upper limit of permissible variation will be obtained from the relation

$$Q_{\max} = E_{l_{\max}} \cdot Q_m$$

when the positive sign is used before the radical and the lower limit from the relation

$$Q_{\min} = E_{l_{\min}} \cdot Q_m$$

when the negative sign is used, further simplification can be achieved, especially if the terms in the numerator and denominator of the group outside the radical be indicated by the symbols f_n and f_d, respectively. Accordingly, if from

$$Q_{\max} = \frac{Q_m\left[2 \cdot R_c + \frac{r}{i} \cdot (1 + R_w)\right] + \frac{r}{v_s \cdot t_s \cdot i} \cdot C'_F \cdot (1 - \rho)}{2 \cdot R_c + \frac{2 \cdot \rho \cdot r}{i} \cdot (1 + R_w)} \cdot$$

$$\left[1 + \sqrt{1 - \frac{\left[4R_c + 4 \cdot \frac{\rho \cdot r}{i} \cdot (1 + R_w)\right] \cdot Q_m{}^2 \cdot R_c}{\left\{Q_m \cdot \left[2R_c + \frac{r}{i} \cdot (1 + R_w)\right] + \frac{r}{v_s \cdot t_s} \cdot i \cdot C'_F \cdot (1 - \rho)\right\}^2}}\right]$$

$$f_n = \sqrt{\frac{P \cdot F}{v_s \cdot t_s \cdot i \cdot R_c}} \cdot \left[2R_c + \frac{r}{i} \cdot (1 + R_w)\right] + \frac{r}{i} \cdot \frac{C'_F}{v_s \cdot t_s} (1 - \rho)$$

where the short-cut method of evaluating Q_m has been employed, as explained in Table XXXV,

and

$$f_d = 2 \cdot R_c + 2\frac{\rho \cdot r}{i}(1 + R_w),$$

the maximum-production quantity can be expressed as

$$Q_{\max} = \frac{f_n}{f_d}\left[1 + \sqrt{1 - \frac{2 \cdot f_d}{f_n^2} \cdot \frac{P \cdot F}{v_s \cdot t_s \cdot i}}\right].$$

In a similar manner the minimum-production quantity may be written when the minus sign is used instead, so that

$$Q_{\min} = \frac{f_n}{f_d}\left[1 - \sqrt{1 - \frac{2 \cdot f_d}{f_n^2} \cdot \frac{P \cdot F}{v_s \cdot t_s \cdot i}}\right].$$

APPENDIX XII

SOLUTION OF A QUADRATIC EQUATION BY FORMULA

In many instances throughout the derivation of certain mathematical relationships having a bearing upon the economic lot size, quadratic equations have been encountered. When such equations are in a form which cannot be solved either by factoring or by completing a perfect square, a mathematical formula may be resorted to. For those who are unfamiliar with mathematical technique, the method employed in the cases referred to above will be outlined in order that these readers may more fully comprehend the steps employed.

If an equation involving both the first and second power of an unknown quantity X can be arranged in the form

$$AX^2 + BX + C = 0,$$

the two roots for X may be obtained from the relation

$$X = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A},$$

when the coefficients A, B, and C are assigned to the groups of constant terms which may be attached to or associated with X^2 and X, in accordance with their respective position in the general form of a quadratic equation, as illustrated at the beginning of this paragraph. When these groups of terms are substituted in the equation for X in place of the coefficients, the resulting combination of the constant terms will be the desired solution for the more complex equation with which one started in the beginning.

APPENDIX XIII

INDEX OF SYMBOLS AND DEFINITIONS

Symbols and their subscripts have been designed to be mnemonic in order to make simpler their interpretation by merely an inspection of their arrangement. Certain general symbols have been used which maintain their identity throughout, their specific reference being indicated by a subscript or a combination of subscripts. Wherever possible, capitals have been employed to designate total items and lower case letters unit items; the chief exception to this rule is the use of U and q. Accents[1] ($'$), ($''$), ($'''$) have been employed to denote terms which have specific characteristics but are closely related to the general term. In this index the subscripts[2] have been classified by groups in order to emphasize their connection with certain particular general terms.

1. ENGLISH ALPHABET

		Page
a	Allowance for spoilage and salvage	182
A	The value accumulated to any unit of production from any phase of manufacture (used with subscripts, group 2)	143
A_c	The area under any curve between specified limits (also used with subscripts, group 10)	330
A_n	Any area illustrated in a diagram (used with subscripts, group 10, *viz.*, A_1, A_2, etc.)	209
A_o	The area of a rectangle, circumscribed about some portion of a diagram	331
A_r	The area of any rectangle	209
b	The overall volume or bulk of a unit of production	243
B	The total commercial and financial costs incidental to the conduct of business	139
B'	The total cost of doing business B exclusive of the total cost of capital invested in manufacturing operations	139
c	The unit-production cost	177
c_o	The unit-production cost, overhead omitted	190

[1] See Sec. 3, p. 355.
[2] See Sec. 4, p. 355.

		Page
c_x	The extra cost per unit of cutting off raw material from bars or sheets or the melting cost of the metal per unit of production	179
c''	The average value that will accumulate to a unit of production while in production	236
C	The average capital invested in any phase of manufacture (used with subscripts, groups 2 and 3)	144
C_e	The total annual expenditure of capital in the manufacture of a final product	143
C_i	The average initial investment of capital in the manufacture and storage of a lot which is repeated each time a lot of the same size and kind is processed	146
d	The rate of wage payment or burden distribution (used with subscripts, group 7)	178
D	The uniform daily rate of withdrawal of articles from work in process (used occasionally with subscripts, group 5)	206
e	The allowable error in the data	288
e_a	The arbitrarily assumed error in the data	293
e_M	The maximum allowable error in the data	288
E	The permissible variation in the lot size	286
E_g	The total engineering or technical expense attributable to the lot as a whole	187
E_l	The permissible limits of variation in the lot size	157
E_M	The maximum permissible variation in the lot size	286
f	Any factor or group of unit charges (used also with accents)	170
f_b	Factor for the volume of a bin in terms of equivalent square feet of floor area	244
f_c	The form index	288
f_m	The flow of material factor	290
f_p	The process factor	290
f_r	The factor designating the identity of the limits of the economic range and the point of maximum return (used also with accents)	285
f_s	The sales factor for correcting the lot size for changes in the rate of consumption (used also with accent)	79
F	Any factor or group of terms relating to the total charges applicable to the lot as a whole (used also with accent)	170
G	The total administrative expense applicable to a lot as a whole	187
h	The height to which storage is permissible on any square foot of floor space	243
i	The interest rate charged on borrowed capital expressed decimally	194

Symbol	Definition	Page
I	The total cost of capital invested in any phase of the manufacture of a lot (used with subscripts, groups 2, 3 and 4)....	160
I_t	The total cost of capital from all phases of the manufacture of a lot of the final product reduced to a unit charge.........	165
J	The total cost of preparing the required tools, jigs, and fixtures for the manufacture of a lot.............................	186
k	Any constant factor	
k_a	The average value factor................................	220
k_b	The bin factor..	248
k_d	The factor for delays or discrepancies between theoretical and actual process time..................................	235
k_M	The value of the ratio M/P............................	222
k_p	The process constant....................................	209
k_r	The constant employed to obtain the economic-production quantity from the minimum-cost quantity................	39
k_s	The average stock factor................................	211
k_v	The constant portion of the unit investment charges (used with subscripts, group 3)...............................	215, 237
K	Any element in the denominator of an expression for the lot size (used with subscripts, groups 3 and 4)................	286
K_c	The sum of all three elements in the denominator of a complete expression for the lot size...........................	286
K_e	The factor of permissible variation......................	300
K_o	The problem index.......................................	300
K_p	The numerator of an expression for the lot size..............	286
K_r	The factor showing the influence of the space charge element K_v upon the economic-production quantity................	87
l	The direct-labor cost per unit.............................	177
L	The total cost of labor (used with accents)..................	188
L_t	The total of all losses incidental to manufacture and storage..	165
L_d	The loss due to deterioration chargeable to the lot as a whole..	256
L_{ob}	The loss due to obsolescence..............................	258
L_r	The loss or increase in the ultmate unit cost due to the production of a quantity within the economic range of production.	155
L_s	The loss or increase in the ultimate unit cost due to simplification..	299
m	The unit cost of material as it enters any phase of manufacture for processing (used with subscripts, group 2)..............	178
m_M	The cost of materials and supplies consumed in machine changeover..	185
M	The total cost of machine changeover for the lot as a whole..	185
n	The number of batches or subdivisions of a lot (used with subscripts, groups 6 and 9)...............................	207

		Page
n_b	The number of bins required to store a given number of articles	249
n_M	The number of machines run in parallel to perform a given operation	235
N	The number of lots, sales turnover, or inventory periods per year (used with subscripts, group 8)	146
o	The overhead or burden distributed to each unit of production	177
O	The total production-control cost applicable to a lot as a whole	186
p	The portion of the minimum-sales price applicable to any unit of production in any phase (used with subscripts, group 2, and accents)	135
p_m	The minimum unit-sales price which will yield the normal unit margin of profit (used also with accents)	308
p_p	The unit-purchase price of raw material	179
p_s	The minimum unit-sales price for the final product (used also with accents)	138
p_u	The unit-sales price which must be obtained to cover the normal unit margin of profit when the lot size is outside the economic range of production	308
P	The total cost of preparing for the manufacture of a lot (used occasionally with accents)	184
P_o	The total preparation costs, overhead omitted	190
q	Any specific quantity of a unit of production (used occasionally with subscripts, groups 8 and 9)	206
q_b	The number of articles of a kind which can be placed in a bin	337
q_c	The number of articles of a kind which can be placed in a container	211
q_d	The quantity in any lot which will deteriorate during the sales period	256
q_p	The constant bank of material, parts, etc., maintained on the production floor during the processing of a lot	33
Q	The lot size (used with subscripts, groups 1, 2, 3, and 4)	168
Q	The economic-production quantity	17, 157
Q_i	The economic lot size determined from one of the complete expressions	286
Q_l	The limits of the economic range of production	155
Q_m	The minimum-cost quantity	172
Q_M	The maximum allowable production quantity	82
Q_R	The point of maximum return on invested capital	18
Q'_s	The quantity held in stock at any instant during the sales period	211
Q''_s	The quantity of articles in stock which will remain in perfect condition throughout the whole sales period	256
Q_t	The economic-turnover quantity	83
Q_u	Any uneconomical lot size	135, 286

		Page
Q_v	The average number of articles of a lot which may be considered as remaining in stores for the full sales period (used with subscripts, group 5)	203
Q'_x	The maximum number of articles which will at any time be held in stores inventories	243
r	The rate of return normally expected on capital invested in manufacturing operations in excess of the interest rate i	142
r_m	The minimum rate of return	78
r_u	The rate of return earned on an uneconomically produced lot	304
R	The index ratio for any element or group of elements in the denominator of an expression for the lot size (used with subscripts, groups 3 and 4)	288
R_c	The index ratio for all three elements of the denominator, in an expression for the lot size, combined	292
R_o	The annual gross return upon the average capital invested in the manufacture of a unit of production for any phase (used with subscripts, group 1 in place of subscript o)	154
R_o/S_y	The unit margin of profit or the spread between the minimum-sales price and the ultmate unit cost for a unit of production in any phase of its manufacture (used with subscripts, group 1 in place of subscript o)	154
R_t	The total annual gross return upon the average capital invested in the manufacture of a final product	139
R_t/S_y	The unit margin of profit for the final product	139
s	The unit-space charge for each square foot of storage area (used occasionally with subscripts, group 3)	241
S	The daily rate of consumption or sales	200
S_a	The average daily rate of consumption for uniform demand	200
S_{a_s}	The average daily rate of consumption for variable demand	200
S_c	The total or cumulative sales over any period T	318
S_s	The total sales during the period T_s	318
S_y	The total estimated sales or consumption of a unit of production for the year	200
t	The unit-process time (used with subscripts, groups 3, 5, 6, and 10)	230
t_d	A time allowance per unit for unavoidable delays of a recurring nature	234
t_o	The unit-operation time (used with subscripts, group 10)	177
t_p	The unit-process time for the average piece	226
t_t	The unit time allowance for transitions between operations due to characteristics of the process	233
T	Any total elapsed time for the lot as a whole (used with subscripts, groups 3 and 8)	148
T_a	The total average time of storage	248

		Page
T_d	The total time required to deliver a lot from work in process to stores	205
T_f	The time any item of fixed capital investment is employed in or contributes toward the manufacture of a lot	147
T_M	The total time required to set up and dismantle the equipment required for the manufacture of a lot	220
T_n	Any designated instant of time (used with subscripts, group 10)	317
T_o	An instant of time used as a starting point	317
T_p	The total time required to process a lot exclusive of the machine changeover time	223
T_s	The duration of the sales period in which any lot can be sold	200
T_W	The total period over which articles are being withdrawn from stock to meet current orders	248
u_m	The unit-manufacturing cost	175
u'	All unit-cost items independent of the lot size	170
U	The ultimate unit cost (used with subscripts, groups 1 and 2)	164
U'	The maximum ultimate unit cost for any lot size which will permit the desired unit margin of profit (occasionally used with subscripts, group 1)	151
U_c	The ultimate unit cost derived from the practical method of solution	167
U_e	The ultimate unit cost for an economic-production quantity	152
U_f	The unit-factory cost of the final product	139
U_i	The ultimate unit cost derived from the exact method of solution	167
U_m	The minimum ultimate unit cost	172
U_R	The ultimate unit cost for the point of maximum return	269
U_t	The total ultimate unit cost for the finished product	159
U_u	The ultimate unit cost for any uneconomically produced lot size	135
v	The value of a unit of production at the moment it is complete for any phase of its manufacture or storage (used with subscripts, group 3)	147
V	The total storage space charge per lot (used with subscripts, group 3)	240

2. GREEK ALPHABET

$\left.\begin{array}{l}\alpha\\ \beta\\ \gamma\\ \epsilon\end{array}\right\}$	Constants in any general equation	
Δ	The coefficient of deterioration	254
δ	The daily rate of deterioration	256
θ	The coefficient of obsolescence	257
λ	The loss factor	300

Page

ρ The allowable variation in the expected rate of return below normal 308

ϕ The plant constant 291

3. MISCELLANEOUS SYMBOLS

∞ Infinity (expressed mathematically as $1/0$)

$\sum_{o}^{n}$ Summation of any series of n terms

$\int_{o}^{n}$ Integral of any function between the limits of o and n

$'$ prime, $''$ second, $'''$ third — Accents are used to designate a specific interpretation or value of any symbol

a, b, c, d, etc., when employed in a diagram, are reference points

4. SUBSCRIPTS

GROUP 1

(Used with symbols Q, R_o, and U)

e Economic (referring to the lower limit of the economic range of production) 152

m Minimum (referring to the upper limit of the economic range of production) 152

R Designates terms related to the point of maximum return 269

u Uneconomic (may designate any term not related to the minimum-cost quantity) 135

GROUP 2

(Used with symbols A, C, I, m, p', Q, U, and U)

a Refers to the assembly phase of manufacture 144

p Refers to the production or fabricating phase of manufacture . . 144

r Refers to raw materials or the purchasing phase 144

GROUP 3

(Used with symbols C, I, k_v, K, Q, R, s, t, T, v, and V)

s Designates terms referring to the storage of articles or inventories 168

w Designates terms referring to work in process 168

v Designates terms referring to the storage space charges 168

f Designates terms referring to fixed capital, investment in land, buildings, machines, and equipment 147

GROUP 4

(Used with symbols I, K, Q, and R)

sw Designates terms referring to both the storage of articles and work in process 285

Page

sv Designates terms referring to both the storage of articles and the storage-space charges 285

wv Designates terms referring to both work in process and the storage-space charges 285

GROUP 5

(Used with symbols D, Q_v, and t separately and sometimes in combinations to indicate a transition between types of processes)

B Denotes batch production 230

C Denotes semicontinuous production 229

N Denotes non-continuous production 229

GROUP 6

(Used with symbols n and t)

c Denotes a concluding operation to a process 234

F Denotes a final operation preceding a transition from one type of process to another 233

i Denotes an initial operation for a process 234

L Denotes the last operation in a series where batch production exists prior to a transition 231

m Denotes the movement of material or work in process 233

S Denotes the first operation or the start of a process of a different type following a transition 233

GROUP 7

(Used with symbol d)

l Designates terms referring to the application of direct labor 178

M Designates terms referring to machine changeover 185

o Designates terms referring to the distribution of overhead 178

p Designates terms referring to piece rates for direct labor 178

u Designates terms referring to the distribution of overhead on the basis of the unit-manufacturing cost u_m 178

GROUP 8

(Used with symbols N, q, S, and T)

ob Designates terms referring to obsolescence 258

GROUP 9

(Used with symbols f, k, n, and q)

b Denotes terms referring to bins for the storage of articles 244

GROUP 10

(Used with symbols A_n and t or in place of the subscript n)

n Designates the nth term in a series or specific item by the numerical order or position it holds in a group or series 230

1, 2, 3, etc., designate the number or the definite position or identity of a term in a group or series 230

APPENDIX XIV

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